Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/1.1.1.6-P-x-a+b-x-^m-c+d-x-^n-e+f-x-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [60]. This is test number [5].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (60)	0.00 (0)
Mathematica	100.00 (60)	0.00 (0)
Maple	100.00 (60)	0.00 (0)
IntegrateAlgebraic	96.67 (58)	3.33 (2)
Fricas	73.33 (44)	26.67 (16)
Mupad	66.67 (40)	33.33 (20)
Giac	55.00 (33)	45.00 (27)
Maxima	45.00 (27)	55.00 (33)
Sympy	23.33 (14)	% 76.67 (46)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

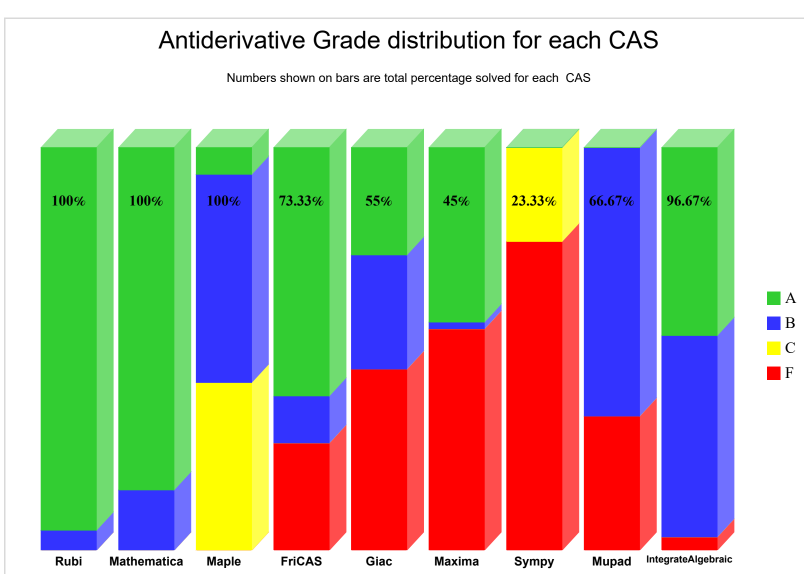
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

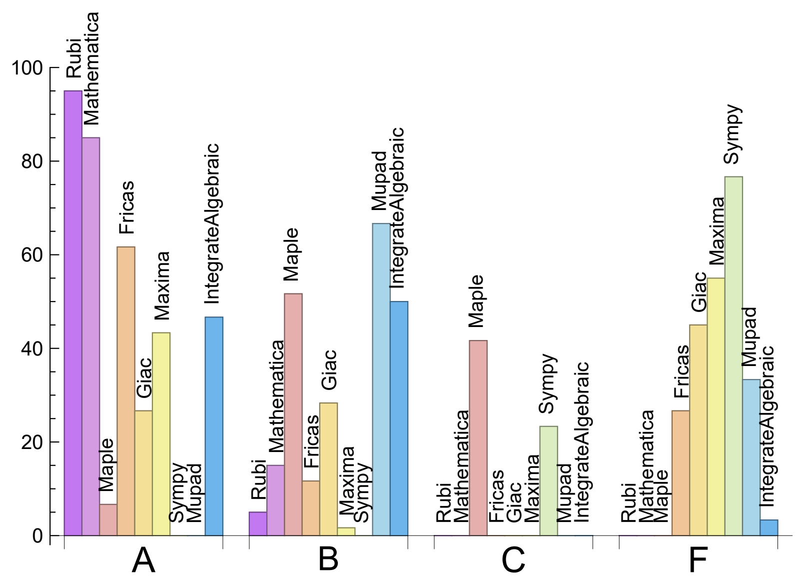
System	% A grade	% B grade	% C grade	% F grade
Rubi	95.00	5.00	0.00	0.00
Mathematica	85.00	15.00	0.00	0.00
Fricas	61.67	11.67	0.00	26.67
IntegrateAlgebraic	46.67	50.00	0.00	3.33
Maxima	43.33	1.67	0.00	55.00
Giac	26.67	28.33	0.00	45.00
Maple	6.67	51.67	41.67	0.00
Mupad	N/A	66.67	0.00	33.33
Sympy	0.00	0.00	23.33	76.67

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	16	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	2	0.00 %	100.00 %	0.00 %
Giac	27	0.00 %	55.56 %	44.44 %
Maxima	33	0.00 %	0.00 %	100.00 %
Sympy	46	19.57 %	80.43 %	0.00 %
Mupad	20	0.00 %	100.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

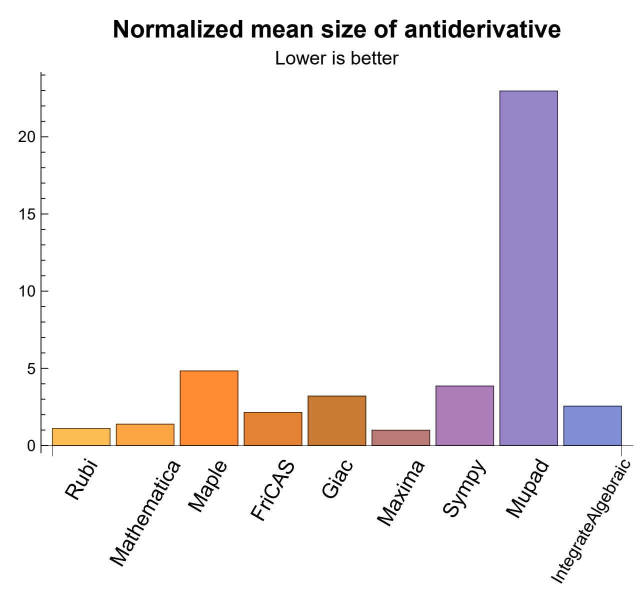
1.3 Performance

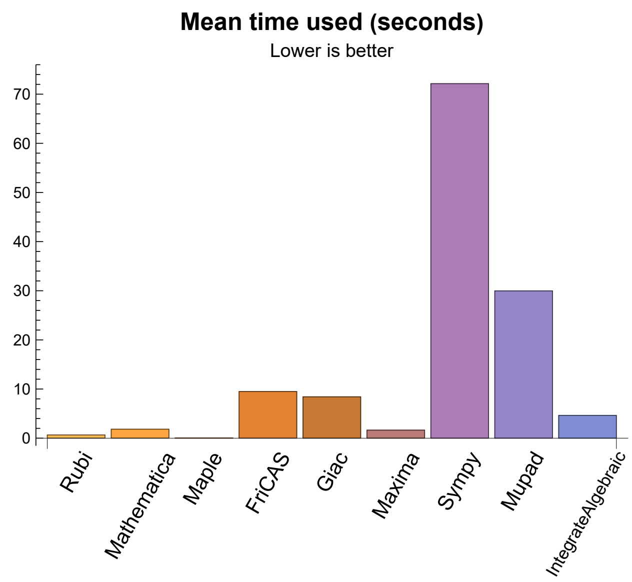
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.66	326.23	1.11	266.00	1.00
Mathematica	1.83	546.15	1.38	273.00	1.01
Maple	0.03	2134.73	4.83	831.00	2.82
Maxima	1.64	187.07	0.99	100.00	1.01
Fricas	9.49	618.57	2.14	393.00	1.48
Sympy	72.14	284.86	3.85	261.00	4.16
Giac	8.41	1314.55	3.20	605.00	1.70
Mupad	29.97	5830.02	22.96	1748.50	6.53
IntegrateAlgebraic	4.64	1000.71	2.55	413.50	2.07

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





Mean time used (seconds)
Lower is better
70
60
50
40
30
20
10
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {34, 35, 36, 37, 40, 45, 46}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

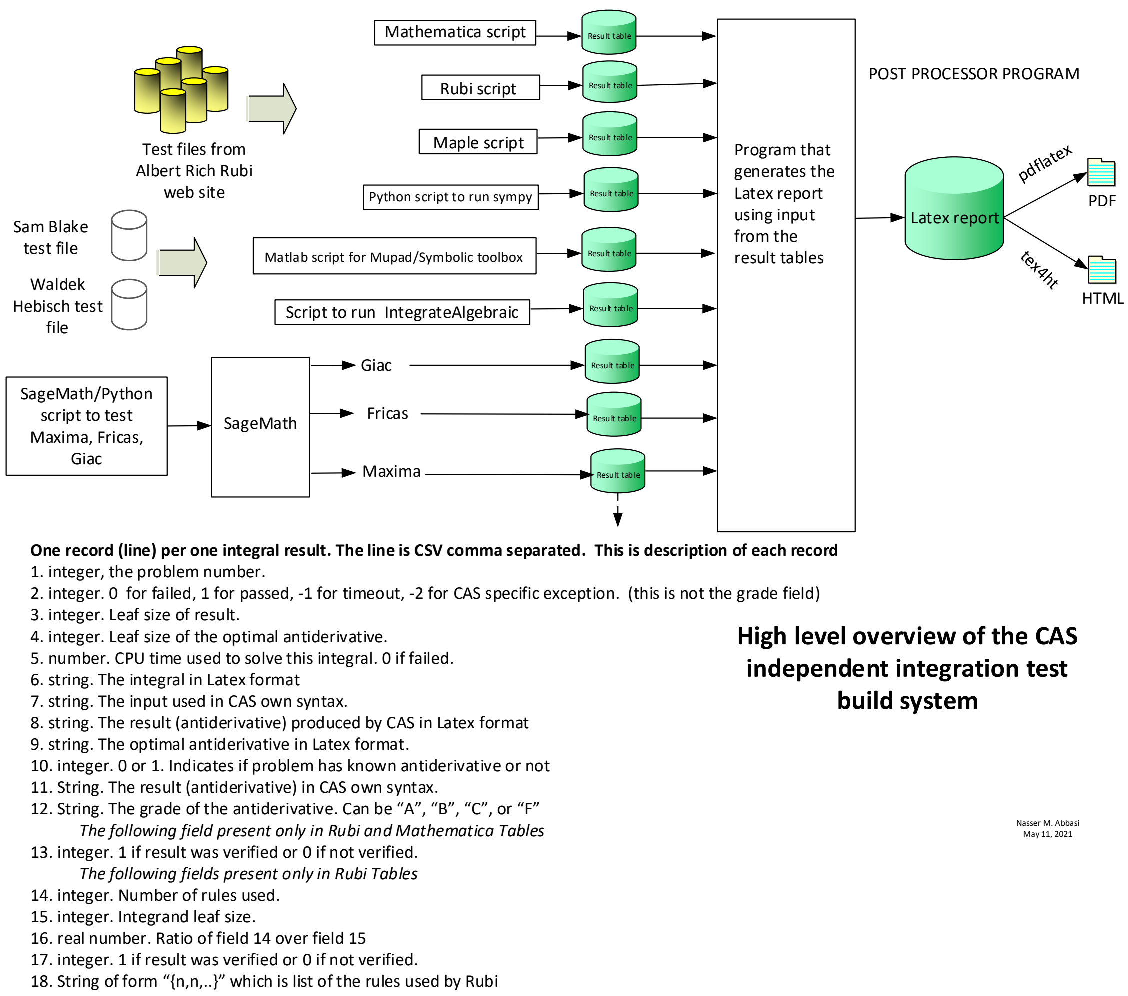
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60 }

B grade: { 35, 36, 37 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 43, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60 }

B grade: { 35, 36, 41, 42, 44, 45, 46, 47, 54 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 23, 28, 29, 30 }

B grade: { 20, 21, 22, 24, 25, 26, 27, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60 }

C grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 34, 35, 36, 37, 38, 39 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 27, 28, 29, 30, 34, 36, 37, 38, 39 }

B grade: { 35 }

C grade: { }

F grade: { 5, 6, 7, 12, 13, 14, 24, 25, 26, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 34, 35, 36, 37, 38, 39, 41, 42, 43, 47, 48, 49, 54, 55, 56 }

B grade: { 5, 6, 7, 12, 13, 14, 40 }

C grade: { }

F grade: { 24, 25, 31, 32, 33, 44, 45, 46, 50, 51, 52, 53, 57, 58, 59, 60 }

2.1.6 Sympy

A grade: { }

B grade: { }

C grade: { 10, 11, 15, 16, 17, 18, 19, 30, 34, 35, 36, 37, 38, 39 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60 }

2.1.7 Giac

A grade: { 8, 9, 10, 11, 15, 16, 34, 35, 36, 37, 47, 48, 49, 54, 55, 56 }

B grade: { 1, 2, 3, 4, 26, 33, 38, 39, 40, 41, 42, 43, 45, 46, 51, 52, 58 }

C grade: { }

F grade: { 5, 6, 7, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 44, 50, 53, 57, 59, 60 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 49, 55, 56 }

C grade: { }

F grade: { 20, 21, 25, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 57, 58, 59, 60 }

2.1.9 IntegrateAlgebraic

A grade: { 5, 6, 11, 12, 13, 16, 17, 18, 19, 23, 24, 25, 26, 29, 30, 31, 32, 33, 36, 37, 38, 39, 43, 45, 49, 56, 57, 58 }

B grade: { 1, 2, 3, 4, 7, 8, 9, 10, 14, 15, 20, 21, 22, 27, 28, 34, 35, 40, 41, 42, 44, 46, 47, 48, 50, 51, 54, 55, 59, 60 }

C grade: { }

F grade: { 52, 53 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	415	415	355	959	444	406	0	1948	3993	1590
N.S.	1	1.00	0.86	2.31	1.07	0.98	0.00	4.69	9.62	3.83
time (sec)	N/A	0.673	0.544	0.035	1.002	0.721	0.000	3.113	47.789	1.063

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	286	286	244	652	307	279	0	1327	2920	1079
N.S.	1	1.00	0.85	2.28	1.07	0.98	0.00	4.64	10.21	3.77
time (sec)	N/A	0.563	0.349	0.018	1.006	0.864	0.000	2.581	36.028	0.713

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	168	170	141	377	174	170	0	782	736	470
N.S.	1	1.01	0.84	2.24	1.04	1.01	0.00	4.65	4.38	2.80
time (sec)	N/A	0.250	0.212	0.013	1.070	0.928	0.000	1.996	12.065	0.392

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	71	185	93	95	0	336	361	242
N.S.	1	1.00	0.75	1.95	0.98	1.00	0.00	3.54	3.80	2.55
time (sec)	N/A	0.073	0.064	0.012	0.981	0.918	0.000	1.536	7.209	0.193

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	B	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	117	373	0	493	0	0	5803	177
N.S.	1	1.00	0.96	3.06	0.00	4.04	0.00	0.00	47.57	1.45
time (sec)	N/A	0.311	0.150	0.049	0.000	15.664	0.000	0.000	25.801	0.582

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	B	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	211	899	0	1025	0	0	10198	235
N.S.	1	1.00	1.29	5.52	0.00	6.29	0.00	0.00	62.56	1.44
time (sec)	N/A	0.331	0.473	0.039	0.000	72.527	0.000	0.000	52.173	1.488

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	273	1449	0	1580	0	0	9097	533
N.S.	1	1.00	1.10	5.84	0.00	6.37	0.00	0.00	36.68	2.15
time (sec)	N/A	0.355	0.416	0.050	0.000	1.238	0.000	0.000	59.182	2.349

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	340	340	241	643	355	286	0	427	2606	1135
N.S.	1	1.00	0.71	1.89	1.04	0.84	0.00	1.26	7.66	3.34
time (sec)	N/A	0.633	0.392	0.029	1.045	1.229	0.000	1.819	35.295	0.771

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	228	228	160	423	231	192	0	277	1732	708
N.S.	1	1.00	0.70	1.86	1.01	0.84	0.00	1.21	7.60	3.11
time (sec)	N/A	0.493	0.221	0.028	1.273	0.824	0.000	1.643	33.636	0.473

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	133	88	235	131	114	617	146	492	275
N.S.	1	1.02	0.68	1.81	1.01	0.88	4.75	1.12	3.78	2.12
time (sec)	N/A	0.230	0.104	0.023	1.315	0.964	158.075	1.310	12.857	0.260

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	45	117	57	67	282	76	232	117
N.S.	1	1.00	0.71	1.86	0.90	1.06	4.48	1.21	3.68	1.86
time (sec)	N/A	0.061	0.036	0.017	1.416	1.451	49.744	1.286	7.525	0.139

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	B	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	117	373	0	493	0	0	5803	177
N.S.	1	1.00	0.96	3.06	0.00	4.04	0.00	0.00	47.57	1.45
time (sec)	N/A	0.283	0.131	0.000	0.000	19.359	0.000	0.000	0.005	0.002

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	B	F(-1)	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	211	899	0	1025	0	0	10198	235
N.S.	1	1.00	1.29	5.52	0.00	6.29	0.00	0.00	62.56	1.44
time (sec)	N/A	0.295	0.431	0.000	0.000	76.120	0.000	0.000	0.008	0.002

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F(-2)	B	F(-1)	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	273	1449	0	1580	0	0	9097	533
N.S.	1	1.00	1.10	5.84	0.00	6.37	0.00	0.00	36.68	2.15
time (sec)	N/A	0.329	0.384	0.000	0.000	0.848	0.000	0.000	0.007	0.002

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	57	139	87	78	313	101	244	179
N.S.	1	1.00	0.72	1.76	1.10	0.99	3.96	1.28	3.09	2.27
time (sec)	N/A	0.139	0.068	0.000	1.270	1.139	82.521	1.305	7.606	0.002

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	45	117	57	67	282	76	232	117
N.S.	1	1.00	0.71	1.86	0.90	1.06	4.48	1.21	3.68	1.86
time (sec)	N/A	0.061	0.034	0.000	1.281	0.975	49.685	1.324	7.411	0.001

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	96	57	81	245	0	122	95
N.S.	1	1.00	1.00	2.00	1.19	1.69	5.10	0.00	2.54	1.98
time (sec)	N/A	0.183	0.055	0.001	1.275	0.998	55.715	0.000	4.331	0.002

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	97	57	84	221	0	114	93
N.S.	1	1.00	1.00	2.02	1.19	1.75	4.60	0.00	2.38	1.94
time (sec)	N/A	0.176	0.061	0.000	1.325	1.314	50.054	0.000	4.266	0.002

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	56	108	98	65	218	0	312	112
N.S.	1	1.00	0.79	1.52	1.38	0.92	3.07	0.00	4.39	1.58
time (sec)	N/A	0.184	0.049	0.000	1.285	0.882	80.629	0.000	6.304	0.002

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	591	584	427	1446	584	1001	0	0	-1	2590
N.S.	1	0.99	0.72	2.45	0.99	1.69	0.00	0.00	-0.00	4.38
time (sec)	N/A	1.517	1.463	0.043	1.461	0.888	0.000	0.000	0.000	2.002

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	451	450	311	987	417	703	0	0	-1	1792
N.S.	1	1.00	0.69	2.19	0.92	1.56	0.00	0.00	-0.00	3.97
time (sec)	N/A	1.010	1.016	0.018	2.067	0.983	0.000	0.000	0.000	1.289

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	300	297	200	588	248	441	0	0	1765	647
N.S.	1	0.99	0.67	1.96	0.83	1.47	0.00	0.00	5.88	2.16
time (sec)	N/A	0.446	0.682	0.014	2.255	1.199	0.000	0.000	30.577	0.640

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	221	221	142	287	140	265	0	0	876	326
N.S.	1	1.00	0.64	1.30	0.63	1.20	0.00	0.00	3.96	1.48
time (sec)	N/A	0.147	0.408	0.013	2.028	0.845	0.000	0.000	16.517	0.410

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	278	278	225	503	0	0	0	0	9298	205
N.S.	1	1.00	0.81	1.81	0.00	0.00	0.00	0.00	33.45	0.74
time (sec)	N/A	0.490	0.768	0.069	0.000	0.000	0.000	0.000	44.562	0.369

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	322	322	309	1200	0	0	0	0	-1	282
N.S.	1	1.00	0.96	3.73	0.00	0.00	0.00	0.00	-0.00	0.88
time (sec)	N/A	0.579	0.852	0.044	0.000	0.000	0.000	0.000	0.000	1.119

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	363	361	492	1848	0	1355	0	1658	9344	610
N.S.	1	0.99	1.36	5.09	0.00	3.73	0.00	4.57	25.74	1.68
time (sec)	N/A	0.677	1.795	0.057	0.000	163.672	0.000	7.021	86.666	1.473

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	501	496	727	965	471	700	0	0	4167	1909
N.S.	1	0.99	1.45	1.93	0.94	1.40	0.00	0.00	8.32	3.81
time (sec)	N/A	1.281	4.902	0.031	1.972	0.777	0.000	0.000	161.428	1.260

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	368	369	555	635	317	482	0	0	2799	1213
N.S.	1	1.00	1.51	1.73	0.86	1.31	0.00	0.00	7.61	3.30
time (sec)	N/A	0.875	2.684	0.029	2.021	1.220	0.000	0.000	81.648	0.816

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	249	390	365	189	302	0	0	1011	356
N.S.	1	1.01	1.59	1.48	0.77	1.23	0.00	0.00	4.11	1.45
time (sec)	N/A	0.400	1.429	0.026	2.055	0.711	0.000	0.000	30.743	0.408

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	169	180	88	196	338	0	489	150
N.S.	1	1.00	0.95	1.02	0.50	1.11	1.91	0.00	2.76	0.85
time (sec)	N/A	0.124	0.437	0.020	2.501	0.772	56.834	0.000	14.952	0.232

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	278	278	225	503	0	0	0	0	9298	205
N.S.	1	1.00	0.81	1.81	0.00	0.00	0.00	0.00	33.45	0.74
time (sec)	N/A	0.464	0.711	0.000	0.000	0.000	0.000	0.000	0.008	0.003

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	322	322	309	1200	0	0	0	0	106511	282
N.S.	1	1.00	0.96	3.73	0.00	0.00	0.00	0.00	330.78	0.88
time (sec)	N/A	0.530	0.794	0.000	0.000	0.000	0.000	0.000	19.397	0.003

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	363	361	492	1848	0	0	0	1658	9344	610
N.S.	1	0.99	1.36	5.09	0.00	0.00	0.00	4.57	25.74	1.68
time (sec)	N/A	0.588	1.310	0.000	0.000	0.000	0.000	9.490	0.008	0.003

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	87	151	149	137	100	73	308	105	318	230
N.S.	1	1.74	1.71	1.57	1.15	0.84	3.54	1.21	3.66	2.64
time (sec)	N/A	0.146	0.358	0.000	1.020	1.286	80.462	1.457	14.762	0.002

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	C	B	A	C	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	52	135	126	120	90	61	277	80	312	112
N.S.	1	2.60	2.42	2.31	1.73	1.17	5.33	1.54	6.00	2.15
time (sec)	N/A	0.071	0.222	0.000	1.107	1.080	48.757	1.386	14.587	0.001

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	C	A	A	C	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	55	135	128	95	56	73	240	71	118	91
N.S.	1	2.45	2.33	1.73	1.02	1.33	4.36	1.29	2.15	1.65
time (sec)	N/A	0.185	0.421	0.000	2.343	0.627	47.371	1.366	5.391	0.001

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	C	A	A	C	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	55	135	89	96	56	82	216	83	118	89
N.S.	1	2.45	1.62	1.75	1.02	1.49	3.93	1.51	2.15	1.62
time (sec)	N/A	0.180	0.182	0.001	2.348	1.027	45.808	1.517	5.151	0.002

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	129	82	103	61	69	212	145	316	107
N.S.	1	1.55	0.99	1.24	0.73	0.83	2.55	1.75	3.81	1.29
time (sec)	N/A	0.191	0.125	0.000	2.468	1.114	75.514	1.442	12.773	0.002

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	171	94	123	86	90	219	197	304	168
N.S.	1	1.47	0.81	1.06	0.74	0.78	1.89	1.70	2.62	1.45
time (sec)	N/A	0.217	0.124	0.000	3.046	1.073	128.739	1.403	11.819	0.002

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	199	242	343	1095	0	1186	0	605	7235	546
N.S.	1	1.22	1.72	5.50	0.00	5.96	0.00	3.04	36.36	2.74
time (sec)	N/A	0.328	0.761	0.053	0.000	0.999	0.000	3.245	66.847	0.669

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F(-2)	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1348	1345	3599	6728	0	3096	0	4708	-1	9831
N.S.	1	1.00	2.67	4.99	0.00	2.30	0.00	3.49	-0.00	7.29
time (sec)	N/A	2.366	7.131	0.053	0.000	8.081	0.000	6.328	0.000	5.303

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F(-2)	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	721	719	2722	3571	0	1620	0	2643	-1	4538
N.S.	1	1.00	3.78	4.95	0.00	2.25	0.00	3.67	-0.00	6.29
time (sec)	N/A	0.963	6.606	0.024	0.000	2.958	0.000	3.389	0.000	2.794

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	330	330	306	1431	0	840	0	1103	-1	643
N.S.	1	1.00	0.93	4.34	0.00	2.55	0.00	3.34	-0.00	1.95
time (sec)	N/A	0.298	1.716	0.020	0.000	0.974	0.000	2.334	0.000	0.977

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F(-2)	F(-1)	F	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	450	453	1936	4227	0	0	0	0	-1	950
N.S.	1	1.01	4.30	9.39	0.00	0.00	0.00	0.00	-0.00	2.11
time (sec)	N/A	1.369	6.215	0.051	0.000	0.000	0.000	0.000	0.000	1.795

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	521	521	2532	5051	0	0	0	1585	-1	942
N.S.	1	1.00	4.86	9.69	0.00	0.00	0.00	3.04	-0.00	1.81
time (sec)	N/A	1.696	6.375	0.048	0.000	0.000	0.000	13.122	0.000	2.758

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F(-2)	F(-1)	F(-1)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	658	657	2150	12065	0	0	0	8347	-1	1687
N.S.	1	1.00	3.27	18.34	0.00	0.00	0.00	12.69	-0.00	2.56
time (sec)	N/A	2.680	6.443	0.072	0.000	0.000	0.000	39.569	0.000	6.129

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F(-2)	A	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1032	1032	3220	3958	0	2176	0	1505	-1	2260
N.S.	1	1.00	3.12	3.84	0.00	2.11	0.00	1.46	-0.00	2.19
time (sec)	N/A	1.788	6.702	0.046	0.000	13.801	0.000	2.759	0.000	9.373

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	540	540	478	2002	0	1114	0	736	-1	1096
N.S.	1	1.00	0.89	3.71	0.00	2.06	0.00	1.36	-0.00	2.03
time (sec)	N/A	0.713	3.539	0.030	0.000	3.758	0.000	1.819	0.000	3.390

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	246	246	225	763	0	576	0	315	1832	357
N.S.	1	1.00	0.91	3.10	0.00	2.34	0.00	1.28	7.45	1.45
time (sec)	N/A	0.230	1.070	0.024	0.000	1.485	0.000	1.348	90.550	1.123

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	290	290	465	1822	0	0	0	0	-1	1375
N.S.	1	1.00	1.60	6.28	0.00	0.00	0.00	0.00	-0.00	4.74
time (sec)	N/A	0.672	3.449	0.039	0.000	0.000	0.000	0.000	0.000	34.326

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	364	364	417	3670	0	0	0	1388	-1	5591
N.S.	1	1.00	1.15	10.08	0.00	0.00	0.00	3.81	-0.00	15.36
time (sec)	N/A	1.097	2.396	0.049	0.000	0.000	0.000	10.820	0.000	169.659

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	484	484	523	9100	0	0	0	8004	-1	0
N.S.	1	1.00	1.08	18.80	0.00	0.00	0.00	16.54	-0.00	0.00
time (sec)	N/A	1.563	5.675	0.095	0.000	0.000	0.000	134.872	0.000	180.012

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	685	685	729	15990	0	0	0	0	-1	0
N.S.	1	1.00	1.06	23.34	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	1.778	6.336	0.159	0.000	0.000	0.000	0.000	0.000	180.006

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F(-2)	A	F(-1)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	718	715	2195	2528	0	1436	0	951	-1	2158
N.S.	1	1.00	3.06	3.52	0.00	2.00	0.00	1.32	-0.00	3.01
time (sec)	N/A	1.336	6.487	0.046	0.000	5.323	0.000	2.512	0.000	1.813

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	371	369	379	1199	0	720	0	447	2621	787
N.S.	1	0.99	1.02	3.23	0.00	1.94	0.00	1.20	7.06	2.12
time (sec)	N/A	0.509	1.963	0.033	0.000	2.266	0.000	1.968	105.189	0.900

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	173	425	0	380	0	194	833	229
N.S.	1	1.00	1.05	2.59	0.00	2.32	0.00	1.18	5.08	1.40
time (sec)	N/A	0.149	0.792	0.023	0.000	1.569	0.000	1.216	25.888	0.378

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	188	304	746	0	0	0	0	-1	227
N.S.	1	1.00	1.62	3.97	0.00	0.00	0.00	0.00	-0.01	1.21
time (sec)	N/A	0.341	0.938	0.034	0.000	0.000	0.000	0.000	0.000	0.616

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	254	254	325	2973	0	0	0	1356	-1	330
N.S.	1	1.00	1.28	11.70	0.00	0.00	0.00	5.34	-0.00	1.30
time (sec)	N/A	0.638	1.863	0.059	0.000	0.000	0.000	9.374	0.000	0.941

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	424	424	512	7119	0	0	0	0	-1	911
N.S.	1	1.00	1.21	16.79	0.00	0.00	0.00	0.00	-0.00	2.15
time (sec)	N/A	0.967	2.090	0.130	0.000	0.000	0.000	0.000	0.000	1.861

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	826	826	794	18802	0	0	0	0	-1	3507
N.S.	1	1.00	0.96	22.76	0.00	0.00	0.00	0.00	-0.00	4.25
time (sec)	N/A	2.433	6.107	0.312	0.000	0.000	0.000	0.000	0.000	5.663

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [36] had the largest ratio of [.2500]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.00	37	0.162
2	A	6	6	1.00	37	0.162
3	A	5	5	1.01	35	0.143
4	A	5	5	1.00	30	0.167
5	A	6	6	1.00	37	0.162
6	A	6	6	1.00	37	0.162
7	A	5	5	1.00	37	0.135
8	A	6	5	1.00	37	0.135
9	A	5	5	1.00	37	0.135
10	A	4	4	1.02	35	0.114
11	A	4	4	1.00	30	0.133
12	A	6	6	1.00	37	0.162
13	A	6	6	1.00	37	0.162
14	A	5	5	1.00	37	0.135
15	A	4	4	1.00	31	0.129
16	A	4	4	1.00	30	0.133
17	A	7	7	1.00	33	0.212
18	A	7	7	1.00	33	0.212
19	A	6	6	1.00	33	0.182
20	A	8	7	0.99	40	0.175
21	A	7	7	1.00	40	0.175
22	A	6	6	0.99	38	0.158
23	A	6	6	1.00	33	0.182
24	A	7	7	1.00	40	0.175
25	A	7	7	1.00	40	0.175
26	A	5	5	0.99	40	0.125
27	A	7	6	0.99	40	0.150
28	A	6	6	1.00	40	0.150
29	A	5	5	1.01	38	0.132
30	A	5	5	1.00	33	0.152
31	A	7	7	1.00	40	0.175
32	A	7	7	1.00	40	0.175
33	A	5	5	0.99	40	0.125
34	A	5	5	1.74	30	0.167
35	B	5	5	2.60	29	0.172
36	B	8	8	2.45	32	0.250
37	B	8	8	2.45	32	0.250
38	A	6	6	1.55	32	0.188
39	A	7	7	1.47	32	0.219

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	5	5	1.22	32	0.156
41	A	8	7	1.00	36	0.194
42	A	7	6	1.00	34	0.176
43	A	7	6	1.00	29	0.207
44	A	9	8	1.01	36	0.222
45	A	9	8	1.00	36	0.222
46	A	9	9	1.00	36	0.250
47	A	7	7	1.00	36	0.194
48	A	6	6	1.00	34	0.176
49	A	6	6	1.00	29	0.207
50	A	8	8	1.00	36	0.222
51	A	8	8	1.00	36	0.222
52	A	8	8	1.00	36	0.222
53	A	6	6	1.00	36	0.167
54	A	6	6	1.00	36	0.167
55	A	5	5	0.99	34	0.147
56	A	5	5	1.00	29	0.172
57	A	7	7	1.00	36	0.194
58	A	7	7	1.00	36	0.194
59	A	5	5	1.00	36	0.139
60	A	6	5	1.00	36	0.139

Chapter 3

Listing of integrals

Local contents

3.1 $\int \sqrt{1-dx}\sqrt{1+dx}(e+fx)^3\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=415

$$-\frac{\left(1-d^2x^2\right)^{3/2}(e+fx)^2\left(7d^2f(2Af+Be)-C\left(3d^2e^2-8f^2\right)\right)}{70d^4f}+\frac{x\sqrt{1-d^2x^2}\left(8Ad^4e^3+6Ad^2ef^2+6Bd^2e^2f+Bf^3\right.}{16d^4}$$

Rubi [A] time = 0.67, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1609, 1654, 833, 780, 195, 216}

$$-\frac{\left(1-d^2x^2\right)^{3/2}(e+fx)^2\left(7d^2f(2Af+Be)-C\left(3d^2e^2-8f^2\right)\right)}{70d^4f}+\frac{\left(1-d^2x^2\right)^{3/2}\left(3d^2fx\left(-98Ad^2ef^2-14Bd^2e^2f-35Bf^3+6Cd^2e^3-41Cef^2\right)+8\left(C\left(-30d^2e^2f^2+3d^4e^4-8f^4\right)-7d^2f\left(2Af\left(6d^2e^2+f^2\right)+B\left(d^2e^3+6ef^2\right)\right)\right)\right)}{840d^6f}+\frac{x\sqrt{1-d^2x^2}\left(8Ad^4e^3+6Ad^2ef^2+6Bd^2e^2f+Bf^3+2Cd^2e^3+3Cef^2\right)}{16d^4}+\frac{\sin^{-1}(dx)\left(8Ad^4e^3+6Ad^2ef^2+6Bd^2e^2f+Bf^3+2Cd^2e^3+3Cef^2\right)}{16d^5}+\frac{\left(1-d^2x^2\right)^{3/2}(e+fx)^3(3Ce-7Bf)}{42d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^4}{7d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] ((2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f + 3*C*e*f^2 + 6*A*d^2*e*f^2 + B
*f^3)*x*Sqrt[1 - d^2*x^2])/(16*d^4) - ((7*d^2*f*(B*e + 2*A*f) - C*(3*d^2*e^
2 - 8*f^2))*(e + f*x)^2*(1 - d^2*x^2)^(3/2))/(70*d^4*f) + ((3*C*e - 7*B*f)*
(e + f*x)^3*(1 - d^2*x^2)^(3/2))/(42*d^2*f) - (C*(e + f*x)^4*(1 - d^2*x^2)^
(3/2))/(7*d^2*f) + ((8*(C*(3*d^4*e^4 - 30*d^2*e^2*f^2 - 8*f^4) - 7*d^2*f*(2
*A*f*(6*d^2*e^2 + f^2) + B*(d^2*e^3 + 6*e*f^2))) + 3*d^2*f*(6*C*d^2*e^3 - 1
4*B*d^2*e^2*f - 41*C*e*f^2 - 98*A*d^2*e*f^2 - 35*B*f^3)*x)*(1 - d^2*x^2)^(3
/2))/(840*d^6*f) + ((2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f + 3*C*e*f^2
+ 6*A*d^2*e*f^2 + B*f^3)*ArcSin[d*x])/(16*d^5)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^3\left(A+Bx+Cx^2\right)dx = \int (e+fx)^3\left(A+Bx+Cx^2\right)\sqrt{1-d^2x^2}\,dx$$

$$= -\frac{C(e+fx)^4\left(1-d^2x^2\right)^{3/2}}{7d^2f} - \frac{\int (e+fx)^3\left(-\left(\left(4C+7Ad^2\right)\right.\right.}{}$$

$$= \frac{(3Ce-7Bf)(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{42d^2f} - \frac{C(e+fx)^4\left(1-d^2x\right.}{7d^2f}$$

$$= -\frac{\left(7d^2f(Be+2Af)-C\left(3d^2e^2-8f^2\right)\right)(e+fx)^2\left(1-d^2x^2\right.}{70d^4f}$$

$$= -\frac{\left(7d^2f(Be+2Af)-C\left(3d^2e^2-8f^2\right)\right)(e+fx)^2\left(1-d^2x^2\right.}{70d^4f}$$

$$= \frac{\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3\right.}{16d^4}$$

$$= \frac{\left(2Cd^2e^3+8Ad^4e^3+6Bd^2e^2f+3Cef^2+6Ad^2ef^2+Bf^3\right.}{16d^4}$$

Mathematica [A] time = 0.54, size = 355, normalized size = 0.86

$$\frac{105d\sin^{-1}(dx)\left(8Ad^4e^3+6Ad^2ef^2+6Bd^2e^2f+Bf^3+2Cd^2e^3+3Cef^2\right)+\sqrt{1-d^2x^2}\left(14Ad^2\left(6d^4x\left(10e^3+20e^2fx+15ef^2x^2+4f^3x^3\right)-d^2f\left(120e^2+45efx+8f^2x^2\right)-16f^3\right)+7B\left(4d^6x^2\left(20e^3+45e^2fx+36ef^2x^2+10f^3x^3\right)-2d^4\left(40e^3+45e^2fx+24ef^2x^2+5f^3x^3\right)-3d^2f^2(32e+5fx)\right)-C\left(-12d^6x^3\left(35e^3+84e^2fx+70ef^2x^2+20f^3x^3\right)+6d^4x\left(35e^3+56e^2fx+35ef^2x^2+8f^3x^3\right)+d^2f\left(672e^2+315efx+64f^2x^2\right)+128f^3\right)\right)}{1680d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[1 - d^2*x^2]*(14*A*d^2*(-16*f^3 - d^2*f*(120*e^2 + 45*e*f*x + 8*f^2*x
^2) + 6*d^4*x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)) + 7*B*(-3*d
^2*f^2*(32*e + 5*f*x) - 2*d^4*(40*e^3 + 45*e^2*f*x + 24*e*f^2*x^2 + 5*f^3*x
^3) + 4*d^6*x^2*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3*x^3)) - C*(128
*f^3 + d^2*f*(672*e^2 + 315*e*f*x + 64*f^2*x^2) + 6*d^4*x*(35*e^3 + 56*e^2*
f*x + 35*e*f^2*x^2 + 8*f^3*x^3) - 12*d^6*x^3*(35*e^3 + 84*e^2*f*x + 70*e*f^
2*x^2 + 20*f^3*x^3))) + 105*d*(2*C*d^2*e^3 + 8*A*d^4*e^3 + 6*B*d^2*e^2*f +
3*C*e*f^2 + 6*A*d^2*e*f^2 + B*f^3)*ArcSin[d*x])/(1680*d^6)
```

IntegrateAlgebraic [B] time = 1.06, size = 1590, normalized size = 3.83

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3*(A + B*x + C*x^2
),x]
```

```
[Out] -1/840*(Sqrt[1 - d*x]*(-210*C*d^3*e^3 - 840*A*d^5*e^3 - 630*B*d^3*e^2*f - 3
15*C*d*e*f^2 - 630*A*d^3*e*f^2 - 105*B*d*f^3 + (210*C*d^3*e^3*(1 - d*x)^6)/
(1 + d*x)^6 + (840*A*d^5*e^3*(1 - d*x)^6)/(1 + d*x)^6 + (630*B*d^3*e^2*f*(1
 - d*x)^6)/(1 + d*x)^6 + (315*C*d*e*f^2*(1 - d*x)^6)/(1 + d*x)^6 + (630*A*d
^3*e*f^2*(1 - d*x)^6)/(1 + d*x)^6 + (105*B*d*f^3*(1 - d*x)^6)/(1 + d*x)^6 -
 (840*C*d^3*e^3*(1 - d*x)^5)/(1 + d*x)^5 + (2240*B*d^4*e^3*(1 - d*x)^5)/(1
+ d*x)^5 + (3360*A*d^5*e^3*(1 - d*x)^5)/(1 + d*x)^5 + (6720*C*d^2*e^2*f*(1
- d*x)^5)/(1 + d*x)^5 - (2520*B*d^3*e^2*f*(1 - d*x)^5)/(1 + d*x)^5 + (6720*
A*d^4*e^2*f*(1 - d*x)^5)/(1 + d*x)^5 - (4620*C*d*e*f^2*(1 - d*x)^5)/(1 + d*
x)^5 + (6720*B*d^2*e*f^2*(1 - d*x)^5)/(1 + d*x)^5 - (2520*A*d^3*e*f^2*(1 -
d*x)^5)/(1 + d*x)^5 + (2240*C*f^3*(1 - d*x)^5)/(1 + d*x)^5 - (1540*B*d*f^3*
(1 - d*x)^5)/(1 + d*x)^5 + (2240*A*d^2*f^3*(1 - d*x)^5)/(1 + d*x)^5 - (2310
*C*d^3*e^3*(1 - d*x)^4)/(1 + d*x)^4 + (8960*B*d^4*e^3*(1 - d*x)^4)/(1 + d*x
)^4 + (4200*A*d^5*e^3*(1 - d*x)^4)/(1 + d*x)^4 + (10752*C*d^2*e^2*f*(1 - d*
x)^4)/(1 + d*x)^4 - (6930*B*d^3*e^2*f*(1 - d*x)^4)/(1 + d*x)^4 + (26880*A*d
^4*e^2*f*(1 - d*x)^4)/(1 + d*x)^4 + (3255*C*d*e*f^2*(1 - d*x)^4)/(1 + d*x)^
4 + (10752*B*d^2*e*f^2*(1 - d*x)^4)/(1 + d*x)^4 - (6930*A*d^3*e*f^2*(1 - d*
x)^4)/(1 + d*x)^4 - (1792*C*f^3*(1 - d*x)^4)/(1 + d*x)^4 + (1085*B*d*f^3*(1
 - d*x)^4)/(1 + d*x)^4 + (3584*A*d^2*f^3*(1 - d*x)^4)/(1 + d*x)^4 + (13440*
B*d^4*e^3*(1 - d*x)^3)/(1 + d*x)^3 + (8064*C*d^2*e^2*f*(1 - d*x)^3)/(1 + d*
x)^3 + (40320*A*d^4*e^2*f*(1 - d*x)^3)/(1 + d*x)^3 + (8064*B*d^2*e*f^2*(1 -
 d*x)^3)/(1 + d*x)^3 + (7296*C*f^3*(1 - d*x)^3)/(1 + d*x)^3 + (2688*A*d^2*f
^3*(1 - d*x)^3)/(1 + d*x)^3 + (2310*C*d^3*e^3*(1 - d*x)^2)/(1 + d*x)^2 + (8
960*B*d^4*e^3*(1 - d*x)^2)/(1 + d*x)^2 - (4200*A*d^5*e^3*(1 - d*x)^2)/(1 +
d*x)^2 + (10752*C*d^2*e^2*f*(1 - d*x)^2)/(1 + d*x)^2 + (6930*B*d^3*e^2*f*(1
 - d*x)^2)/(1 + d*x)^2 + (26880*A*d^4*e^2*f*(1 - d*x)^2)/(1 + d*x)^2 - (325
5*C*d*e*f^2*(1 - d*x)^2)/(1 + d*x)^2 + (10752*B*d^2*e*f^2*(1 - d*x)^2)/(1 +
 d*x)^2 + (6930*A*d^3*e*f^2*(1 - d*x)^2)/(1 + d*x)^2 - (1792*C*f^3*(1 - d*x
)^2)/(1 + d*x)^2 - (1085*B*d*f^3*(1 - d*x)^2)/(1 + d*x)^2 + (3584*A*d^2*f^3
*(1 - d*x)^2)/(1 + d*x)^2 + (840*C*d^3*e^3*(1 - d*x))/(1 + d*x) + (2240*B*d
^4*e^3*(1 - d*x))/(1 + d*x) - (3360*A*d^5*e^3*(1 - d*x))/(1 + d*x) + (6720*
C*d^2*e^2*f*(1 - d*x))/(1 + d*x) + (2520*B*d^3*e^2*f*(1 - d*x))/(1 + d*x) +
 (6720*A*d^4*e^2*f*(1 - d*x))/(1 + d*x) + (4620*C*d*e*f^2*(1 - d*x))/(1 + d
*x) + (6720*B*d^2*e*f^2*(1 - d*x))/(1 + d*x) + (2520*A*d^3*e*f^2*(1 - d*x))
/(1 + d*x) + (2240*C*f^3*(1 - d*x))/(1 + d*x) + (1540*B*d*f^3*(1 - d*x))/(1
 + d*x) + (2240*A*d^2*f^3*(1 - d*x))/(1 + d*x)))/(d^6*Sqrt[1 + d*x]*(1 + (1
 - d*x)/(1 + d*x))^7) + ((-2*C*d^2*e^3 - 8*A*d^4*e^3 - 6*B*d^2*e^2*f - 3*C*
e*f^2 - 6*A*d^2*e*f^2 - B*f^3)*ArcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/(8*d^5)
```

fricas [A] time = 0.72, size = 406, normalized size = 0.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/1680*((240*C*d^6*f^3*x^6 - 560*B*d^4*e^3 - 672*B*d^2*e*f^2 + 280*(3*C*d^6
*e*f^2 + B*d^6*f^3)*x^5 + 48*(21*C*d^6*e^2*f + 21*B*d^6*e*f^2 + (7*A*d^6 -
C*d^4)*f^3)*x^4 - 336*(5*A*d^4 + 2*C*d^2)*e^2*f - 32*(7*A*d^2 + 4*C)*f^3 +
70*(6*C*d^6*e^3 + 18*B*d^6*e^2*f - B*d^4*f^3 + 3*(6*A*d^6 - C*d^4)*e*f^2)*x
```

```
^3 + 16*(35*B*d^6*e^3 - 21*B*d^4*e*f^2 + 21*(5*A*d^6 - C*d^4)*e^2*f - (7*A*
d^4 + 4*C*d^2)*f^3)*x^2 - 105*(6*B*d^4*e^2*f + B*d^2*f^3 - 2*(4*A*d^6 - C*d
^4)*e^3 + 3*(2*A*d^4 + C*d^2)*e*f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 210*
(6*B*d^3*e^2*f + B*d*f^3 + 2*(4*A*d^5 + C*d^3)*e^3 + 3*(2*A*d^3 + C*d)*e*f^
2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^6
```

giac [B] time = 3.11, size = 1948, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] 1/1680*(14*(((2*(d*x + 1)*(3*(d*x + 1)*(4*(d*x + 1)/d^4 - 21/d^4) + 133/d^4
) - 295/d^4)*(d*x + 1) + 195/d^4)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(
1/2*sqrt(2)*sqrt(d*x + 1))/d^4)*A*d*f^3 + 7*(((2*((d*x + 1)*(4*(d*x + 1)*(5
*(d*x + 1)/d^5 - 31/d^5) + 321/d^5) - 451/d^5)*(d*x + 1) + 745/d^5)*(d*x +
1) - 405/d^5)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 150*arcsin(1/2*sqrt(2)*sqrt(d*
x + 1))/d^5)*B*d*f^3 + (((2*((4*(d*x + 1)*(5*(d*x + 1)*(6*(d*x + 1)/d^6 - 4
3/d^6) + 661/d^6) - 4551/d^6)*(d*x + 1) + 4781/d^6)*(d*x + 1) - 6335/d^6)*(
d*x + 1) + 2835/d^6)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 1050*arcsin(1/2*sqrt(2)
*sqrt(d*x + 1))/d^6)*C*d*f^3 + 210*(((d*x + 1)*(2*(d*x + 1)*(3*(d*x + 1)/d^
3 - 13/d^3) + 43/d^3) - 39/d^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/
2*sqrt(2)*sqrt(d*x + 1))/d^3)*A*d*f^2*e + 42*(((2*(d*x + 1)*(3*(d*x + 1)*(4
*(d*x + 1)/d^4 - 21/d^4) + 133/d^4) - 295/d^4)*(d*x + 1) + 195/d^4)*sqrt(d*
x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^4)*B*d*f^2*e
 + 21*(((2*((d*x + 1)*(4*(d*x + 1)*(5*(d*x + 1)/d^5 - 31/d^5) + 321/d^5) -
451/d^5)*(d*x + 1) + 745/d^5)*(d*x + 1) - 405/d^5)*sqrt(d*x + 1)*sqrt(-d*x
+ 1) - 150*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^5)*C*d*f^2*e + 70*(((d*x + 1
)*(2*(d*x + 1)*(3*(d*x + 1)/d^3 - 13/d^3) + 43/d^3) - 39/d^3)*sqrt(d*x + 1)
*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^3)*A*f^3 + 14*(((2
*(d*x + 1)*(3*(d*x + 1)*(4*(d*x + 1)/d^4 - 21/d^4) + 133/d^4) - 295/d^4)*(d
*x + 1) + 195/d^4)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*sqr
t(d*x + 1))/d^4)*B*f^3 + 7*(((2*((d*x + 1)*(4*(d*x + 1)*(5*(d*x + 1)/d^5 -
31/d^5) + 321/d^5) - 451/d^5)*(d*x + 1) + 745/d^5)*(d*x + 1) - 405/d^5)*sqr
t(d*x + 1)*sqrt(-d*x + 1) - 150*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^5)*C*f^
3 + 840*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)/d^2 - 7/d^2)
+ 9/d^2) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*A*d*f*e^2 + 210*(((d*x
+ 1)*(2*(d*x + 1)*(3*(d*x + 1)/d^3 - 13/d^3) + 43/d^3) - 39/d^3)*sqrt(d*x +
 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^3)*B*d*f*e^2 +
42*(((2*(d*x + 1)*(3*(d*x + 1)*(4*(d*x + 1)/d^4 - 21/d^4) + 133/d^4) - 295/
d^4)*(d*x + 1) + 195/d^4)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt
(2)*sqrt(d*x + 1))/d^4)*C*d*f*e^2 + 840*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x
 + 1)*(2*(d*x + 1)/d^2 - 7/d^2) + 9/d^2) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x +
1))/d^2)*A*f^2*e + 210*(((d*x + 1)*(2*(d*x + 1)*(3*(d*x + 1)/d^3 - 13/d^3)
+ 43/d^3) - 39/d^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sq
rt(d*x + 1))/d^3)*B*f^2*e + 42*(((2*(d*x + 1)*(3*(d*x + 1)*(4*(d*x + 1)/d^4
 - 21/d^4) + 133/d^4) - 295/d^4)*(d*x + 1) + 195/d^4)*sqrt(d*x + 1)*sqrt(-d
*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^4)*C*f^2*e + 280*(sqrt(d*x
 + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)/d^2 - 7/d^2) + 9/d^2) + 6*arcs
in(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*B*d*e^3 + 70*(((d*x + 1)*(2*(d*x + 1)*(3
*(d*x + 1)/d^3 - 13/d^3) + 43/d^3) - 39/d^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) -
 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^3)*C*d*e^3 + 840*(sqrt(d*x + 1)*sqr
t(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)/d^2 - 7/d^2) + 9/d^2) + 6*arcsin(1/2*sq
rt(2)*sqrt(d*x + 1))/d^2)*B*f*e^2 + 210*(((d*x + 1)*(2*(d*x + 1)*(3*(d*x +
1)/d^3 - 13/d^3) + 43/d^3) - 39/d^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcs
in(1/2*sqrt(2)*sqrt(d*x + 1))/d^3)*C*f*e^2 + 840*(sqrt(d*x + 1)*(d*x - 2)*s
qrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*e^3 + 1680*(sqrt(d*x
 + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*e^3 + 280*(sq
```

```
rt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)/d^2 - 7/d^2) + 9/d^2) +
6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*C*e^3 + 2520*(sqrt(d*x + 1)*(d*x -
 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*f*e^2/d + 840*(
sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)
))*B*e^3/d)/d
```

maple [C] time = 0.04, size = 959, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x)
```

```
[Out] 1/1680*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(-128*C*csgn(d)*(-d^2*x^2+1)^(1/2)*f^3+
840*A*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*d^5*e^3+210*C*arctan(1/(-d^2
*x^2+1)^(1/2)*d*x*csgn(d))*d^3*e^3+105*B*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*cs
gn(d))*d*f^3-560*B*csgn(d)*(-d^2*x^2+1)^(1/2)*d^4*e^3-224*A*csgn(d)*(-d^2*x
^2+1)^(1/2)*d^2*f^3+630*A*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*d^3*e*f^
2+630*B*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*d^3*e^2*f+315*C*arctan(1/(
-d^2*x^2+1)^(1/2)*d*x*csgn(d))*d*e*f^2+336*A*csgn(d)*x^4*d^6*f^3*(-d^2*x^2+
1)^(1/2)+420*C*csgn(d)*x^3*d^6*e^3*(-d^2*x^2+1)^(1/2)+560*B*csgn(d)*x^2*d^6
*e^3*(-d^2*x^2+1)^(1/2)-48*C*csgn(d)*(-d^2*x^2+1)^(1/2)*x^4*d^4*f^3-70*B*cs
gn(d)*(-d^2*x^2+1)^(1/2)*x^3*d^4*f^3-112*A*csgn(d)*(-d^2*x^2+1)^(1/2)*x^2*d
^4*f^3-1680*A*csgn(d)*(-d^2*x^2+1)^(1/2)*d^4*e^2*f-64*C*csgn(d)*(-d^2*x^2+1
)^(1/2)*x^2*d^2*f^3+840*A*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^6*e^3-210*C*csgn(d
)*(-d^2*x^2+1)^(1/2)*x*d^4*e^3-105*B*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^2*f^3-6
72*B*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*e*f^2-672*C*csgn(d)*(-d^2*x^2+1)^(1/2)*
d^2*e^2*f+240*C*csgn(d)*x^6*d^6*f^3*(-d^2*x^2+1)^(1/2)+280*B*csgn(d)*x^5*d^
6*f^3*(-d^2*x^2+1)^(1/2)-630*A*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^4*e*f^2-630*B
*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^4*e^2*f-315*C*csgn(d)*(-d^2*x^2+1)^(1/2)*x*
d^2*e*f^2+840*C*csgn(d)*x^5*d^6*e*f^2*(-d^2*x^2+1)^(1/2)+1008*B*csgn(d)*x^4
*d^6*e*f^2*(-d^2*x^2+1)^(1/2)+1008*C*csgn(d)*x^4*d^6*e^2*f*(-d^2*x^2+1)^(1/
2)+1260*A*csgn(d)*x^3*d^6*e*f^2*(-d^2*x^2+1)^(1/2)+1260*B*csgn(d)*x^3*d^6*e
^2*f*(-d^2*x^2+1)^(1/2)+1680*A*csgn(d)*x^2*d^6*e^2*f*(-d^2*x^2+1)^(1/2)-210
*C*csgn(d)*(-d^2*x^2+1)^(1/2)*x^3*d^4*e*f^2-336*B*csgn(d)*(-d^2*x^2+1)^(1/2
)*x^2*d^4*e*f^2-336*C*csgn(d)*(-d^2*x^2+1)^(1/2)*x^2*d^4*e^2*f)*csgn(d)/d^6
/(-d^2*x^2+1)^(1/2)
```

maxima [A] time = 1.00, size = 444, normalized size = 1.07

$$-\frac{(-d^2x^2+1)^{\frac{3}{2}}Cf^3x^4}{7d^2}+\frac{1}{2}\sqrt{-d^2x^2+1}\,Ae^3x+\frac{Ae^3\arcsin(dx)}{2d}-\frac{(-d^2x^2+1)^{\frac{3}{2}}Be^3}{3d^2}-\frac{(-d^2x^2+1)^{\frac{3}{2}}Ae^2f}{d^2}-\frac{4(-d^2x^2+1)^{\frac{3}{2}}Cf^3x^2}{35d^4}-\frac{(3Cef^2+Bf^3)(-d^2x^2+1)^{\frac{3}{2}}x^3}{6d^2}-\frac{(3Ce^2f+3Bef^2+Af^3)(-d^2x^2+1)^{\frac{3}{2}}x^2}{5d^2}-\frac{(Ce^3+3Be^2f+3Aef^2)(-d^2x^2+1)^{\frac{3}{2}}x}{4d^2}+\frac{(Ce^3+3Be^2f+3Aef^2)\sqrt{-d^2x^2+1}\,x}{8d^2}-\frac{8(-d^2x^2+1)^{\frac{3}{2}}Cf^3}{105d^6}-\frac{(3Cef^2+Bf^3)(-d^2x^2+1)^{\frac{3}{2}}x}{8d^4}+\frac{(Ce^3+3Be^2f+3Aef^2)\arcsin(dx)}{8d^3}-\frac{2(3Ce^2f+3Bef^2+Af^3)(-d^2x^2+1)^{\frac{3}{2}}}{15d^4}+\frac{(3Cef^2+Bf^3)\sqrt{-d^2x^2+1}\,x}{16d^4}+\frac{(3Cef^2+Bf^3)\arcsin(dx)}{16d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/7*(-d^2*x^2 + 1)^(3/2)*C*f^3*x^4/d^2 + 1/2*sqrt(-d^2*x^2 + 1)*A*e^3*x +
1/2*A*e^3*arcsin(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e^3/d^2 - (-d^2*x^2 +
1)^(3/2)*A*e^2*f/d^2 - 4/35*(-d^2*x^2 + 1)^(3/2)*C*f^3*x^2/d^4 - 1/6*(3*C*e
*f^2 + B*f^3)*(-d^2*x^2 + 1)^(3/2)*x^3/d^2 - 1/5*(3*C*e^2*f + 3*B*e*f^2 + A
*f^3)*(-d^2*x^2 + 1)^(3/2)*x^2/d^2 - 1/4*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*(-
d^2*x^2 + 1)^(3/2)*x/d^2 + 1/8*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*sqrt(-d^2*x^
2 + 1)*x/d^2 - 8/105*(-d^2*x^2 + 1)^(3/2)*C*f^3/d^6 - 1/8*(3*C*e*f^2 + B*f^
3)*(-d^2*x^2 + 1)^(3/2)*x/d^4 + 1/8*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*arcsin(
d*x)/d^3 - 2/15*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*(-d^2*x^2 + 1)^(3/2)/d^4 +
1/16*(3*C*e*f^2 + B*f^3)*sqrt(-d^2*x^2 + 1)*x/d^4 + 1/16*(3*C*e*f^2 + B*f^3
)*arcsin(d*x)/d^5
```

mupad [B] time = 47.79, size = 3993, normalized size = 9.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] - ((((2048*C*f^3)/3 - 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^
(1/2) - 1)^6 + (((2048*C*f^3)/3 - 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^22
)/((d*x + 1)^(1/2) - 1)^22 - (((20480*C*f^3)/3 - 448*C*d^2*e^2*f)*((1 - d*x
)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 - (((20480*C*f^3)/3 - 448*C*d^2*e^2
*f)*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) - 1)^20 + (((458752*C*f^3)/1
5 + (27136*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^
10 + (((458752*C*f^3)/15 + (27136*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^18)
/((d*x + 1)^(1/2) - 1)^18 - (((1011712*C*f^3)/15 - (13184*C*d^2*e^2*f)/5)*(
(1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 - (((1011712*C*f^3)/15 -
(13184*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 +
 (((9293824*C*f^3)/105 - (15104*C*d^2*e^2*f)/5)*((1 - d*x)^(1/2) - 1)^14)/(
(d*x + 1)^(1/2) - 1)^14 + (((1 - d*x)^(1/2) - 1)^3*((29*C*d^3*e^3)/2 - (41*
C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^3 - (((1 - d*x)^(1/2) - 1)^25*((29*C*d
^3*e^3)/2 - (41*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^25 - (((1 - d*x)^(1/2)
 - 1)^5*(39*C*d^3*e^3 - (1099*C*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^5 + (((1
 - d*x)^(1/2) - 1)^23*(39*C*d^3*e^3 - (1099*C*d*e*f^2)/2))/((d*x + 1)^(1/2)
 - 1)^23 - (((1 - d*x)^(1/2) - 1)^7*(209*C*d^3*e^3 + (8755*C*d*e*f^2)/2))/(
(d*x + 1)^(1/2) - 1)^7 + (((1 - d*x)^(1/2) - 1)^21*(209*C*d^3*e^3 + (8755*C
*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^21 + (((1 - d*x)^(1/2) - 1)^11*((1767*C
*d^3*e^3)/2 - (8267*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^11 - (((1 - d*x)^(
1/2) - 1)^17*((1767*C*d^3*e^3)/2 - (8267*C*d*e*f^2)/4))/((d*x + 1)^(1/2) -
1)^17 + (((1 - d*x)^(1/2) - 1)^13*(646*C*d^3*e^3 - 17527*C*d*e*f^2))/((d*x
+ 1)^(1/2) - 1)^13 - (((1 - d*x)^(1/2) - 1)^15*(646*C*d^3*e^3 - 17527*C*d*e
*f^2))/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^9*((165*C*d^3*e^3)
/2 + (42095*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^9 - (((1 - d*x)^(1/2) - 1)
^19*((165*C*d^3*e^3)/2 + (42095*C*d*e*f^2)/4))/((d*x + 1)^(1/2) - 1)^19 - (
d*(2*C*d^2*e^3 + 3*C*e*f^2)*((1 - d*x)^(1/2) - 1))/(4*((d*x + 1)^(1/2) - 1)
) + (d*(2*C*d^2*e^3 + 3*C*e*f^2)*((1 - d*x)^(1/2) - 1)^27)/(4*((d*x + 1)^(1
/2) - 1)^27) + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) -
 1)^4 + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^24)/((d*x + 1)^(1/2) - 1)^24
)/(d^6 + (14*d^6*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (91*d^6
*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (364*d^6*((1 - d*x)^(1/
2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (1001*d^6*((1 - d*x)^(1/2) - 1)^8)/((d
*x + 1)^(1/2) - 1)^8 + (2002*d^6*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2)
 - 1)^10 + (3003*d^6*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (
3432*d^6*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (3003*d^6*((1
 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (2002*d^6*((1 - d*x)^(1/2
) - 1)^18)/((d*x + 1)^(1/2) - 1)^18 + (1001*d^6*((1 - d*x)^(1/2) - 1)^20)/(
(d*x + 1)^(1/2) - 1)^20 + (364*d^6*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/
2) - 1)^22 + (91*d^6*((1 - d*x)^(1/2) - 1)^24)/((d*x + 1)^(1/2) - 1)^24 + (
14*d^6*((1 - d*x)^(1/2) - 1)^26)/((d*x + 1)^(1/2) - 1)^26 + (d^6*((1 - d*x)
^(1/2) - 1)^28)/((d*x + 1)^(1/2) - 1)^28) - ((((4928*A*f^3)/3 + 512*A*d^2*e
^2*f)*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 - (((1408*A*f^3)/3 -
 32*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 - (((14
08*A*f^3)/3 - 32*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1
)^6 + (((4928*A*f^3)/3 + 512*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^12)/((d*x +
 1)^(1/2) - 1)^12 - (((11008*A*f^3)/5 - 912*A*d^2*e^2*f)*((1 - d*x)^(1/2) -
 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (((1 - d*x)^(1/2) - 1)*(2*A*d^3*e^3 - (3
*A*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^19*(2*A*d^3*
e^3 - (3*A*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^19 - (((1 - d*x)^(1/2) - 1)^3
*(2*A*d^3*e^3 - (99*A*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^3 + (((1 - d*x)^(1
/2) - 1)^17*(2*A*d^3*e^3 - (99*A*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)^17 - ((
(1 - d*x)^(1/2) - 1)^5*(40*A*d^3*e^3 + 306*A*d*e*f^2))/((d*x + 1)^(1/2) - 1
)^5 + (((1 - d*x)^(1/2) - 1)^15*(40*A*d^3*e^3 + 306*A*d*e*f^2))/((d*x + 1)^
(1/2) - 1)^15 - (((1 - d*x)^(1/2) - 1)^7*(88*A*d^3*e^3 - 306*A*d*e*f^2))/((
d*x + 1)^(1/2) - 1)^7 + (((1 - d*x)^(1/2) - 1)^13*(88*A*d^3*e^3 - 306*A*d*e
```

```
*f^2))/((d*x + 1)^(1/2) - 1)^13 - (((1 - d*x)^(1/2) - 1)^9*(52*A*d^3*e^3 -
663*A*d*e*f^2))/((d*x + 1)^(1/2) - 1)^9 + (((1 - d*x)^(1/2) - 1)^11*(52*A*d
^3*e^3 - 663*A*d*e*f^2))/((d*x + 1)^(1/2) - 1)^11 + (64*A*f^3*((1 - d*x)^(1
/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (64*A*f^3*((1 - d*x)^(1/2) - 1)^16)/(
(d*x + 1)^(1/2) - 1)^16 + (24*A*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2 + (24*A*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^18)/((d*x + 1)^(1/2
) - 1)^18)/(d^4 + (10*d^4*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2
+ (45*d^4*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (120*d^4*((1 -
 d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (210*d^4*((1 - d*x)^(1/2) - 1
)^8)/((d*x + 1)^(1/2) - 1)^8 + (252*d^4*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1
)^(1/2) - 1)^10 + (210*d^4*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^
12 + (120*d^4*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (45*d^4*
((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (10*d^4*((1 - d*x)^(1/
2) - 1)^18)/((d*x + 1)^(1/2) - 1)^18 + (d^4*((1 - d*x)^(1/2) - 1)^20)/((d*x
 + 1)^(1/2) - 1)^20) - ((((B*f^3)/4 + (3*B*d^2*e^2*f)/2)*((1 - d*x)^(1/2) -
 1)^23)/((d*x + 1)^(1/2) - 1)^23 - (((35*B*f^3)/12 - (93*B*d^2*e^2*f)/2)*((
1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (((35*B*f^3)/12 - (93*B*d^
2*e^2*f)/2)*((1 - d*x)^(1/2) - 1)^21)/((d*x + 1)^(1/2) - 1)^21 + (((757*B*f
^3)/4 - (417*B*d^2*e^2*f)/2)*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)
^5 - (((757*B*f^3)/4 - (417*B*d^2*e^2*f)/2)*((1 - d*x)^(1/2) - 1)^19)/((d*x
 + 1)^(1/2) - 1)^19 - (((7339*B*f^3)/4 + (513*B*d^2*e^2*f)/2)*((1 - d*x)^(1
/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (((7339*B*f^3)/4 + (513*B*d^2*e^2*f)/
2)*((1 - d*x)^(1/2) - 1)^17)/((d*x + 1)^(1/2) - 1)^17 - (((25661*B*f^3)/2 -
 969*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^11)/((d*x + 1)^(1/2) - 1)^11 + (((2
5661*B*f^3)/2 - 969*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^13)/((d*x + 1)^(1/2)
 - 1)^13 + (((41929*B*f^3)/6 + 969*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^9)/((
d*x + 1)^(1/2) - 1)^9 - (((41929*B*f^3)/6 + 969*B*d^2*e^2*f)*((1 - d*x)^(1/
2) - 1)^15)/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^4*(16*B*d^3*e
^3 + 192*B*d*e*f^2))/((d*x + 1)^(1/2) - 1)^4 + (((1 - d*x)^(1/2) - 1)^20*(1
6*B*d^3*e^3 + 192*B*d*e*f^2))/((d*x + 1)^(1/2) - 1)^20 + (((1 - d*x)^(1/2)
- 1)^6*((56*B*d^3*e^3)/3 - 1024*B*d*e*f^2))/((d*x + 1)^(1/2) - 1)^6 + (((1
- d*x)^(1/2) - 1)^18*((56*B*d^3*e^3)/3 - 1024*B*d*e*f^2))/((d*x + 1)^(1/2)
- 1)^18 + (((1 - d*x)^(1/2) - 1)^8*(192*B*d^3*e^3 + 2304*B*d*e*f^2))/((d*x
+ 1)^(1/2) - 1)^8 + (((1 - d*x)^(1/2) - 1)^16*(192*B*d^3*e^3 + 2304*B*d*e*f
^2))/((d*x + 1)^(1/2) - 1)^16 + (((1 - d*x)^(1/2) - 1)^10*(656*B*d^3*e^3 +
(9216*B*d*e*f^2)/5))/((d*x + 1)^(1/2) - 1)^10 + (((1 - d*x)^(1/2) - 1)^14*(
656*B*d^3*e^3 + (9216*B*d*e*f^2)/5))/((d*x + 1)^(1/2) - 1)^14 + (((1 - d*x)
^(1/2) - 1)^12*((2848*B*d^3*e^3)/3 - (16768*B*d*e*f^2)/5))/((d*x + 1)^(1/2)
 - 1)^12 - (((B*f^3)/4 + (3*B*d^2*e^2*f)/2)*((1 - d*x)^(1/2) - 1))/((d*x +
1)^(1/2) - 1) + (8*B*d^3*e^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
^2 + (8*B*d^3*e^3*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/2) - 1)^22)/(d^5
+ (12*d^5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (66*d^5*((1 -
d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (220*d^5*((1 - d*x)^(1/2) - 1)
^6)/((d*x + 1)^(1/2) - 1)^6 + (495*d^5*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^
(1/2) - 1)^8 + (792*d^5*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10
+ (924*d^5*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (792*d^5*((
1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (495*d^5*((1 - d*x)^(1/2
) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (220*d^5*((1 - d*x)^(1/2) - 1)^18)/((
d*x + 1)^(1/2) - 1)^18 + (66*d^5*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2)
 - 1)^20 + (12*d^5*((1 - d*x)^(1/2) - 1)^22)/((d*x + 1)^(1/2) - 1)^22 + (d^
5*((1 - d*x)^(1/2) - 1)^24)/((d*x + 1)^(1/2) - 1)^24) - (B*f*atan((B*f*(f^2
 + 6*d^2*e^2)*((1 - d*x)^(1/2) - 1))/((B*f^3 + 6*B*d^2*e^2*f)*((d*x + 1)^(1
/2) - 1)))*(f^2 + 6*d^2*e^2))/(4*d^5) - (A*e*atan((A*e*((1 - d*x)^(1/2) - 1
)*(3*f^2 + 4*d^2*e^2))/((4*A*d^2*e^3 + 3*A*e*f^2)*((d*x + 1)^(1/2) - 1)))*(
3*f^2 + 4*d^2*e^2))/(2*d^3) - (C*e*atan((C*e*((1 - d*x)^(1/2) - 1)*(3*f^2 +
 2*d^2*e^2))/((2*C*d^2*e^3 + 3*C*e*f^2)*((d*x + 1)^(1/2) - 1)))*(3*f^2 + 2*
d^2*e^2))/(4*d^5)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
[Out] Timed out
```

3.2 $\int \sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^2\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=286

$$\frac{\sin^{-1}(dx)\left(2d^2\left(A\left(4d^2e^2+f^2\right)+2Bef\right)+C\left(2d^2e^2+f^2\right)\right)}{16d^5}+\frac{x\sqrt{1-d^2x^2}\left(2d^2\left(A\left(4d^2e^2+f^2\right)+2Bef\right)+C\left(2d^2e\right.\right.}{16d^4}$$

Rubi [A] time = 0.56, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1609, 1654, 833, 780, 195, 216}

$$\frac{\left(1-d^2x^2\right)^{3/2}\left(8\left(C\left(d^2e^3-4ef^2\right)-2f\left(5Ad^2ef+B\left(d^2e^2+f^2\right)\right)\right)-3fx\left(5f^2\left(2Ad^2+C\right)-2d^2e(Ce-2Bf)\right)\right)}{120d^4f}+\frac{x\sqrt{1-d^2x^2}\left(2d^2\left(A\left(4d^2e^2+f^2\right)+2Bef\right)+C\left(2d^2e^2+f^2\right)\right)}{16d^4}+\frac{\sin^{-1}(dx)\left(2d^2\left(A\left(4d^2e^2+f^2\right)+2Bef\right)+C\left(2d^2e^2+f^2\right)\right)}{16d^5}+\frac{\left(1-d^2x^2\right)^{3/2}(e+fx)^2(Ce-2Bf)}{10d^2f}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^3}{6d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] ((C*(2*d^2*e^2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*x*Sqrt[1 - d
^2*x^2])/(16*d^4) + ((C*e - 2*B*f)*(e + f*x)^2*(1 - d^2*x^2)^(3/2))/(10*d^2
*f) - (C*(e + f*x)^3*(1 - d^2*x^2)^(3/2))/(6*d^2*f) + ((8*(C*(d^2*e^3 - 4*e
*f^2) - 2*f*(5*A*d^2*e*f + B*(d^2*e^2 + f^2))) - 3*f*(5*(C + 2*A*d^2)*f^2 -
 2*d^2*e*(C*e - 2*B*f))*x)*(1 - d^2*x^2)^(3/2))/(120*d^4*f) + ((C*(2*d^2*e^
2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*ArcSin[d*x])/(16*d^5)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
```

```
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^2\left(A+Bx+Cx^2\right)dx = \int (e+fx)^2\left(A+Bx+Cx^2\right)\sqrt{1-d^2x^2}\,dx$$

$$= -\frac{C(e+fx)^3\left(1-d^2x^2\right)^{3/2}}{6d^2f} - \frac{\int (e+fx)^2\left(-3\left(C+2Ad^2\right)f \ldots\right.}{\ldots}$$

$$= \frac{(Ce-2Bf)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{10d^2f} - \frac{C(e+fx)^3\left(1-d^2x^2\right. \ldots}{6d^2f}$$

$$= \frac{(Ce-2Bf)(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{10d^2f} - \frac{C(e+fx)^3\left(1-d^2x^2\right. \ldots}{6d^2f}$$

$$= \frac{\left(C\left(2d^2e^2+f^2\right)+2d^2\left(2Bef+A\left(4d^2e^2+f^2\right)\right)\right)x\sqrt{1-d^2 \ldots}}{16d^4}$$

$$= \frac{\left(C\left(2d^2e^2+f^2\right)+2d^2\left(2Bef+A\left(4d^2e^2+f^2\right)\right)\right)x\sqrt{1-d^2 \ldots}}{16d^4}$$

Mathematica [A] time = 0.35, size = 244, normalized size = 0.85

$$\frac{15\sin^{-1}(dx)\left(2d^2\left(A\left(4d^2e^2+f^2\right)+2Bef\right)+C\left(2d^2e^2+f^2\right)\right)+d\sqrt{1-d^2x^2}\left(10Ad^2\left(12d^2e^2x+16ef\left(d^2x^2-1\right)+3f^2x\left(2d^2x^2-1\right)\right)+4B\left(2d^4x^2\left(10e^2+15efx+6f^2x^2\right)-d^2\left(20e^2+15efx+4f^2x^2\right)-8f^2\right)+C\left(30d^2e^2x\left(2d^2x^2-1\right)+32ef\left(3d^4x^4-d^2x^2-2\right)+5f^2x\left(8d^4x^4-2d^2x^2-3\right)\right)\right)}{240d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] (d*Sqrt[1 - d^2*x^2]*(10*A*d^2*(12*d^2*e^2*x + 16*e*f*(-1 + d^2*x^2) + 3*f^
2*x*(-1 + 2*d^2*x^2)) + 4*B*(-8*f^2 - d^2*(20*e^2 + 15*e*f*x + 4*f^2*x^2) +
 2*d^4*x^2*(10*e^2 + 15*e*f*x + 6*f^2*x^2)) + C*(30*d^2*e^2*x*(-1 + 2*d^2*x
^2) + 32*e*f*(-2 - d^2*x^2 + 3*d^4*x^4) + 5*f^2*x*(-3 - 2*d^2*x^2 + 8*d^4*x
^4))) + 15*(C*(2*d^2*e^2 + f^2) + 2*d^2*(2*B*e*f + A*(4*d^2*e^2 + f^2)))*Ar
cSin[d*x])/(240*d^5)
```

IntegrateAlgebraic [B] time = 0.71, size = 1079, normalized size = 3.77

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2*(A + B*x + C*x^2
),x]
```

```
[Out] -1/120*(Sqrt[1 - d*x]*(-30*C*d^2*e^2 - 120*A*d^4*e^2 - 60*B*d^2*e*f - 15*C*
f^2 - 30*A*d^2*f^2 + (30*C*d^2*e^2*(1 - d*x)^5)/(1 + d*x)^5 + (120*A*d^4*e^
2*(1 - d*x)^5)/(1 + d*x)^5 + (60*B*d^2*e*f*(1 - d*x)^5)/(1 + d*x)^5 + (15*C
*f^2*(1 - d*x)^5)/(1 + d*x)^5 + (30*A*d^2*f^2*(1 - d*x)^5)/(1 + d*x)^5 - (1
50*C*d^2*e^2*(1 - d*x)^4)/(1 + d*x)^4 + (320*B*d^3*e^2*(1 - d*x)^4)/(1 + d*
x)^4 + (360*A*d^4*e^2*(1 - d*x)^4)/(1 + d*x)^4 + (640*C*d*e*f*(1 - d*x)^4)/
(1 + d*x)^4 - (300*B*d^2*e*f*(1 - d*x)^4)/(1 + d*x)^4 + (640*A*d^3*e*f*(1 -
 d*x)^4)/(1 + d*x)^4 - (235*C*f^2*(1 - d*x)^4)/(1 + d*x)^4 + (320*B*d*f^2*(
1 - d*x)^4)/(1 + d*x)^4 - (150*A*d^2*f^2*(1 - d*x)^4)/(1 + d*x)^4 - (180*C*
d^2*e^2*(1 - d*x)^3)/(1 + d*x)^3 + (960*B*d^3*e^2*(1 - d*x)^3)/(1 + d*x)^3
+ (240*A*d^4*e^2*(1 - d*x)^3)/(1 + d*x)^3 + (384*C*d*e*f*(1 - d*x)^3)/(1 +
d*x)^3 - (360*B*d^2*e*f*(1 - d*x)^3)/(1 + d*x)^3 + (1920*A*d^3*e*f*(1 - d*x
)^3)/(1 + d*x)^3 + (390*C*f^2*(1 - d*x)^3)/(1 + d*x)^3 + (192*B*d*f^2*(1 -
d*x)^3)/(1 + d*x)^3 - (180*A*d^2*f^2*(1 - d*x)^3)/(1 + d*x)^3 + (180*C*d^2*
e^2*(1 - d*x)^2)/(1 + d*x)^2 + (960*B*d^3*e^2*(1 - d*x)^2)/(1 + d*x)^2 - (2
40*A*d^4*e^2*(1 - d*x)^2)/(1 + d*x)^2 + (384*C*d*e*f*(1 - d*x)^2)/(1 + d*x)
^2 + (360*B*d^2*e*f*(1 - d*x)^2)/(1 + d*x)^2 + (1920*A*d^3*e*f*(1 - d*x)^2)
/(1 + d*x)^2 - (390*C*f^2*(1 - d*x)^2)/(1 + d*x)^2 + (192*B*d*f^2*(1 - d*x)
^2)/(1 + d*x)^2 + (180*A*d^2*f^2*(1 - d*x)^2)/(1 + d*x)^2 + (150*C*d^2*e^2*
(1 - d*x))/(1 + d*x) + (320*B*d^3*e^2*(1 - d*x))/(1 + d*x) - (360*A*d^4*e^2
*(1 - d*x))/(1 + d*x) + (640*C*d*e*f*(1 - d*x))/(1 + d*x) + (300*B*d^2*e*f*
(1 - d*x))/(1 + d*x) + (640*A*d^3*e*f*(1 - d*x))/(1 + d*x) + (235*C*f^2*(1
- d*x))/(1 + d*x) + (320*B*d*f^2*(1 - d*x))/(1 + d*x) + (150*A*d^2*f^2*(1 -
 d*x))/(1 + d*x)))/(d^5*Sqrt[1 + d*x]*(1 + (1 - d*x)/(1 + d*x))^6) + ((-2*C
*d^2*e^2 - 8*A*d^4*e^2 - 4*B*d^2*e*f - C*f^2 - 2*A*d^2*f^2)*ArcTan[Sqrt[1 -
 d*x]/Sqrt[1 + d*x]])/(8*d^5)
```

fricas [A] time = 0.86, size = 279, normalized size = 0.98

$$\frac{\left(40 Cd^5 f^2 x^5 - 80 Bd^3 e^2 + 48 \left(2 Cd^5 ef + Bd^5 f^2\right) x^4 - 32 Bdf^2 + 10 \left(6 Cd^5 e^2 + 12 Bd^5 ef + \left(6 Ad^5 - Cd^3\right) f^2\right) x^3 - 32 \left(5 Ad^3 + 2 Cd\right) ef + 16 \left(5 Bd^5 e^2 - Bd^3 f^2 + 2 \left(5 Ad^5 - Cd^3\right) ef\right) x^2 - 15 \left(4 Bd^3 ef - 2 \left(4 Ad^5 - Cd^3\right) e^2 + \left(2 Ad^3 + Cd\right) f^2\right) x\right) \sqrt{dx+1} \sqrt{-dx+1} - 30 \left(4 Bd^2 ef + 2 \left(4 Ad^4 + Cd^2\right) e^2 + \left(2 Ad^2 + C\right) f^2\right) \arctan\left(\frac{\sqrt{dx+1} \sqrt{-dx+1} - 1}{dx}\right)}{240 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/240*((40*C*d^5*f^2*x^5 - 80*B*d^3*e^2 + 48*(2*C*d^5*e*f + B*d^5*f^2)*x^4
- 32*B*d*f^2 + 10*(6*C*d^5*e^2 + 12*B*d^5*e*f + (6*A*d^5 - C*d^3)*f^2)*x^3
- 32*(5*A*d^3 + 2*C*d)*e*f + 16*(5*B*d^5*e^2 - B*d^3*f^2 + 2*(5*A*d^5 - C*d
^3)*e*f)*x^2 - 15*(4*B*d^3*e*f - 2*(4*A*d^5 - C*d^3)*e^2 + (2*A*d^3 + C*d)*
f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30*(4*B*d^2*e*f + 2*(4*A*d^4 + C*d^2
)*e^2 + (2*A*d^2 + C)*f^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x))
)/d^5
```

giac [B] time = 2.58, size = 1327, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] 1/240*(10*(((d*x + 1)*(2*(d*x + 1)*(3*(d*x + 1)/d^3 - 13/d^3) + 43/d^3) - 3
9/d^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/
d^3)*A*d*f^2 + 2*(((2*(d*x + 1)*(3*(d*x + 1)*(4*(d*x + 1)/d^4 - 21/d^4) + 1
33/d^4) - 295/d^4)*(d*x + 1) + 195/d^4)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*a
rcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^4)*B*d*f^2 + (((2*((d*x + 1)*(4*(d*x + 1
)*(5*(d*x + 1)/d^5 - 31/d^5) + 321/d^5) - 451/d^5)*(d*x + 1) + 745/d^5)*(d*
x + 1) - 405/d^5)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 150*arcsin(1/2*sqrt(2)*sqr
t(d*x + 1))/d^5)*C*d*f^2 + 80*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(
d*x + 1)/d^2 - 7/d^2) + 9/d^2) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*A
*d*f*e + 20*(((d*x + 1)*(2*(d*x + 1)*(3*(d*x + 1)/d^3 - 13/d^3) + 43/d^3) -
```

```
39/d^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)
)/d^3)*B*d*f*e + 4*(((2*(d*x + 1)*(3*(d*x + 1)*(4*(d*x + 1)/d^4 - 21/d^4) +
 133/d^4) - 295/d^4)*(d*x + 1) + 195/d^4)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90
*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^4)*C*d*f*e + 40*(sqrt(d*x + 1)*sqrt(-d
*x + 1)*((d*x + 1)*(2*(d*x + 1)/d^2 - 7/d^2) + 9/d^2) + 6*arcsin(1/2*sqrt(2
)*sqrt(d*x + 1))/d^2)*A*f^2 + 10*(((d*x + 1)*(2*(d*x + 1)*(3*(d*x + 1)/d^3
- 13/d^3) + 43/d^3) - 39/d^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*
sqrt(2)*sqrt(d*x + 1))/d^3)*B*f^2 + 2*(((2*(d*x + 1)*(3*(d*x + 1)*(4*(d*x +
 1)/d^4 - 21/d^4) + 133/d^4) - 295/d^4)*(d*x + 1) + 195/d^4)*sqrt(d*x + 1)*
sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^4)*C*f^2 + 40*(sqrt
(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)/d^2 - 7/d^2) + 9/d^2) + 6*
arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*B*d*e^2 + 10*(((d*x + 1)*(2*(d*x + 1
)*(3*(d*x + 1)/d^3 - 13/d^3) + 43/d^3) - 39/d^3)*sqrt(d*x + 1)*sqrt(-d*x +
1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^3)*C*d*e^2 + 80*(sqrt(d*x + 1)*
sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)/d^2 - 7/d^2) + 9/d^2) + 6*arcsin(1/2
*sqrt(2)*sqrt(d*x + 1))/d^2)*B*f*e + 20*(((d*x + 1)*(2*(d*x + 1)*(3*(d*x +
1)/d^3 - 13/d^3) + 43/d^3) - 39/d^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcs
in(1/2*sqrt(2)*sqrt(d*x + 1))/d^3)*C*f*e + 120*(sqrt(d*x + 1)*(d*x - 2)*sqr
t(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*e^2 + 240*(sqrt(d*x +
1)*sqrt(-d*x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*e^2 + 40*(sqrt(d
*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)/d^2 - 7/d^2) + 9/d^2) + 6*ar
csin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*C*e^2 + 240*(sqrt(d*x + 1)*(d*x - 2)*s
qrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*f*e/d + 120*(sqrt(d*
x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B*e^
2/d)/d
```

maple [C] time = 0.02, size = 652, normalized size = 2.28

$$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(40\sqrt{-d^2x^2+1}\,C d^5 f^2 x^5 \mathrm{csgn}(d) + 48\sqrt{-d^2x^2+1}\,B d^5 f^2 x^4 \mathrm{csgn}(d) + 96\sqrt{-d^2x^2+1}\,C d^5 e f x^4 \mathrm{csgn}(d) + 60\sqrt{-d^2x^2+1}\,A d^5 f^2 x^3 \mathrm{csgn}(d) + 120\sqrt{-d^2x^2+1}\,B d^5 e f x^3 \mathrm{csgn}(d) + 60\sqrt{-d^2x^2+1}\,C d^5 e^2 x^3 \mathrm{csgn}(d) + 160\sqrt{-d^2x^2+1}\,A d^5 e f x^2 \mathrm{csgn}(d) + 80\sqrt{-d^2x^2+1}\,B d^5 e^2 x^2 \mathrm{csgn}(d) + 120\sqrt{-d^2x^2+1}\,A d^5 e^2 x \,\mathrm{csgn}(d) - 10\sqrt{-d^2x^2+1}\,C d^3 f^2 x^3 \mathrm{csgn}(d) - 16\sqrt{-d^2x^2+1}\,B d^3 f^2 x^2 \mathrm{csgn}(d) - 32\sqrt{-d^2x^2+1}\,C d^3 e f x^2 \mathrm{csgn}(d) + 120 A d^4 e^2 \arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right) - 30\sqrt{-d^2x^2+1}\,A d^3 f^2 x \,\mathrm{csgn}(d) - 60\sqrt{-d^2x^2+1}\,B d^3 e f x \,\mathrm{csgn}(d) - 30\sqrt{-d^2x^2+1}\,C d^3 e^2 x \,\mathrm{csgn}(d) - 160\sqrt{-d^2x^2+1}\,A d^3 e f \,\mathrm{csgn}(d) - 80\sqrt{-d^2x^2+1}\,B d^3 e^2 \mathrm{csgn}(d) + 30 A d^2 f^2 \arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right) + 60 B d^2 e f \arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right) + 30 C d^2 e^2 \arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right) - 15\sqrt{-d^2x^2+1}\,C d f^2 x \,\mathrm{csgn}(d) - 32\sqrt{-d^2x^2+1}\,B d f^2 \mathrm{csgn}(d) - 64\sqrt{-d^2x^2+1}\,C d e f \,\mathrm{csgn}(d) + 15 C f^2 \arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)\right) \mathrm{csgn}(d)}{240\sqrt{-d^2x^2+1}\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x)
```

```
[Out] 1/240*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(-160*A*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*e
*f-64*C*csgn(d)*d*(-d^2*x^2+1)^(1/2)*e*f+40*C*csgn(d)*x^5*d^5*f^2*(-d^2*x^2
+1)^(1/2)+48*B*csgn(d)*x^4*d^5*f^2*(-d^2*x^2+1)^(1/2)+60*A*csgn(d)*x^3*d^5*
f^2*(-d^2*x^2+1)^(1/2)+60*C*csgn(d)*x^3*d^5*e^2*(-d^2*x^2+1)^(1/2)+30*A*arc
tan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*d^2*f^2+30*C*arctan(1/(-d^2*x^2+1)^(1
/2)*d*x*csgn(d))*d^2*e^2+120*A*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*d^4
*e^2+15*C*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*f^2+60*B*arctan(1/(-d^2*
x^2+1)^(1/2)*d*x*csgn(d))*d^2*e*f+80*B*csgn(d)*x^2*d^5*e^2*(-d^2*x^2+1)^(1/
2)-32*B*csgn(d)*d*(-d^2*x^2+1)^(1/2)*f^2-80*B*csgn(d)*d^3*(-d^2*x^2+1)^(1/2
)*e^2-10*C*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x^3*f^2-16*B*csgn(d)*d^3*(-d^2*x^
2+1)^(1/2)*x^2*f^2-30*C*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x*e^2-30*A*csgn(d)*d
^3*(-d^2*x^2+1)^(1/2)*x*f^2+120*A*csgn(d)*d^5*(-d^2*x^2+1)^(1/2)*x*e^2-15*C
*csgn(d)*d*(-d^2*x^2+1)^(1/2)*x*f^2+160*A*csgn(d)*x^2*d^5*e*f*(-d^2*x^2+1)^
(1/2)-60*B*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x*e*f-32*C*csgn(d)*d^3*(-d^2*x^2+
1)^(1/2)*x^2*e*f+96*C*csgn(d)*x^4*d^5*e*f*(-d^2*x^2+1)^(1/2)+120*B*csgn(d)*
x^3*d^5*e*f*(-d^2*x^2+1)^(1/2))*csgn(d)/(-d^2*x^2+1)^(1/2)/d^5
```

maxima [A] time = 1.01, size = 307, normalized size = 1.07

$$-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}} C f^2 x^3}{6\,d^2} + \frac{1}{2}\sqrt{-d^2x^2+1}\,A e^2 x + \frac{A e^2 \arcsin\left(dx\right)}{2\,d} - \frac{\left(-d^2x^2+1\right)^{\frac{3}{2}} B e^2}{3\,d^2} - \frac{2\left(-d^2x^2+1\right)^{\frac{3}{2}} A e f}{3\,d^2} - \frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}\left(2\,C e f + B f^2\right) x^2}{5\,d^2} - \frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}\left(C e^2 + 2\,B e f + A f^2\right) x}{4\,d^2} - \frac{\left(-d^2x^2+1\right)^{\frac{3}{2}} C f^2 x}{8\,d^4} + \frac{\sqrt{-d^2x^2+1}\left(C e^2 + 2\,B e f + A f^2\right) x}{8\,d^2} + \frac{\sqrt{-d^2x^2+1}\,C f^2 x}{16\,d^4} + \frac{\left(C e^2 + 2\,B e f + A f^2\right)\arcsin\left(dx\right)}{8\,d^3} + \frac{C f^2 \arcsin\left(dx\right)}{16\,d^5} - \frac{2\left(-d^2x^2+1\right)^{\frac{3}{2}}\left(2\,C e f + B f^2\right)}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/6*(-d^2*x^2 + 1)^(3/2)*C*f^2*x^3/d^2 + 1/2*sqrt(-d^2*x^2 + 1)*A*e^2*x +
1/2*A*e^2*arcsin(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e^2/d^2 - 2/3*(-d^2*x^
```

```
2 + 1)^(3/2)*A*e*f/d^2 - 1/5*(-d^2*x^2 + 1)^(3/2)*(2*C*e*f + B*f^2)*x^2/d^2
 - 1/4*(-d^2*x^2 + 1)^(3/2)*(C*e^2 + 2*B*e*f + A*f^2)*x/d^2 - 1/8*(-d^2*x^2
 + 1)^(3/2)*C*f^2*x/d^4 + 1/8*sqrt(-d^2*x^2 + 1)*(C*e^2 + 2*B*e*f + A*f^2)*
x/d^2 + 1/16*sqrt(-d^2*x^2 + 1)*C*f^2*x/d^4 + 1/8*(C*e^2 + 2*B*e*f + A*f^2)
*arcsin(d*x)/d^3 + 1/16*C*f^2*arcsin(d*x)/d^5 - 2/15*(-d^2*x^2 + 1)^(3/2)*(
2*C*e*f + B*f^2)/d^4
```

mupad [B] time = 36.03, size = 2920, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] - ((((1 - d*x)^(1/2) - 1)^8*((4928*B*f^2)/3 + (512*B*d^2*e^2)/3))/((d*x + 1
)^(1/2) - 1)^8 - (((1 - d*x)^(1/2) - 1)^14*((1408*B*f^2)/3 - (32*B*d^2*e^2)
/3))/((d*x + 1)^(1/2) - 1)^14 - (((1 - d*x)^(1/2) - 1)^6*((1408*B*f^2)/3 -
(32*B*d^2*e^2)/3))/((d*x + 1)^(1/2) - 1)^6 + (((1 - d*x)^(1/2) - 1)^12*((49
28*B*f^2)/3 + (512*B*d^2*e^2)/3))/((d*x + 1)^(1/2) - 1)^12 - (((1 - d*x)^(1
/2) - 1)^10*((11008*B*f^2)/5 - 304*B*d^2*e^2))/((d*x + 1)^(1/2) - 1)^10 + (
64*B*f^2*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (64*B*f^2*((1 -
 d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (8*B*d^2*e^2*((1 - d*x)^(1/
2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (8*B*d^2*e^2*((1 - d*x)^(1/2) - 1)^18)
/((d*x + 1)^(1/2) - 1)^18 + (33*B*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)
^(1/2) - 1)^3 - (204*B*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)
^5 + (204*B*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (442*B
*d*e*f*((1 - d*x)^(1/2) - 1)^9)/((d*x + 1)^(1/2) - 1)^9 - (442*B*d*e*f*((1
- d*x)^(1/2) - 1)^11)/((d*x + 1)^(1/2) - 1)^11 - (204*B*d*e*f*((1 - d*x)^(1
/2) - 1)^13)/((d*x + 1)^(1/2) - 1)^13 + (204*B*d*e*f*((1 - d*x)^(1/2) - 1)^
15)/((d*x + 1)^(1/2) - 1)^15 - (33*B*d*e*f*((1 - d*x)^(1/2) - 1)^17)/((d*x
+ 1)^(1/2) - 1)^17 + (B*d*e*f*((1 - d*x)^(1/2) - 1)^19)/((d*x + 1)^(1/2) -
1)^19 - (B*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^4 + (10*d
^4*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (45*d^4*((1 - d*x)^(1
/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (120*d^4*((1 - d*x)^(1/2) - 1)^6)/((d
*x + 1)^(1/2) - 1)^6 + (210*d^4*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) -
 1)^8 + (252*d^4*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (210*
d^4*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (120*d^4*((1 - d*x
)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (45*d^4*((1 - d*x)^(1/2) - 1)^1
6)/((d*x + 1)^(1/2) - 1)^16 + (10*d^4*((1 - d*x)^(1/2) - 1)^18)/((d*x + 1)^
(1/2) - 1)^18 + (d^4*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) - 1)^20) -
((((1 - d*x)^(1/2) - 1)^15*((A*f^2)/2 - 2*A*d^2*e^2))/((d*x + 1)^(1/2) - 1)
^15 - (((1 - d*x)^(1/2) - 1)*((A*f^2)/2 - 2*A*d^2*e^2))/((d*x + 1)^(1/2) -
1) + (((1 - d*x)^(1/2) - 1)^3*((35*A*f^2)/2 - 6*A*d^2*e^2))/((d*x + 1)^(1/2
) - 1)^3 - (((1 - d*x)^(1/2) - 1)^13*((35*A*f^2)/2 - 6*A*d^2*e^2))/((d*x +
1)^(1/2) - 1)^13 - (((1 - d*x)^(1/2) - 1)^5*((273*A*f^2)/2 + 30*A*d^2*e^2))
/((d*x + 1)^(1/2) - 1)^5 + (((1 - d*x)^(1/2) - 1)^11*((273*A*f^2)/2 + 30*A*
d^2*e^2))/((d*x + 1)^(1/2) - 1)^11 + (((1 - d*x)^(1/2) - 1)^7*((715*A*f^2)/
2 - 22*A*d^2*e^2))/((d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^9*((715
*A*f^2)/2 - 22*A*d^2*e^2))/((d*x + 1)^(1/2) - 1)^9 + (16*A*d*e*f*((1 - d*x)
^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (32*A*d*e*f*((1 - d*x)^(1/2) - 1)^
4)/((d*x + 1)^(1/2) - 1)^4 + (208*A*d*e*f*((1 - d*x)^(1/2) - 1)^6)/(3*((d*x
 + 1)^(1/2) - 1)^6) + (704*A*d*e*f*((1 - d*x)^(1/2) - 1)^8)/(3*((d*x + 1)^(
1/2) - 1)^8) + (208*A*d*e*f*((1 - d*x)^(1/2) - 1)^10)/(3*((d*x + 1)^(1/2) -
 1)^10) - (32*A*d*e*f*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 +
(16*A*d*e*f*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14)/(d^3 + (8*d
^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^3*((1 - d*x)^(1
/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (56*d^3*((1 - d*x)^(1/2) - 1)^6)/((d*
x + 1)^(1/2) - 1)^6 + (70*d^3*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1
)^8 + (56*d^3*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (28*d^3*
((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (8*d^3*((1 - d*x)^(1/2
```

```
) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (d^3*((1 - d*x)^(1/2) - 1)^16)/((d*x
+ 1)^(1/2) - 1)^16) - ((((1 - d*x)^(1/2) - 1)^23*((C*f^2)/4 + (C*d^2*e^2)/2
))/((d*x + 1)^(1/2) - 1)^23 - (((1 - d*x)^(1/2) - 1)*((C*f^2)/4 + (C*d^2*e^
2)/2))/((d*x + 1)^(1/2) - 1) - (((1 - d*x)^(1/2) - 1)^3*((35*C*f^2)/12 - (3
1*C*d^2*e^2)/2))/((d*x + 1)^(1/2) - 1)^3 + (((1 - d*x)^(1/2) - 1)^21*((35*C
*f^2)/12 - (31*C*d^2*e^2)/2))/((d*x + 1)^(1/2) - 1)^21 + (((1 - d*x)^(1/2)
- 1)^5*((757*C*f^2)/4 - (139*C*d^2*e^2)/2))/((d*x + 1)^(1/2) - 1)^5 - (((1
- d*x)^(1/2) - 1)^19*((757*C*f^2)/4 - (139*C*d^2*e^2)/2))/((d*x + 1)^(1/2)
- 1)^19 - (((1 - d*x)^(1/2) - 1)^7*((7339*C*f^2)/4 + (171*C*d^2*e^2)/2))/((
d*x + 1)^(1/2) - 1)^7 + (((1 - d*x)^(1/2) - 1)^17*((7339*C*f^2)/4 + (171*C*
d^2*e^2)/2))/((d*x + 1)^(1/2) - 1)^17 - (((1 - d*x)^(1/2) - 1)^11*((25661*C
*f^2)/2 - 323*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^11 + (((1 - d*x)^(1/2) - 1)
^13*((25661*C*f^2)/2 - 323*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 - d*
x)^(1/2) - 1)^9*((41929*C*f^2)/6 + 323*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^9
- (((1 - d*x)^(1/2) - 1)^15*((41929*C*f^2)/6 + 323*C*d^2*e^2))/((d*x + 1)^(
1/2) - 1)^15 + (128*C*d*e*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^
4 - (2048*C*d*e*f*((1 - d*x)^(1/2) - 1)^6)/(3*((d*x + 1)^(1/2) - 1)^6) + (1
536*C*d*e*f*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (6144*C*d*e*
f*((1 - d*x)^(1/2) - 1)^10)/(5*((d*x + 1)^(1/2) - 1)^10) - (33536*C*d*e*f*(
(1 - d*x)^(1/2) - 1)^12)/(15*((d*x + 1)^(1/2) - 1)^12) + (6144*C*d*e*f*((1
- d*x)^(1/2) - 1)^14)/(5*((d*x + 1)^(1/2) - 1)^14) + (1536*C*d*e*f*((1 - d*
x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 - (2048*C*d*e*f*((1 - d*x)^(1/2)
 - 1)^18)/(3*((d*x + 1)^(1/2) - 1)^18) + (128*C*d*e*f*((1 - d*x)^(1/2) - 1)
^20)/((d*x + 1)^(1/2) - 1)^20)/(d^5 + (12*d^5*((1 - d*x)^(1/2) - 1)^2)/((d*
x + 1)^(1/2) - 1)^2 + (66*d^5*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1
)^4 + (220*d^5*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (495*d^5*
((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (792*d^5*((1 - d*x)^(1/2
) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (924*d^5*((1 - d*x)^(1/2) - 1)^12)/((
d*x + 1)^(1/2) - 1)^12 + (792*d^5*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2
) - 1)^14 + (495*d^5*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (
220*d^5*((1 - d*x)^(1/2) - 1)^18)/((d*x + 1)^(1/2) - 1)^18 + (66*d^5*((1 -
d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) - 1)^20 + (12*d^5*((1 - d*x)^(1/2) - 1
)^22)/((d*x + 1)^(1/2) - 1)^22 + (d^5*((1 - d*x)^(1/2) - 1)^24)/((d*x + 1)^
(1/2) - 1)^24) - (A*atan((A*(f^2 + 4*d^2*e^2)*((1 - d*x)^(1/2) - 1))/(((d*x
 + 1)^(1/2) - 1)*(A*f^2 + 4*A*d^2*e^2)))*(f^2 + 4*d^2*e^2))/(2*d^3) - (C*at
an((C*(f^2 + 2*d^2*e^2)*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(C*f^
2 + 2*C*d^2*e^2)))*(f^2 + 2*d^2*e^2))/(4*d^5) - (B*e*f*atan(((1 - d*x)^(1/2
) - 1)/((d*x + 1)^(1/2) - 1)))/d^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
[Out] Timed out
```

3.3 $\int \sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=168

$$\frac{x\sqrt{1-d^2x^2}\left(4Ad^2e+Bf+Ce\right)}{8d^2}-\frac{\left(1-d^2x^2\right)^{3/2}\left(4\left(5d^2f(Af+Be)-C\left(3d^2e^2-2f^2\right)\right)-3d^2fx(3Ce-5Bf)\right)}{60d^4f}+\frac{\sin}{}$$

Rubi [A] time = 0.25, antiderivative size = 170, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1609, 1654, 780, 195, 216}

$$-\frac{\left(1-d^2x^2\right)^{3/2}\left(4\left(5d^2f(Af+Be)-\frac{1}{4}C\left(12d^2e^2-8f^2\right)\right)-3d^2fx(3Ce-5Bf)\right)}{60d^4f}+\frac{x\sqrt{1-d^2x^2}\left(4Ad^2e+Bf+Ce\right)}{8d^2}+\frac{\sin^{-1}(dx)\left(4Ad^2e+Bf+Ce\right)}{8d^3}-\frac{C\left(1-d^2x^2\right)^{3/2}(e+fx)^2}{5d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] ((C*e + 4*A*d^2*e + B*f)*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (C*(e + f*x)^2*(1 -
 d^2*x^2)^(3/2))/(5*d^2*f) - ((4*(5*d^2*f*(B*e + A*f) - (C*(12*d^2*e^2 - 8*
f^2))/4) - 3*d^2*f*(3*C*e - 5*B*f)*x)*(1 - d^2*x^2)^(3/2))/(60*d^4*f) + ((C
*e + 4*A*d^2*e + B*f)*ArcSin[d*x])/(8*d^3)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
```

```
e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && !(EqQ[d, 0] && T
rue) && !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)\left(A+Bx+Cx^2\right)dx = \int (e+fx)\left(A+Bx+Cx^2\right)\sqrt{1-d^2x^2}\,dx$$

$$= -\frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f} - \frac{\int (e+fx)\left(-\left(\left(2C+5Ad^2\right)f^2 \right.\right.}{}$$

$$= -\frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f} - \frac{\left(4\left(5d^2f(Be+Af)-\frac{1}{4}C\left(12d \right.\right.\right.}{}$$

$$= \frac{\left(Ce+4Ad^2e+Bf\right)x\sqrt{1-d^2x^2}}{8d^2} - \frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f}$$

$$= \frac{\left(Ce+4Ad^2e+Bf\right)x\sqrt{1-d^2x^2}}{8d^2} - \frac{C(e+fx)^2\left(1-d^2x^2\right)^{3/2}}{5d^2f}$$

Mathematica [A] time = 0.21, size = 141, normalized size = 0.84

$$\frac{15d\sin^{-1}(dx)\left(4Ad^2e+Bf+Ce\right)+\sqrt{1-d^2x^2}\left(60Ad^4ex+40Ad^2f\left(d^2x^2-1\right)+5Bd^2\left(8d^2ex^2+6d^2fx^3-8e-3fx\right)+15Cd^2ex\left(2d^2x^2-1\right)+8Cf\left(3d^4x^4-d^2x^2-2\right)\right)}{120d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[1 - d^2*x^2]*(60*A*d^4*e*x + 40*A*d^2*f*(-1 + d^2*x^2) + 15*C*d^2*e*x
*(-1 + 2*d^2*x^2) + 5*B*d^2*(-8*e - 3*f*x + 8*d^2*e*x^2 + 6*d^2*f*x^3) + 8*
C*f*(-2 - d^2*x^2 + 3*d^4*x^4)) + 15*d*(C*e + 4*A*d^2*e + B*f)*ArcSin[d*x])
/(120*d^4)
```

IntegrateAlgebraic [B] time = 0.39, size = 470, normalized size = 2.80

$$\frac{\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)\left(-4Ad^2e-Bf-Ce\right)}{4d^3}-\frac{\sqrt{1-dx}\left(\frac{60Ad^3e(1-dx)^4}{(dx+1)^4}+\frac{120Ad^3e(1-dx)^3}{(dx+1)^3}-\frac{120Ad^3e(1-dx)}{dx+1}-60Ad^3e+\frac{160Ad^2f(1-dx)^3}{(dx+1)^3}+\frac{320Ad^2f(1-dx)^2}{(dx+1)^2}+\frac{160Ad^2f(1-dx)}{dx+1}+\frac{160Bd^2e(1-dx)^3}{(dx+1)^3}+\frac{320Bd^2e(1-dx)^2}{(dx+1)^2}+\frac{160Bd^2e(1-dx)}{dx+1}+\frac{15Bdf(1-dx)^4}{(dx+1)^4}-\frac{90Bdf(1-dx)^3}{(dx+1)^3}+\frac{90Bdf(1-dx)}{dx+1}-15Bdf+\frac{15Cde(1-dx)^4}{(dx+1)^4}-\frac{90Cde(1-dx)^3}{(dx+1)^3}+\frac{90Cde(1-dx)}{dx+1}-15Cde+\frac{160Cf(1-dx)^3}{(dx+1)^3}-\frac{64Cf(1-dx)^2}{(dx+1)^2}+\frac{160Cf(1-dx)}{dx+1}\right)}{60d^4\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)*(A + B*x + C*x^2),
x]
```

```
[Out] -1/60*(Sqrt[1 - d*x]*(-15*C*d*e - 60*A*d^3*e - 15*B*d*f + (15*C*d*e*(1 - d*
x)^4)/(1 + d*x)^4 + (60*A*d^3*e*(1 - d*x)^4)/(1 + d*x)^4 + (15*B*d*f*(1 - d
*x)^4)/(1 + d*x)^4 - (90*C*d*e*(1 - d*x)^3)/(1 + d*x)^3 + (160*B*d^2*e*(1 -
 d*x)^3)/(1 + d*x)^3 + (120*A*d^3*e*(1 - d*x)^3)/(1 + d*x)^3 + (160*C*f*(1
- d*x)^3)/(1 + d*x)^3 - (90*B*d*f*(1 - d*x)^3)/(1 + d*x)^3 + (160*A*d^2*f*(
1 - d*x)^3)/(1 + d*x)^3 + (320*B*d^2*e*(1 - d*x)^2)/(1 + d*x)^2 - (64*C*f*(
1 - d*x)^2)/(1 + d*x)^2 + (320*A*d^2*f*(1 - d*x)^2)/(1 + d*x)^2 + (90*C*d*e
*(1 - d*x))/(1 + d*x) + (160*B*d^2*e*(1 - d*x))/(1 + d*x) - (120*A*d^3*e*(1
 - d*x))/(1 + d*x) + (160*C*f*(1 - d*x))/(1 + d*x) + (90*B*d*f*(1 - d*x))/(
1 + d*x) + (160*A*d^2*f*(1 - d*x))/(1 + d*x)))/(d^4*Sqrt[1 + d*x]*(1 + (1 -
 d*x)/(1 + d*x))^5) + ((-(C*e) - 4*A*d^2*e - B*f)*ArcTan[Sqrt[1 - d*x]/Sqrt
[1 + d*x]])/(4*d^3)
```

fricas [A] time = 0.93, size = 170, normalized size = 1.01

$$\frac{\left(24\,Cd^4fx^4-40\,Bd^2e+30\left(Cd^4e+Bd^4f\right)x^3+8\left(5\,Bd^4e+\left(5\,Ad^4-Cd^2\right)f\right)x^2-8\left(5\,Ad^2+2\,C\right)f-15\left(Bd^2f-\left(4\,Ad^4-Cd^2\right)e\right)x\right)\sqrt{dx+1}\,\sqrt{-dx+1}-30\left(Bdf+\left(4\,Ad^3+Cd\right)e\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{120\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/120*((24*C*d^4*f*x^4 - 40*B*d^2*e + 30*(C*d^4*e + B*d^4*f)*x^3 + 8*(5*B*d
^4*e + (5*A*d^4 - C*d^2)*f)*x^2 - 8*(5*A*d^2 + 2*C)*f - 15*(B*d^2*f - (4*A*
d^4 - C*d^2)*e)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 30*(B*d*f + (4*A*d^3 + C*
d)*e)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^4
```

giac [B] time = 2.00, size = 782, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] 1/120*(20*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)/d^2 - 7/d^2
) + 9/d^2) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*A*d*f + 5*(((d*x + 1)
*(2*(d*x + 1)*(3*(d*x + 1)/d^3 - 13/d^3) + 43/d^3) - 39/d^3)*sqrt(d*x + 1)*
sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^3)*B*d*f + (((2*(d*
x + 1)*(3*(d*x + 1)*(4*(d*x + 1)/d^4 - 21/d^4) + 133/d^4) - 295/d^4)*(d*x +
 1) + 195/d^4)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 90*arcsin(1/2*sqrt(2)*sqrt(d*
x + 1))/d^4)*C*d*f + 20*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x +
1)/d^2 - 7/d^2) + 9/d^2) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*B*d*e +
 5*(((d*x + 1)*(2*(d*x + 1)*(3*(d*x + 1)/d^3 - 13/d^3) + 43/d^3) - 39/d^3)*
sqrt(d*x + 1)*sqrt(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^3)*C*
d*e + 20*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)/d^2 - 7/d^2)
 + 9/d^2) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*B*f + 5*(((d*x + 1)*(2
*(d*x + 1)*(3*(d*x + 1)/d^3 - 13/d^3) + 43/d^3) - 39/d^3)*sqrt(d*x + 1)*sqr
t(-d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^3)*C*f + 60*(sqrt(d*x
+ 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*e +
120*(sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*
e + 20*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)/d^2 - 7/d^2) +
 9/d^2) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*C*e + 60*(sqrt(d*x + 1)*
(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A*f/d + 60*
(sqrt(d*x + 1)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1
)))*B*e/d)/d
```

maple [C] time = 0.01, size = 377, normalized size = 2.24

$$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(24\sqrt{-d^2x^2+1}\,C d^4 f x^4 \mathrm{csgn}(d)+30\sqrt{-d^2x^2+1}\,B d^4 f x^3 \mathrm{csgn}(d)+30\sqrt{-d^2x^2+1}\,C d^4 e x^3 \mathrm{csgn}(d)+40\sqrt{-d^2x^2+1}\,A d^4 f x^2 \mathrm{csgn}(d)+40\sqrt{-d^2x^2+1}\,B d^4 e x^2 \mathrm{csgn}(d)+60\sqrt{-d^2x^2+1}\,A d^4 e x\,\mathrm{csgn}(d)-8\sqrt{-d^2x^2+1}\,C d^2 f x^2 \mathrm{csgn}(d)+60 A d^3 e \arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)-15\sqrt{-d^2x^2+1}\,B d^2 f x\,\mathrm{csgn}(d)-15\sqrt{-d^2x^2+1}\,C d^2 e x\,\mathrm{csgn}(d)-40\sqrt{-d^2x^2+1}\,A d^2 f\,\mathrm{csgn}(d)-40\sqrt{-d^2x^2+1}\,B d^2 e\,\mathrm{csgn}(d)+15 B d f \arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+15 C d e \arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)-16\sqrt{-d^2x^2+1}\,C f\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{120\sqrt{-d^2x^2+1}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x)
```

```
[Out] 1/120*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(24*C*csgn(d)*x^4*d^4*f*(-d^2*x^2+1)^(1/
2)+30*B*csgn(d)*x^3*d^4*f*(-d^2*x^2+1)^(1/2)+30*C*csgn(d)*x^3*d^4*e*(-d^2*x
^2+1)^(1/2)+40*A*csgn(d)*x^2*d^4*f*(-d^2*x^2+1)^(1/2)+40*B*csgn(d)*x^2*d^4*
e*(-d^2*x^2+1)^(1/2)+60*A*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^4*e-8*C*csgn(d)*(-
d^2*x^2+1)^(1/2)*x^2*d^2*f-15*B*csgn(d)*(-d^2*x^2+1)^(1/2)*x*d^2*f-15*C*csg
n(d)*(-d^2*x^2+1)^(1/2)*x*d^2*e-40*A*csgn(d)*(-d^2*x^2+1)^(1/2)*d^2*f+60*A*
arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*d^3*e-40*B*csgn(d)*(-d^2*x^2+1)^(1
/2)*d^2*e+15*B*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*d*f-16*C*csgn(d)*(-
d^2*x^2+1)^(1/2)*f+15*C*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*d*e)*csgn(
d)/d^4/(-d^2*x^2+1)^(1/2)
```

maxima [A] time = 1.07, size = 174, normalized size = 1.04

$$\frac{1}{2}\sqrt{-d^2x^2+1}\,Aex-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Cfx^2}{5\,d^2}+\frac{Ae\arcsin\left(dx\right)}{2\,d}-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Be}{3\,d^2}-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Af}{3\,d^2}-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}\left(Ce+Bf\right)x}{4\,d^2}+\frac{\sqrt{-d^2x^2+1}\left(Ce+Bf\right)x}{8\,d^2}-\frac{2\left(-d^2x^2+1\right)^{\frac{3}{2}}Cf}{15\,d^4}+\frac{\left(Ce+Bf\right)\arcsin\left(dx\right)}{8\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/2*sqrt(-d^2*x^2 + 1)*A*e*x - 1/5*(-d^2*x^2 + 1)^(3/2)*C*f*x^2/d^2 + 1/2*A
*e*arcsin(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B*e/d^2 - 1/3*(-d^2*x^2 + 1)^(3
/2)*A*f/d^2 - 1/4*(-d^2*x^2 + 1)^(3/2)*(C*e + B*f)*x/d^2 + 1/8*sqrt(-d^2*x^
2 + 1)*(C*e + B*f)*x/d^2 - 2/15*(-d^2*x^2 + 1)^(3/2)*C*f/d^4 + 1/8*(C*e + B
*f)*arcsin(d*x)/d^3
```

mupad [B] time = 12.06, size = 736, normalized size = 4.38

$$\frac{\frac{Bf\left(\sqrt{1-dx}-1\right)}{2\left(\sqrt{dx+1}-1\right)}-\frac{35Bf\left(\sqrt{1-dx}-1\right)^3}{2\left(\sqrt{dx+1}-1\right)^3}+\frac{273Bf\left(\sqrt{1-dx}-1\right)^5}{2\left(\sqrt{dx+1}-1\right)^5}-\frac{715Bf\left(\sqrt{1-dx}-1\right)^7}{2\left(\sqrt{dx+1}-1\right)^7}+\frac{715Bf\left(\sqrt{1-dx}-1\right)^9}{2\left(\sqrt{dx+1}-1\right)^9}-\frac{273Bf\left(\sqrt{1-dx}-1\right)^{11}}{2\left(\sqrt{dx+1}-1\right)^{11}}+\frac{35Bf\left(\sqrt{1-dx}-1\right)^{13}}{2\left(\sqrt{dx+1}-1\right)^{13}}-\frac{Bf\left(\sqrt{1-dx}-1\right)^{15}}{2\left(\sqrt{dx+1}-1\right)^{15}}}{d^3\left(\frac{\left(\sqrt{1-dx}-1\right)^2}{\left(\sqrt{dx+1}-1\right)^2}+1\right)^8}-\sqrt{1-dx}\left(\frac{2Cf\sqrt{dx+1}}{15d^4}-\frac{Cfx^4\sqrt{dx+1}}{5}+\frac{Cfx^2\sqrt{dx+1}}{15d^2}\right)+\frac{\frac{Ce\left(\sqrt{1-dx}-1\right)}{2\left(\sqrt{dx+1}-1\right)}-\frac{35Ce\left(\sqrt{1-dx}-1\right)^3}{2\left(\sqrt{dx+1}-1\right)^3}+\frac{273Ce\left(\sqrt{1-dx}-1\right)^5}{2\left(\sqrt{dx+1}-1\right)^5}-\frac{715Ce\left(\sqrt{1-dx}-1\right)^7}{2\left(\sqrt{dx+1}-1\right)^7}+\frac{715Ce\left(\sqrt{1-dx}-1\right)^9}{2\left(\sqrt{dx+1}-1\right)^9}-\frac{273Ce\left(\sqrt{1-dx}-1\right)^{11}}{2\left(\sqrt{dx+1}-1\right)^{11}}+\frac{35Ce\left(\sqrt{1-dx}-1\right)^{13}}{2\left(\sqrt{dx+1}-1\right)^{13}}-\frac{Ce\left(\sqrt{1-dx}-1\right)^{15}}{2\left(\sqrt{dx+1}-1\right)^{15}}}{d^3\left(\frac{\left(\sqrt{1-dx}-1\right)^2}{\left(\sqrt{dx+1}-1\right)^2}+1\right)^8}-\frac{Bf\operatorname{atan}\left(\frac{\sqrt{1-dx}-1}{\sqrt{dx+1}-1}\right)}{2d^3}-\frac{Ce\operatorname{atan}\left(\frac{\sqrt{1-dx}-1}{\sqrt{dx+1}-1}\right)}{2d^3}+\frac{Aex\sqrt{1-dx}\sqrt{dx+1}}{2}-\frac{A\sqrt{d}\,e\ln\left(\sqrt{-d}\sqrt{1-dx}\sqrt{dx+1}-d^{3/2}x\right)}{2(-d)^{3/2}}+\frac{Af\left(d^2x^2-1\right)\sqrt{1-dx}\sqrt{dx+1}}{3d^2}+\frac{Be\left(d^2x^2-1\right)\sqrt{1-dx}\sqrt{dx+1}}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] ((B*f*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) - 1)) - (35*B*f*((1 - d*x)
^(1/2) - 1)^3)/(2*((d*x + 1)^(1/2) - 1)^3) + (273*B*f*((1 - d*x)^(1/2) - 1)
^5)/(2*((d*x + 1)^(1/2) - 1)^5) - (715*B*f*((1 - d*x)^(1/2) - 1)^7)/(2*((d*
x + 1)^(1/2) - 1)^7) + (715*B*f*((1 - d*x)^(1/2) - 1)^9)/(2*((d*x + 1)^(1/2
) - 1)^9) - (273*B*f*((1 - d*x)^(1/2) - 1)^11)/(2*((d*x + 1)^(1/2) - 1)^11)
 + (35*B*f*((1 - d*x)^(1/2) - 1)^13)/(2*((d*x + 1)^(1/2) - 1)^13) - (B*f*((
1 - d*x)^(1/2) - 1)^15)/(2*((d*x + 1)^(1/2) - 1)^15))/(d^3*(((1 - d*x)^(1/2
) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8) - (1 - d*x)^(1/2)*((2*C*f*(d*x + 1
)^(1/2))/(15*d^4) - (C*f*x^4*(d*x + 1)^(1/2))/5 + (C*f*x^2*(d*x + 1)^(1/2))
/(15*d^2)) + ((C*e*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) - 1)) - (35*C
*e*((1 - d*x)^(1/2) - 1)^3)/(2*((d*x + 1)^(1/2) - 1)^3) + (273*C*e*((1 - d*
x)^(1/2) - 1)^5)/(2*((d*x + 1)^(1/2) - 1)^5) - (715*C*e*((1 - d*x)^(1/2) -
1)^7)/(2*((d*x + 1)^(1/2) - 1)^7) + (715*C*e*((1 - d*x)^(1/2) - 1)^9)/(2*((
d*x + 1)^(1/2) - 1)^9) - (273*C*e*((1 - d*x)^(1/2) - 1)^11)/(2*((d*x + 1)^(
1/2) - 1)^11) + (35*C*e*((1 - d*x)^(1/2) - 1)^13)/(2*((d*x + 1)^(1/2) - 1)^
13) - (C*e*((1 - d*x)^(1/2) - 1)^15)/(2*((d*x + 1)^(1/2) - 1)^15))/(d^3*(((
1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8) - (B*f*atan(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) - (C*e*atan(((1 - d*x)^(1/2) -
 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) + (A*e*x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2
))/2 - (A*d^(1/2)*e*log((-d)^(1/2)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2) - d^(3/2
)*x))/(2*(-d)^(3/2)) + (A*f*(d^2*x^2 - 1)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/
(3*d^2) + (B*e*(d^2*x^2 - 1)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/(3*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.4 $\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=95

$$\frac{x\sqrt{1-d^2x^2}\left(4Ad^2+C\right)}{8d^2}+\frac{\left(4Ad^2+C\right)\sin^{-1}(dx)}{8d^3}-\frac{B\left(1-d^2x^2\right)^{3/2}}{3d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}$$

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {899, 1815, 641, 195, 216}

$$\frac{x\sqrt{1-d^2x^2}\left(4Ad^2+C\right)}{8d^2}+\frac{\left(4Ad^2+C\right)\sin^{-1}(dx)}{8d^3}-\frac{B\left(1-d^2x^2\right)^{3/2}}{3d^2}-\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((C + 4*A*d^2)*x*Sqrt[1 - d^2*x^2])/(8*d^2) - (B*(1 - d^2*x^2)^(3/2))/(3*d^
2) - (C*x*(1 - d^2*x^2)^(3/2))/(4*d^2) + ((C + 4*A*d^2)*ArcSin[d*x])/(8*d^3
)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{1-dx}\sqrt{1+dx}\left(A+Bx+Cx^2\right)dx = \int \left(A+Bx+Cx^2\right)\sqrt{1-d^2x^2}\,dx$$
$$= -\frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2} - \frac{\int \left(-C-4Ad^2-4Bd^2x\right)\sqrt{1-d^2x^2}\,dx}{4d^2}$$
$$= -\frac{B\left(1-d^2x^2\right)^{3/2}}{3d^2} - \frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2} - \frac{\left(-C-4Ad^2\right)\int \sqrt{1-d^2x^2}\,d}{4d^2}$$
$$= \frac{\left(C+4Ad^2\right)x\sqrt{1-d^2x^2}}{8d^2} - \frac{B\left(1-d^2x^2\right)^{3/2}}{3d^2} - \frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2} + \frac{(C}{}$$
$$= \frac{\left(C+4Ad^2\right)x\sqrt{1-d^2x^2}}{8d^2} - \frac{B\left(1-d^2x^2\right)^{3/2}}{3d^2} - \frac{Cx\left(1-d^2x^2\right)^{3/2}}{4d^2} + \frac{(C}{}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.75

$$\frac{d\sqrt{1-d^2x^2}\left(12Ad^2x+8Bd^2x^2-8B+6Cd^2x^3-3Cx\right)+3\left(4Ad^2+C\right)\sin^{-1}(dx)}{24d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2),x]
```

```
[Out] (d*Sqrt[1 - d^2*x^2]*(-8*B - 3*C*x + 12*A*d^2*x + 8*B*d^2*x^2 + 6*C*d^2*x^3
) + 3*(C + 4*A*d^2)*ArcSin[d*x])/(24*d^3)
```

IntegrateAlgebraic [B] time = 0.19, size = 242, normalized size = 2.55

$$\frac{\left(-4Ad^2-C\right)\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)}{4d^3} - \frac{\sqrt{1-dx}\left(\frac{12Ad^2(1-dx)^3}{(dx+1)^3}+\frac{12Ad^2(1-dx)^2}{(dx+1)^2}-\frac{12Ad^2(1-dx)}{dx+1}-12Ad^2+\frac{32Bd(1-dx)^2}{(dx+1)^2}+\frac{32Bd(1-dx)}{dx+1}+\frac{3C(1-dx)^3}{(dx+1)^3}-\frac{21C(1-dx)^2}{(dx+1)^2}+\frac{21C(1-dx)}{dx+1}-3C\right)}{12d^3\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - d*x]*Sqrt[1 + d*x]*(A + B*x + C*x^2),x]
```

```
[Out] -1/12*(Sqrt[1 - d*x]*(-3*C - 12*A*d^2 + (3*C*(1 - d*x)^3)/(1 + d*x)^3 + (12
*A*d^2*(1 - d*x)^3)/(1 + d*x)^3 - (21*C*(1 - d*x)^2)/(1 + d*x)^2 + (32*B*d*
(1 - d*x)^2)/(1 + d*x)^2 + (12*A*d^2*(1 - d*x)^2)/(1 + d*x)^2 + (21*C*(1 -
d*x))/(1 + d*x) + (32*B*d*(1 - d*x))/(1 + d*x) - (12*A*d^2*(1 - d*x))/(1 +
d*x)))/(d^3*Sqrt[1 + d*x]*(1 + (1 - d*x)/(1 + d*x))^4) + ((-C - 4*A*d^2)*Ar
cTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/(4*d^3)
```

fricas [A] time = 0.92, size = 95, normalized size = 1.00

$$\frac{\left(6\,Cd^3x^3+8\,Bd^3x^2-8\,Bd+3\left(4\,Ad^3-Cd\right)x\right)\sqrt{dx+1}\,\sqrt{-dx+1}-6\left(4\,Ad^2+C\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{24\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*((6*C*d^3*x^3 + 8*B*d^3*x^2 - 8*B*d + 3*(4*A*d^3 - C*d)*x)*sqrt(d*x +
1)*sqrt(-d*x + 1) - 6*(4*A*d^2 + C)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) -
1)/(d*x)))/d^3
```

giac [B] time = 1.54, size = 336, normalized size = 3.54

$$\frac{4\left(\sqrt{dx+1}\,\sqrt{-dx+1}\left((dx+1)\left(\frac{2\,(dx+1)}{d^2}-\frac{7}{d^2}\right)+\frac{9}{d^2}\right)+\frac{6\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^2}\right)Bd+\left(\left((dx+1)\left(2\,(dx+1)\left(\frac{3\,(dx+1)}{d^3}-\frac{13}{d^3}\right)+\frac{43}{d^3}\right)-\frac{39}{d^3}\right)\sqrt{dx+1}\,\sqrt{-dx+1}-\frac{18\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^3}\right)Cd+12\left(\sqrt{dx+1}\,(dx-2)\sqrt{-dx+1}-2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)\right)A+24\left(\sqrt{dx+1}\,\sqrt{-dx+1}+2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)\right)A+4\left(\sqrt{dx+1}\,\sqrt{-dx+1}\left((dx+1)\left(\frac{2\,(dx+1)}{d^2}-\frac{7}{d^2}\right)+\frac{9}{d^2}\right)+\frac{6\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^2}\right)C+\frac{12\left(\sqrt{dx+1}\,(dx-2)\sqrt{-dx+1}-2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)\right)B}{d}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(4*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)/d^2 - 7/d^2)
+ 9/d^2) + 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*B*d + (((d*x + 1)*(2*(d
*x + 1)*(3*(d*x + 1)/d^3 - 13/d^3) + 43/d^3) - 39/d^3)*sqrt(d*x + 1)*sqrt(-
d*x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^3)*C*d + 12*(sqrt(d*x + 1
)*(d*x - 2)*sqrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A + 24*(s
qrt(d*x + 1)*sqrt(-d*x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*A + 4*(s
qrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)/d^2 - 7/d^2) + 9/d^2) +
 6*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)*C + 12*(sqrt(d*x + 1)*(d*x - 2)*s
qrt(-d*x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)))*B/d)/d
```

maple [C] time = 0.01, size = 185, normalized size = 1.95

$$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(6\sqrt{-d^2x^2+1}\,C\,d^3x^3\operatorname{csgn}(d)+8\sqrt{-d^2x^2+1}\,B\,d^3x^2\operatorname{csgn}(d)+12\sqrt{-d^2x^2+1}\,A\,d^3x\operatorname{csgn}(d)+12A\,d^2\arctan\left(\frac{dx\operatorname{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)-3\sqrt{-d^2x^2+1}\,Cdx\operatorname{csgn}(d)-8\sqrt{-d^2x^2+1}\,Bd\operatorname{csgn}(d)+3C\arctan\left(\frac{dx\operatorname{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)\right)\operatorname{csgn}(d)}{24\sqrt{-d^2x^2+1}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x)
```

```
[Out] 1/24*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(6*C*csgn(d)*x^3*d^3*(-d^2*x^2+1)^(1/2)+8
*B*csgn(d)*x^2*d^3*(-d^2*x^2+1)^(1/2)+12*A*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x
-3*C*csgn(d)*d*(-d^2*x^2+1)^(1/2)*x+12*A*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*cs
gn(d))*d^2-8*B*(-d^2*x^2+1)^(1/2)*csgn(d)*d+3*C*arctan(1/(-d^2*x^2+1)^(1/2)
*d*x*csgn(d)))*csgn(d)/(-d^2*x^2+1)^(1/2)/d^3
```

maxima [A] time = 0.98, size = 93, normalized size = 0.98

$$\frac{1}{2}\sqrt{-d^2x^2+1}\,Ax-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}Cx}{4\,d^2}+\frac{A\arcsin\left(dx\right)}{2\,d}-\frac{\left(-d^2x^2+1\right)^{\frac{3}{2}}B}{3\,d^2}+\frac{\sqrt{-d^2x^2+1}\,Cx}{8\,d^2}+\frac{C\arcsin\left(dx\right)}{8\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(-d*x+1)^(1/2)*(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-d^2*x^2 + 1)*A*x - 1/4*(-d^2*x^2 + 1)^(3/2)*C*x/d^2 + 1/2*A*arcsi
n(d*x)/d - 1/3*(-d^2*x^2 + 1)^(3/2)*B/d^2 + 1/8*sqrt(-d^2*x^2 + 1)*C*x/d^2
+ 1/8*C*arcsin(d*x)/d^3
```

mupad [B] time = 7.21, size = 361, normalized size = 3.80

$$\frac{A\,x\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{2}-\frac{\frac{35\,C\,{\left(\sqrt{1-d\,x}-1\right)}^3}{2\,{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{273\,C\,{\left(\sqrt{1-d\,x}-1\right)}^5}{2\,{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{715\,C\,{\left(\sqrt{1-d\,x}-1\right)}^7}{2\,{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{715\,C\,{\left(\sqrt{1-d\,x}-1\right)}^9}{2\,{\left(\sqrt{d\,x+1}-1\right)}^9}+\frac{273\,C\,{\left(\sqrt{1-d\,x}-1\right)}^{11}}{2\,{\left(\sqrt{d\,x+1}-1\right)}^{11}}-\frac{35\,C\,{\left(\sqrt{1-d\,x}-1\right)}^{13}}{2\,{\left(\sqrt{d\,x+1}-1\right)}^{13}}+\frac{C\,{\left(\sqrt{1-d\,x}-1\right)}^{15}}{2\,{\left(\sqrt{d\,x+1}-1\right)}^{15}}-\frac{C\,\left(\sqrt{1-d\,x}-1\right)}{2\,\left(\sqrt{d\,x+1}-1\right)}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^8}-\frac{C\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2\,d^3}-\frac{A\,\sqrt{d}\,\ln\left(\sqrt{-d}\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}-d^{3/2}\,x\right)}{2\,{\left(-d\right)}^{3/2}}+\frac{B\,\left(d^2\,x^2-1\right)\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - d*x)^(1/2)*(d*x + 1)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] (A*x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/2 - ((35*C*((1 - d*x)^(1/2) - 1)^3)/(
2*((d*x + 1)^(1/2) - 1)^3) - (273*C*((1 - d*x)^(1/2) - 1)^5)/(2*((d*x + 1)^
(1/2) - 1)^5) + (715*C*((1 - d*x)^(1/2) - 1)^7)/(2*((d*x + 1)^(1/2) - 1)^7)
 - (715*C*((1 - d*x)^(1/2) - 1)^9)/(2*((d*x + 1)^(1/2) - 1)^9) + (273*C*((1
 - d*x)^(1/2) - 1)^11)/(2*((d*x + 1)^(1/2) - 1)^11) - (35*C*((1 - d*x)^(1/2
) - 1)^13)/(2*((d*x + 1)^(1/2) - 1)^13) + (C*((1 - d*x)^(1/2) - 1)^15)/(2*(
(d*x + 1)^(1/2) - 1)^15) - (C*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) -
1)))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^8) - (C*ata
n(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/(2*d^3) - (A*d^(1/2)*log((-
d)^(1/2)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2) - d^(3/2)*x))/(2*(-d)^(3/2)) + (B*
(d^2*x^2 - 1)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/(3*d^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(-d*x+1)**(1/2)*(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.5 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$

Optimal. Leaf size=122

$$\frac{\left(Af^2-Bef+Ce^2\right)\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}-\frac{\sin^{-1}(dx)(Ce-Bf)}{df^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

Rubi [A] time = 0.31, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1609, 1654, 844, 216, 725, 204}

$$\frac{\left(Af^2-Bef+Ce^2\right)\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}-\frac{\sin^{-1}(dx)(Ce-Bf)}{df^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] -((C*Sqrt[1 - d^2*x^2])/(d^2*f)) - ((C*e - B*f)*ArcSin[d*x])/(d*f^2) + ((C*
e^2 - B*e*f + A*f^2)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2
*x^2])])/(f^2*Sqrt[d^2*e^2 - f^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
```

```
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}\,(e+fx)}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{\int \frac{-Ad^2f^2+d^2f(Ce-Bf)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2f^2}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{\left(Ce^2-Bef+Af^2\right)\int \frac{1}{(e+fx)\sqrt{1-d\ldots}}}{f^2}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} - \frac{\left(Ce^2-Bef+Af^2\right)\text{Subst}\left(\int \frac{1}{-d^2e^2+\ldots}\right.}{f^2}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} + \frac{\left(Ce^2-Bef+Af^2\right)\tan^{-1}\left(\frac{f+d\ldots}{\sqrt{d^2e^2-f^2}\ldots}\right.}{f^2\sqrt{d^2e^2-f^2}}$$

Mathematica [A] time = 0.15, size = 117, normalized size = 0.96

$$\frac{\frac{\left(f(Af-Be)+Ce^2\right)\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{\sqrt{d^2e^2-f^2}} + \frac{\sin^{-1}(dx)(Bf-Ce)}{d} - \frac{Cf\sqrt{1-d^2x^2}}{d^2}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] (-((C*f*Sqrt[1 - d^2*x^2])/d^2) + ((-(C*e) + B*f)*ArcSin[d*x])/d + ((C*e^2
+ f*(-(B*e) + A*f))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*
x^2])])/Sqrt[d^2*e^2 - f^2])/f^2
```

IntegrateAlgebraic [A] time = 0.58, size = 177, normalized size = 1.45

$$-\frac{2\left(Af^2-Bef+Ce^2\right)\tan^{-1}\left(\frac{\sqrt{1-dx}\sqrt{-de-f}\sqrt{f-de}}{\sqrt{dx+1}\,(de+f)}\right)}{f^2\sqrt{-de-f}\sqrt{f-de}} - \frac{2\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)(Bf-Ce)}{df^2} - \frac{2C\sqrt{1-dx}}{d^2f\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)
),x]
```

```
[Out] (-2*C*Sqrt[1 - d*x])/(d^2*f*Sqrt[1 + d*x]*(1 + (1 - d*x)/(1 + d*x))) - (2*(
-(C*e) + B*f)*ArcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/(d*f^2) - (2*(C*e^2 - B*
e*f + A*f^2)*ArcTan[(Sqrt[-(d*e) - f]*Sqrt[-(d*e) + f]*Sqrt[1 - d*x])/((d*e
 + f)*Sqrt[1 + d*x])])/(Sqrt[-(d*e) - f]*f^2*Sqrt[-(d*e) + f])
```

fricas [B] time = 15.66, size = 493, normalized size = 4.04

$$\left[-\frac{\left(Cd^2e^2 - Bd^2ef + Ad^2f^2\right)\sqrt{-d^2e^2 + f^2}\log\left(\frac{d^2efx+f^2-\sqrt{-d^2e^2+f^2}\left(d^2ex+f\right)-\left(\sqrt{-d^2e^2+f^2}\sqrt{-dx+1}f+\left(d^2e^2-f^2\right)\sqrt{-dx+1}\right)\sqrt{dx+1}}{fx+e}\right) + \left(Cd^2e^2f - Cf^3\right)\sqrt{dx+1}\sqrt{-dx+1} - 2\left(Cd^3e^3 - Bd^3e^2f - Cdef^2 + Bdf^3\right)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{d^4e^2f^2 - d^2f^4}, \frac{2\left(Cd^2e^2 - Bd^2ef + Ad^2f^2\right)\sqrt{d^2e^2 - f^2}\arctan\left(-\frac{\sqrt{d^2e^2-f^2}\sqrt{dx+1}\sqrt{-dx+1}e-\sqrt{d^2e^2-f^2}\left(fx+e\right)}{\left(d^2e^2-f^2\right)x}\right) - \left(Cd^2e^2f - Cf^3\right)\sqrt{dx+1}\sqrt{-dx+1} + 2\left(Cd^3e^3 - Bd^3e^2f - Cdef^2 + Bdf^3\right)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{d^4e^2f^2 - d^2f^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] [-((C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
+ f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (sqrt(-d^2*e^2 + f^2)*sqrt(-d*
x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (C*d
^2*e^2*f - C*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(C*d^3*e^3 - B*d^3*e^2*f
 - C*d*e*f^2 + B*d*f^3)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(
d^4*e^2*f^2 - d^2*f^4), (2*(C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(d^2*e^2
 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(
d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) - (C*d^2*e^2*f - C*f^3)*sqrt
(d*x + 1)*sqrt(-d*x + 1) + 2*(C*d^3*e^3 - B*d^3*e^2*f - C*d*e*f^2 + B*d*f^3
)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^4*e^2*f^2 - d^2*f^4)
]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Undef/Unsigned Inf encountered in limit
```

maple [C] time = 0.05, size = 373, normalized size = 3.06

$$\frac{\left(-A\,d^2f^2\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)+B\,d^2ef\,\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)-C\,d^2e^2\mathrm{csgn}(d)\ln\left(\frac{2d^2ex+2\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,f+2f}{fx+e}\right)+\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,Bd\,f^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)-\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,Cdef\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)-\sqrt{-d^2x^2+1}\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,C\,f^2\mathrm{csgn}(d)\right)\sqrt{-dx+1}\,\sqrt{dx+1}\,\mathrm{csgn}(d)}{\sqrt{-\frac{d^2e^2-f^2}{f^2}}\,\sqrt{-d^2x^2+1}\,d^2f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f
)/(f*x+e))*d^2*f^2+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^
2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e*f-C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1
/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^2+B*arctan(1/(-d^2*x^2+1
)^(1/2)*d*x*csgn(d))*d*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*csgn(d)*f^2*(-d^2*x
^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*cs
gn(d))*d*e*f*(-(d^2*e^2-f^2)/f^2)^(1/2))*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(
d)/(-(d^2*e^2-f^2)/f^2)^(1/2)/f^3/(-d^2*x^2+1)^(1/2)/d^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(f-d*e>0)', see `assume?` for more d
etails)Is f-d*e positive, negative or zero?
```

mupad [B] time = 25.80, size = 5803, normalized size = 47.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (4*C*e*atan((37748736*C^5*d^4*e^10*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
 - 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8
*f^2)) + (67108864*C^5*e^6*f^4*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1
)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8*f^2
)) - (100663296*C^5*d^2*e^8*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) -
1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8*f^
2))))/(d*f^2) - (4*B*atan((67108864*B^5*e*f^4*((1 - d*x)^(1/2) - 1))/(((d*x
 + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5
*d^2*e^3*f^2)) + (37748736*B^5*d^4*e^5*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(
1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5*d^2*e^
3*f^2)) - (100663296*B^5*d^2*e^3*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/
2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5*d^2*e^3*
f^2))))/(d*f) - (8*C*((1 - d*x)^(1/2) - 1)^2)/(f*((d*x + 1)^(1/2) - 1)^2*(d
^2 + (2*d^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (d^2*((1 - d
*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4)) - (A*atan((f^2*1i - d^2*e^2*1i
 - (f^2*((1 - d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2 + (d^2*e^2*((1 -
 d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2)/(f*(f + d*e)^(1/2)*(f - d*e
)^(1/2) - (f*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))/((d*x
 + 1)^(1/2) - 1)^2 + (2*d*e*((1 - d*x)^(1/2) - 1)*(f + d*e)^(1/2)*(f - d*e)
^(1/2))/((d*x + 1)^(1/2) - 1)))*2i)/((f + d*e)^(1/2)*(f - d*e)^(1/2)) - (C*
e^2*atan(((C*e^2*((4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (409
6*((1 - d*x)^(1/2) - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x
 + 1)^(1/2) - 1)^2) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1
)^(1/2) - 1)) + (C*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4)
 + (16384*((1 - d*x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d
*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 -
144*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) - (C*
e^2*((4096*(24*C*d^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*
x)^(1/2) - 1)*(20*C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)
) + (4096*(96*C*d^2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d
*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^
6))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))
/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^
8 - 9*d^6*e^5*f^6))/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*
(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(
1/2)*(f - d*e)^(1/2)))*1i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)) + (C*e^2*(
(4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d*x)^(1/2)
 - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/2) - 1)^2
) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) - 1)) - (C
*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384*((1 - d*
x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(1/2) - 1)
) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e^3*f^6 +
9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(24*C*d
^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(20*
C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*(96*C*d^
2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*x + 1)^(1
/2) - 1)^2) - (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4) + (163
84*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d*x + 1)^(
1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6*e^5*f^6)
)/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))
```

```
/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/
2)))*1i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)))/((131072*C^4*e^7)/(d*f^4) +
 (C*e^2*((4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d
*x)^(1/2) - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/
2) - 1)^2) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) -
 1)) + (C*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384
*((1 - d*x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(
1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e
^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) - (C*e^2*((409
6*(24*C*d^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2)
- 1)*(20*C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096
*(96*C*d^2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*
x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^
4) + (16384*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d
*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6
*e^5*f^6))/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*(f - d*e)
^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f -
 d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)) - (C*e^2*((4096*(32*C^
3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d*x)^(1/2) - 1)^2*(32*
C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (458752*
C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) - 1)) - (C*e^2*((4096*
(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1
)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*((
1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e^3*f^6 + 9*C^2*d^4*e^
7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(24*C*d^2*e^3*f^7 -
 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(20*C*e^2*f^6 -
22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*(96*C*d^2*e^3*f^7 -
90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*x + 1)^(1/2) - 1)^2)
- (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4) + (16384*((1 - d*x
)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d*x + 1)^(1/2) - 1)) +
 (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4*((d
*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d
*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(
f + d*e)^(1/2)*(f - d*e)^(1/2)) + (917504*C^4*e^7*((1 - d*x)^(1/2) - 1)^2)/
(d*f^4*((d*x + 1)^(1/2) - 1)^2)))*2i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))
 + (B*e*atan(((B*e*((4096*(24*B^3*d^2*e^4 + 32*B^3*e^2*f^2))/d + (4096*((1
- d*x)^(1/2) - 1)^2*(96*B^3*d^2*e^4 - 32*B^3*e^2*f^2))/(d*((d*x + 1)^(1/2)
- 1)^2) + (458752*B^3*e^3*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) +
(B*e*((4096*(16*B^2*e*f^4 + 9*B^2*d^4*e^5))/d + (((1 - d*x)^(1/2) - 1)*(131
072*B^2*e^2*f^3 + 49152*B^2*d^2*e^4*f))/((d*x + 1)^(1/2) - 1) + (4096*((1 -
 d*x)^(1/2) - 1)^2*(9*B^2*d^4*e^5 - 144*B^2*e*f^4 + 128*B^2*d^2*e^3*f^2))/(
d*((d*x + 1)^(1/2) - 1)^2) - (B*e*((4096*(24*B*d^2*e^2*f^4 - 30*B*d^4*e^4*f
^2))/d + ((327680*B*e*f^5 - 360448*B*d^2*e^3*f^3)*((1 - d*x)^(1/2) - 1))/((
d*x + 1)^(1/2) - 1) + (4096*(96*B*d^2*e^2*f^4 - 90*B*d^4*e^4*f^2)*((1 - d*x
)^(1/2) - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) + (B*e*((4096*(7*d^4*e^3*f^4 -
9*d^6*e^5*f^2))/d + (((1 - d*x)^(1/2) - 1)*(81920*d^2*e^2*f^5 - 98304*d^4*e
^4*f^3))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*
f^4 - 9*d^6*e^5*f^2))/(d*((d*x + 1)^(1/2) - 1)^2)))/(f*(f + d*e)^(1/2)*(f -
 d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f
- d*e)^(1/2)))*1i)/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2)) + (B*e*((4096*(24*B^
3*d^2*e^4 + 32*B^3*e^2*f^2))/d + (4096*((1 - d*x)^(1/2) - 1)^2*(96*B^3*d^2*
e^4 - 32*B^3*e^2*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) + (458752*B^3*e^3*f*((1
- d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (B*e*((4096*(16*B^2*e*f^4 + 9*B^
2*d^4*e^5))/d + (((1 - d*x)^(1/2) - 1)*(131072*B^2*e^2*f^3 + 49152*B^2*d^2*
e^4*f))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(9*B^2*d^4*e^
5 - 144*B^2*e*f^4 + 128*B^2*d^2*e^3*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) + (B*
e*((4096*(24*B*d^2*e^2*f^4 - 30*B*d^4*e^4*f^2))/d + ((327680*B*e*f^5 - 3604
48*B*d^2*e^3*f^3)*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (4096*(96*
B*d^2*e^2*f^4 - 90*B*d^4*e^4*f^2)*((1 - d*x)^(1/2) - 1)^2)/(d*((d*x + 1)^(1
```

```
/2) - 1)^2) - (B*e*((4096*(7*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/d + (((1 - d*x)^
(1/2) - 1)*(81920*d^2*e^2*f^5 - 98304*d^4*e^4*f^3))/((d*x + 1)^(1/2) - 1) +
 (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/(d*((d*x +
 1)^(1/2) - 1)^2)))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2
)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2)))*1i)/(f*(f + d*e)^
(1/2)*(f - d*e)^(1/2)))/((131072*B^4*e^3)/d + (917504*B^4*e^3*((1 - d*x)^(1
/2) - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) + (B*e*((4096*(24*B^3*d^2*e^4 + 32*
B^3*e^2*f^2))/d + (4096*((1 - d*x)^(1/2) - 1)^2*(96*B^3*d^2*e^4 - 32*B^3*e^
2*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) + (458752*B^3*e^3*f*((1 - d*x)^(1/2) -
1))/((d*x + 1)^(1/2) - 1) + (B*e*((4096*(16*B^2*e*f^4 + 9*B^2*d^4*e^5))/d +
 (((1 - d*x)^(1/2) - 1)*(131072*B^2*e^2*f^3 + 49152*B^2*d^2*e^4*f))/((d*x +
 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(9*B^2*d^4*e^5 - 144*B^2*e*f
^4 + 128*B^2*d^2*e^3*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) - (B*e*((4096*(24*B*
d^2*e^2*f^4 - 30*B*d^4*e^4*f^2))/d + ((327680*B*e*f^5 - 360448*B*d^2*e^3*f^
3)*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (4096*(96*B*d^2*e^2*f^4 -
 90*B*d^4*e^4*f^2)*((1 - d*x)^(1/2) - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) + (
B*e*((4096*(7*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/d + (((1 - d*x)^(1/2) - 1)*(819
20*d^2*e^2*f^5 - 98304*d^4*e^4*f^3))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*
x)^(1/2) - 1)^2*(11*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/(d*((d*x + 1)^(1/2) - 1)^
2)))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/
2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1
/2)) - (B*e*((4096*(24*B^3*d^2*e^4 + 32*B^3*e^2*f^2))/d + (4096*((1 - d*x)^
(1/2) - 1)^2*(96*B^3*d^2*e^4 - 32*B^3*e^2*f^2))/(d*((d*x + 1)^(1/2) - 1)^2)
 + (458752*B^3*e^3*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (B*e*((
4096*(16*B^2*e*f^4 + 9*B^2*d^4*e^5))/d + (((1 - d*x)^(1/2) - 1)*(131072*B^2
*e^2*f^3 + 49152*B^2*d^2*e^4*f))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(
1/2) - 1)^2*(9*B^2*d^4*e^5 - 144*B^2*e*f^4 + 128*B^2*d^2*e^3*f^2))/(d*((d*x
 + 1)^(1/2) - 1)^2) + (B*e*((4096*(24*B*d^2*e^2*f^4 - 30*B*d^4*e^4*f^2))/d
+ ((327680*B*e*f^5 - 360448*B*d^2*e^3*f^3)*((1 - d*x)^(1/2) - 1))/((d*x + 1
)^(1/2) - 1) + (4096*(96*B*d^2*e^2*f^4 - 90*B*d^4*e^4*f^2)*((1 - d*x)^(1/2)
 - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) - (B*e*((4096*(7*d^4*e^3*f^4 - 9*d^6*e
^5*f^2))/d + (((1 - d*x)^(1/2) - 1)*(81920*d^2*e^2*f^5 - 98304*d^4*e^4*f^3)
)/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^4 - 9
*d^6*e^5*f^2))/(d*((d*x + 1)^(1/2) - 1)^2)))/(f*(f + d*e)^(1/2)*(f - d*e)^(
1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^
(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))*2i)/(f*(f + d*e)^(1/2)*(f -
d*e)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
[Out] Timed out
```

3.6 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx$

Optimal. Leaf size=163

$$\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}-\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{f^2\left(d^2e^2-f^2\right)^{3/2}}+\frac{C\sin^{-1}(dx)}{df^2}$$

Rubi [A] time = 0.33, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1609, 1651, 844, 216, 725, 204}

$$\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}-\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{f^2\left(d^2e^2-f^2\right)^{3/2}}+\frac{C\sin^{-1}(dx)}{df^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x)) +
 (C*ArcSin[d*x])/(d*f^2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*A
rcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x^2])])/(f^2*(d^2*e^2
 - f^2)^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{\int \frac{Ce+Ad^2e-Bf+C\left(\frac{d^2e^2}{f}-f\right)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2e^2-f^2}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{C\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2} -\right.}{d^2e^2}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} - \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right.}{}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} + \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right.}{\left(d^2e^2-\right.}$$

Mathematica [A] time = 0.47, size = 211, normalized size = 1.29

$$\frac{-\frac{f\sqrt{1-d^2x^2}\left(f(Af-Be)+Ce^2\right)}{\left(f^2-d^2e^2\right)(e+fx)} - \frac{\log\left(\sqrt{1-d^2x^2}\sqrt{f^2-d^2e^2}+d^2ex+f\right)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{\left(f^2-d^2e^2\right)^{3/2}} + \frac{\log(e+fx)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{\left(f^2-d^2e^2\right)^{3/2}} + \frac{C\sin^{-1}(dx)}{d}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

```
[Out] (-((f*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[1 - d^2*x^2])/((-(d^2*e^2) + f^2)*(e
+ f*x))) + (C*ArcSin[d*x])/d + ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^
3)*Log[e + f*x])/(-(d^2*e^2) + f^2)^(3/2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2
*e*f^2 + B*f^3)*Log[f + d^2*e*x + Sqrt[-(d^2*e^2) + f^2]*Sqrt[1 - d^2*x^2]]
)/(-(d^2*e^2) + f^2)^(3/2))/f^2
```

IntegrateAlgebraic [A] time = 1.49, size = 235, normalized size = 1.44

$$\frac{2\tan^{-1}\left(\frac{\sqrt{1-dx}\sqrt{-de-f}\sqrt{f-de}}{\sqrt{dx+1}(de+f)}\right)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{f^2(-de-f)^{3/2}(f-de)^{3/2}} + \frac{2d\sqrt{1-dx}\left(Af^2-Bef+Ce^2\right)}{f\sqrt{dx+1}(de-f)(de+f)\left(\frac{de(1-dx)}{dx+1}+de-\frac{f(1-dx)}{dx+1}+f\right)} - \frac{2C\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)}{df^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)
^2),x]
```

```
[Out] (2*d*(C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d*x])/((d*e - f)*f*(d*e + f)*Sqrt[1 +
 d*x]*(d*e + f + (d*e*(1 - d*x))/(1 + d*x) - (f*(1 - d*x))/(1 + d*x))) - (2
```

```
*C*ArcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/(d*f^2) + (2*(C*d^2*e^3 - 2*C*e*f^2
 - A*d^2*e*f^2 + B*f^3)*ArcTan[(Sqrt[-(d*e) - f]*Sqrt[-(d*e) + f]*Sqrt[1 -
d*x])/((d*e + f)*Sqrt[1 + d*x])])/((-(d*e) - f)^(3/2)*f^2*(-(d*e) + f)^(3/2
))
```

fricas [B] time = 72.53, size = 1025, normalized size = 6.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [(C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3
*f^3 - (C*d^3*e^5 + B*d*e^2*f^3 - (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f +
B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*f^3)*x)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
 + f^2 + sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) + (sqrt(-d^2*e^2 + f^2)*sqrt(-d
*x + 1)*f - (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (C*
d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3
)*sqrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*d*e*f^5
 - A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*f^2 + C*
e^2*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqrt(d*x + 1
)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^2*f^6 + (d
^5*e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x), (C*d^3*e^5*f - B*d^3*e^4*f^2 + B*
d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3 - 2*(C*d^3*e^5 + B*d*e^2*f^3
- (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f + B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*
f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt
(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) + (C*d^3
*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3)*s
qrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*d*e*f^5 -
A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*f^2 + C*e^2
*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqrt(d*x + 1)*s
qrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^2*f^6 + (d^5*
e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Undef/Unsigned Inf encountered in limit
```

maple [C] time = 0.04, size = 899, normalized size = 5.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f
)/(f*x+e))*x*d^3*e*f^3+C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^
2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*x*d^3*e^3*f-A*csgn(d)*ln(2*(d^2*e*x+(-d^2*x
^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^3*e^2*f^2+C*csgn(d)*
ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d
^3*e^4+C*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x*d^2*e^2*f^2*(-(d^2*e^2-
```

```
f^2)/f^2)^(1/2)+A*csgn(d)*d*f^4*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/
2)+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+
f)/(f*x+e))*x*d*f^4-B*csgn(d)*d*e*f^3*(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^
2)^(1/2)-2*C*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^
(1/2)*f+f)/(f*x+e))*x*d*e*f^3+C*csgn(d)*d*e^2*f^2*(-d^2*x^2+1)^(1/2)*(-(d^2
*e^2-f^2)/f^2)^(1/2)+C*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*d^2*e^3*f*(
-(d^2*e^2-f^2)/f^2)^(1/2)+B*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2
*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e*f^3-2*C*csgn(d)*ln(2*(d^2*e*x+(-d^2*
x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d*e^2*f^2-C*arctan(1/
(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x*f^4*(-(d^2*e^2-f^2)/f^2)^(1/2)-C*arctan(1
/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2))*csgn(d)*
(d*x+1)^(1/2)*(-d*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e+f)/d/(d*e-f)/(f*x+e)/(
-(d^2*e^2-f^2)/f^2)^(1/2)/f^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(f-d*e>0)', see `assume?` for more d
etails)Is f-d*e positive, negative or zero?
```

mupad [B] time = 52.17, size = 10198, normalized size = 62.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2
*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2)
 - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2))*2i - A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*
1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1
)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)
^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (
2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d
*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i + (4*A*f^2*((1 - d*x)^(1/2)
- 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (A*d^5*e^5*at
an(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)
^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3
*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x
+ 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 + (A*d^5*e^5*atan(
((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3
/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((
1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2
) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1
)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2)
)*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 - (4*A*f^2*((1 - d*x)
^(1/2) - 1)^3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 - (A
*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) -
 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d
```

```
^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e
^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/
2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
+ 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 + (
A*d^2*e^2*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
 + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 -
(A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
 - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
 d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3
*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*
x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 +
 (A*d^4*e^4*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
 + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) - 1) - (A*d
^2*e^2*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2
*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2)
 - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) - 1) - (A*d^4*
e^4*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(
f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*
f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1
- d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1)
)/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1
/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 + (8*A*d*e
*f*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2
) - 1)^2)/(d^3*e^4*(f + d*e)^(3/2)*(f - d*e)^(3/2) - d*e^2*f^2*(f + d*e)^(3
/2)*(f - d*e)^(3/2) - (4*e*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d
*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (4*e*f^3*((1 - d*x)^(1/2) - 1)^3*(f + d*
e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 + (2*d^3*e^4*((1 - d*x)^(
1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (d^3
*e^4*((1 - d*x)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1
/2) - 1)^4 - (2*d*e^2*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)
^(3/2))/((d*x + 1)^(1/2) - 1)^2 - (4*d^2*e^3*f*((1 - d*x)^(1/2) - 1)^3*(f +
 d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 - (d*e^2*f^2*((1 - d*x
)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^4 + (
4*d^2*e^3*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x +
1)^(1/2) - 1)) - (B*d^3*e^3*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - ((
(1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2)
 - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2)
- 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f
^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/
2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i - (B*f^4*atan(((f + d*e)^(3/2)*(f -
 d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1
i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2
)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1
/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e
^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) -
1)*8i)/((d*x + 1)^(1/2) - 1) + (B*f^4*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)
*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x +
```

```
1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1
)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) +
(2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 -
d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((
d*x + 1)^(1/2) - 1)^3 - B*d*e*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i
- (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(
1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1
/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d
*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)
^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i - (4*B*f*((1 - d*x)^(1/2) - 1)^
3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 + (4*B*f*((1 - d
*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (B*
d^2*e^2*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) -
1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^
2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^
3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2
) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 - (B
*d*e*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^
2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e
^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*(
(1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) -
 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)
^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 - (B*d*
e*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(
f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*
f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1
- d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1)
)/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1
/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 + (8*B*d*e
*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2)
- 1)^2 + (B*d^2*e^2*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d
*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^
2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2
 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1
 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1
)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) -
 1) + (B*d^3*e^3*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(
1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f
^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2
*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*
x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/
((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)
^2 + (B*d^3*e^3*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1
/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^
3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*
d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x
)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/(
(d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^
4)/(d^3*e^3*(f + d*e)^(3/2)*(f - d*e)^(3/2) + (4*f^3*((1 - d*x)^(1/2) - 1)^
3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 - d*e*f^2*(f + d
*e)^(3/2)*(f - d*e)^(3/2) - (4*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f
 - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (2*d^3*e^3*((1 - d*x)^(1/2) - 1)^2*(
f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (d^3*e^3*((1 - d*
x)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^4 -
(4*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x
 + 1)^(1/2) - 1)^3 + (4*d^2*e^2*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f
- d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (2*d*e*f^2*((1 - d*x)^(1/2) - 1)^2*(f
 + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 - (d*e*f^2*((1 - d*x
```

```
)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^4) -
((4*C*d*e*((1 - d*x)^(1/2) - 1))/((f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)) -
(4*C*d*e*((1 - d*x)^(1/2) - 1)^3)/((f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)^3)
 + (8*C*d^2*e^2*((1 - d*x)^(1/2) - 1)^2)/(f*(f^2 - d^2*e^2)*((d*x + 1)^(1/2
) - 1)^2))/(d^2*e + (4*d*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (
4*d*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (2*d^2*e*((1 - d*x
)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (d^2*e*((1 - d*x)^(1/2) - 1)^4)/(
(d*x + 1)^(1/2) - 1)^4) + (4*C*atan((((((1 - d*x)^(1/2) - 1)*((2097152*(288
*e^3*f^11 - 6*d^10*e^13*f - 912*d^2*e^5*f^9 + 1048*d^4*e^7*f^7 - 532*d^6*e^
9*f^5 + 112*d^8*e^11*f^3))/(d*f^2*(d*f^13 - 4*d^3*e^2*f^11 + 6*d^5*e^4*f^9
- 4*d^7*e^6*f^7 + d^9*e^8*f^5)) - (33554432*(20*d^2*e*f^21 - 103*d^4*e^3*f^
19 + 215*d^6*e^5*f^17 - 230*d^8*e^7*f^15 + 130*d^10*e^9*f^13 - 35*d^12*e^11
*f^11 + 3*d^14*e^13*f^9))/(d^5*f^10*(d*f^13 - 4*d^3*e^2*f^11 + 6*d^5*e^4*f^
9 - 4*d^7*e^6*f^7 + d^9*e^8*f^5)) + (8388608*(72*e*f^17 - 452*d^2*e^3*f^15
+ 1024*d^4*e^5*f^13 - 1106*d^6*e^7*f^11 + 597*d^8*e^9*f^9 - 144*d^10*e^11*f
^7 + 9*d^12*e^13*f^5))/(d^3*f^6*(d*f^13 - 4*d^3*e^2*f^11 + 6*d^5*e^4*f^9 -
4*d^7*e^6*f^7 + d^9*e^8*f^5))))/((d*x + 1)^(1/2) - 1) - (33554432*(7*d^2*e^
2*f^19 - 35*d^4*e^4*f^17 + 70*d^6*e^6*f^15 - 70*d^8*e^8*f^13 + 35*d^10*e^10
*f^11 - 7*d^12*e^12*f^9))/(d^5*f^10*(f^12 - 4*d^2*e^2*f^10 + 6*d^4*e^4*f^8
- 4*d^6*e^6*f^6 + d^8*e^8*f^4)) + (2097152*(112*e^4*f^9 + 28*d^8*e^12*f - 3
36*d^2*e^6*f^7 + 364*d^4*e^8*f^5 - 168*d^6*e^10*f^3))/(d*f^2*(f^12 - 4*d^2*
e^2*f^10 + 6*d^4*e^4*f^8 - 4*d^6*e^6*f^6 + d^8*e^8*f^4)) + (8388608*(28*e^2
*f^15 - 168*d^2*e^4*f^13 + 364*d^4*e^6*f^11 - 371*d^6*e^8*f^9 + 182*d^8*e^1
0*f^7 - 35*d^10*e^12*f^5))/(d^3*f^6*(f^12 - 4*d^2*e^2*f^10 + 6*d^4*e^4*f^8
- 4*d^6*e^6*f^6 + d^8*e^8*f^4)))*(d^4*f^14 - 4*d^6*e^2*f^12 + 6*d^8*e^4*f^1
0 - 4*d^10*e^6*f^8 + d^12*e^8*f^6))/(67108864*e*f^12 + 37748736*d^12*e^13 -
 268435456*d^2*e^3*f^10 + 536870912*d^4*e^5*f^8 - 637534208*d^6*e^7*f^6 + 4
69762048*d^8*e^9*f^4 - 201326592*d^10*e^11*f^2)))/(d*f^2) + (log(16*f^15 -
9*d^14*e^14*f - (16*f^15*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 -
 92*d^2*e^2*f^13 + 236*d^4*e^4*f^11 - 352*d^6*e^6*f^9 + 329*d^8*e^8*f^7 - 1
91*d^10*e^10*f^5 + 63*d^12*e^12*f^3 + 16*f^6*(f + d*e)^(9/2)*(f - d*e)^(9/2
) + 12*d^6*e^6*(f + d*e)^(9/2)*(f - d*e)^(9/2) + 15*d^12*e^12*(f + d*e)^(3/
2)*(f - d*e)^(3/2) - (6*d^15*e^15*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
 1) + (16*d*e*f^14*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (92*d^2*e
^2*f^13*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (236*d^4*e^4*f^1
1*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (352*d^6*e^6*f^9*((1 -
 d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (329*d^8*e^8*f^7*((1 - d*x)^(
1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (191*d^10*e^10*f^5*((1 - d*x)^(1/2)
- 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (63*d^12*e^12*f^3*((1 - d*x)^(1/2) - 1)^2
)/((d*x + 1)^(1/2) - 1)^2 - (16*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)
*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 - 24*d^2*e^2*f^10*(f + d*e)^(3/2)
*(f - d*e)^(3/2) + 120*d^4*e^4*f^8*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 228*d^
6*e^6*f^6*(f + d*e)^(3/2)*(f - d*e)^(3/2) + 4*d^2*e^2*f^4*(f + d*e)^(9/2)*(
f - d*e)^(9/2) + 207*d^8*e^8*f^4*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 28*d^4*e
^4*f^2*(f + d*e)^(9/2)*(f - d*e)^(9/2) - 90*d^10*e^10*f^2*(f + d*e)^(3/2)*(
f - d*e)^(3/2) - (88*d^3*e^3*f^12*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
 1) + (216*d^5*e^5*f^10*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (308
*d^7*e^7*f^8*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (274*d^9*e^9*f^
6*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (150*d^11*e^11*f^4*((1 - d
*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (46*d^13*e^13*f^2*((1 - d*x)^(1/2)
- 1))/((d*x + 1)^(1/2) - 1) + (9*d^14*e^14*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
 + 1)^(1/2) - 1)^2 + (48*d^6*e^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f
 - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 + (45*d^12*e^12*((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (376*d^3*e^3
*f^9*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2
) - 1) - (688*d^5*e^5*f^7*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(
3/2))/((d*x + 1)^(1/2) - 1) + (612*d^7*e^7*f^5*((1 - d*x)^(1/2) - 1)*(f + d
*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (152*d^3*e^3*f^3*((1 - d
*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) - (26
```

4*d^9*e^9*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (80*d*e*f^11*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (96*d*e*f^5*((1 - d*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) - (136*d^2*e^2*f^10*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (560*d^4*e^4*f^8*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 - (912*d^6*e^6*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (156*d^2*e^2*f^4*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 + (733*d^8*e^8*f^4*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 - (172*d^4*e^4*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 - (290*d^10*e^10*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (56*d^5*e^5*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) + (44*d^11*e^11*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1))*(C*d^2*e^3 - 2*C*e*f^2))/(f^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)) + (C*e*log(9*d^14*e^14*f - 16*f^15 + (16*f^15*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + 92*d^2*e^2*f^13 - 236*d^4*e^4*f^11 + 352*d^6*e^6*f^9 - 329*d^8*e^8*f^7 + 191*d^10*e^10*f^5 - 63*d^12*e^12*f^3 + 16*f^6*(f + d*e)^(9/2)*(f - d*e)^(9/2) + 12*d^6*e^6*(f + d*e)^(9/2)*(f - d*e)^(9/2) + 15*d^12*e^12*(f + d*e)^(3/2)*(f - d*e)^(3/2) + (6*d^15*e^15*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (16*d*e*f^14*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (92*d^2*e^2*f^13*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (236*d^4*e^4*f^11*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (352*d^6*e^6*f^9*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (329*d^8*e^8*f^7*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (191*d^10*e^10*f^5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (63*d^12*e^12*f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (16*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 - 24*d^2*e^2*f^10*(f + d*e)^(3/2)*(f - d*e)^(3/2) + 120*d^4*e^4*f^8*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 228*d^6*e^6*f^6*(f + d*e)^(3/2)*(f - d*e)^(3/2) + 4*d^2*e^2*f^4*(f + d*e)^(9/2)*(f - d*e)^(9/2) + 207*d^8*e^8*f^4*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 28*d^4*e^4*f^2*(f + d*e)^(9/2)*(f - d*e)^(9/2) - 90*d^10*e^10*f^2*(f + d*e)^(3/2)*(f - d*e)^(3/2) + (88*d^3*e^3*f^12*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (216*d^5*e^5*f^10*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (308*d^7*e^7*f^8*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (274*d^9*e^9*f^6*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (150*d^11*e^11*f^4*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (46*d^13*e^13*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (9*d^14*e^14*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (48*d^6*e^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 + (45*d^12*e^12*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (376*d^3*e^3*f^9*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (688*d^5*e^5*f^7*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (612*d^7*e^7*f^5*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (152*d^3*e^3*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) - (264*d^9*e^9*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (80*d*e*f^11*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (96*d*e*f^5*((1 - d*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) - (136*d^2*e^2*f^10*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (560*d^4*e^4*f^8*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 - (912*d^6*e^6*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (156*d^2*e^2*f^4*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 + (733*d^8*e^8*f^4*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 - (172*d^4*e^4*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))

```
/((d*x + 1)^(1/2) - 1)^2 - (290*d^10*e^10*f^2*((1 - d*x)^(1/2) - 1)^2*(f +
d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (56*d^5*e^5*f*((1 - d
*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) + (44
*d^11*e^11*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x +
 1)^(1/2) - 1))*(2*f^2 - d^2*e^2))/(f^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.7 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^3}\,dx$

Optimal. Leaf size=248

$$\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}+\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\,\sqrt{d^2e^2-f^2}}\right)\left(C\left(d^2e^2+2f^2\right)-d^2\left(3Bef-A\left(2d^2e^2+f^2\right)\right)\right)}{2\left(d^2e^2-f^2\right)^{5/2}}-\frac{\sqrt{1}}{}$$

Rubi [A] time = 0.36, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {1609, 1651, 807, 725, 204}

$$\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}-\frac{\sqrt{1-d^2x^2}\left(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2\right)}{2f\left(d^2e^2-f^2\right)^2(e+fx)}+\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\,\sqrt{d^2e^2-f^2}}\right)\left(C\left(d^2e^2+2f^2\right)-d^2\left(3Bef-A\left(2d^2e^2+f^2\right)\right)\right)}{2\left(d^2e^2-f^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)*(e + f*x)^
2) - ((C*d^2*e^3 + B*d^2*e^2*f - 4*C*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3)*Sqrt[
1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)^2*(e + f*x)) + ((C*(d^2*e^2 + 2*f^2) - d
^2*(3*B*e*f - A*(2*d^2*e^2 + f^2)))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^
2]*Sqrt[1 - d^2*x^2])])/(2*(d^2*e^2 - f^2)^(5/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
```

```
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}\,(e+fx)^3}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} + \frac{\int \frac{2\left(Ce+Ad^2e-Bf\right)+\left(Bd^2e+\frac{Cd^2e^2}{f}-2Cf-Ad^2f\right)x}{(e+fx)^2\sqrt{1-d^2x^2}}\,dx}{2\left(d^2e^2-f^2\right)}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2B\right.}{2f\left(d^2e^2-f^2\right)^2(e+fx)}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2B\right.}{2f\left(d^2e^2-f^2\right)^2(e+fx)}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+2B\right.}{2f\left(d^2e^2-f^2\right)^2(e+fx)}$$

Mathematica [A]   time = 0.42, size = 273, normalized size = 1.10

$$\frac{1}{2}\left(-\frac{\log\left(\sqrt{1-d^2x^2}\sqrt{f^2-d^2e^2}+d^2ex+f\right)\left(d^2\left(A\left(2d^2e^2+f^2\right)-3Bef\right)+C\left(d^2e^2+2f^2\right)\right)}{\left(f^2-d^2e^2\right)^{5/2}}+\frac{\log(e+fx)\left(d^2\left(A\left(2d^2e^2+f^2\right)-3Bef\right)+C\left(d^2e^2+2f^2\right)\right)}{\left(f^2-d^2e^2\right)^{5/2}}-\frac{\sqrt{1-d^2x^2}\left(-Ad^2ef(4e+3fx)+Af^3+Bd^2e^2(2e+fx)+Bf^2(e+2fx)+Ce\left(d^2e^2x-3ef-4f^2x\right)\right)}{\left(f^2-d^2e^2\right)^2(e+fx)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] (-((Sqrt[1 - d^2*x^2]*(A*f^3 + B*d^2*e^2*(2*e + f*x) + B*f^2*(e + 2*f*x) -
A*d^2*e*f*(4*e + 3*f*x) + C*e*(-3*e*f + d^2*e^2*x - 4*f^2*x)))/((-(d^2*e^2)
 + f^2)^2*(e + f*x)^2)) + ((C*(d^2*e^2 + 2*f^2) + d^2*(-3*B*e*f + A*(2*d^2*
e^2 + f^2)))*Log[e + f*x])/(-(d^2*e^2) + f^2)^(5/2) - ((C*(d^2*e^2 + 2*f^2)
 + d^2*(-3*B*e*f + A*(2*d^2*e^2 + f^2)))*Log[f + d^2*e*x + Sqrt[-(d^2*e^2)
+ f^2]*Sqrt[1 - d^2*x^2]])/(-(d^2*e^2) + f^2)^(5/2))/2
```

IntegrateAlgebraic [B]   time = 2.35, size = 533, normalized size = 2.15

$$\frac{\tan^{-1}\left(\frac{\sqrt{1-dx}\sqrt{-de-f}\sqrt{f-de}}{\sqrt{dx+1}\,(de+f)}\right)\left(2Ad^4e^2\sqrt{f-de}+Ad^2f^2\sqrt{f-de}-3Bd^2ef\sqrt{f-de}+Cd^2e^2\sqrt{f-de}+2Cf^2\sqrt{f-de}\right)}{(-de-f)^{5/2}(de-f)^3}-\frac{d\sqrt{1-dx}\left(-\frac{4Ad^3e^2f(1-dx)}{dx+1}-4Ad^3e^2f+\frac{3Ad^2ef^2(1-dx)}{dx+1}-3Ad^2ef^2+\frac{Adf^3(1-dx)}{dx+1}+Adf^3+\frac{2Bd^3e^3(1-dx)}{dx+1}+2Bd^3e^3-\frac{Bd^2e^2f(1-dx)}{dx+1}+Bd^2e^2f+\frac{Bdef^2(1-dx)}{dx+1}+Bdef^2-\frac{2Bf^3(1-dx)}{dx+1}+2Bf^3-\frac{Cd^2e^3(1-dx)}{dx+1}+Cd^2e^3-\frac{3Cde^2f(1-dx)}{dx+1}-3Cde^2f+\frac{4Cef^2(1-dx)}{dx+1}-4Cef^2\right)}{\sqrt{dx+1}\,(de-f)^2(de+f)^2\left(\frac{de(1-dx)}{dx+1}+de-\frac{f(1-dx)}{dx+1}+f\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)
^3),x]
```

```
[Out] -((d*Sqrt[1 - d*x]*(C*d^2*e^3 + 2*B*d^3*e^3 - 3*C*d*e^2*f + B*d^2*e^2*f - 4
*A*d^3*e^2*f - 4*C*e*f^2 + B*d*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3 + A*d*f^3 -
(C*d^2*e^3*(1 - d*x))/(1 + d*x) + (2*B*d^3*e^3*(1 - d*x))/(1 + d*x) - (3*C*
d*e^2*f*(1 - d*x))/(1 + d*x) - (B*d^2*e^2*f*(1 - d*x))/(1 + d*x) - (4*A*d^3
*e^2*f*(1 - d*x))/(1 + d*x) + (4*C*e*f^2*(1 - d*x))/(1 + d*x) + (B*d*e*f^2*
(1 - d*x))/(1 + d*x) + (3*A*d^2*e*f^2*(1 - d*x))/(1 + d*x) - (2*B*f^3*(1 -
d*x))/(1 + d*x) + (A*d*f^3*(1 - d*x))/(1 + d*x)))/((d*e - f)^2*(d*e + f)^2*
Sqrt[1 + d*x]*(d*e + f + (d*e*(1 - d*x))/(1 + d*x) - (f*(1 - d*x))/(1 + d*x
))^2)) + ((C*d^2*e^2*Sqrt[-(d*e) + f] + 2*A*d^4*e^2*Sqrt[-(d*e) + f] - 3*B*
```

```
d^2*e*f*Sqrt[-(d*e) + f] + 2*C*f^2*Sqrt[-(d*e) + f] + A*d^2*f^2*Sqrt[-(d*e)
 + f])*ArcTan[(Sqrt[-(d*e) - f]*Sqrt[-(d*e) + f]*Sqrt[1 - d*x])/((d*e + f)*
Sqrt[1 + d*x])]])/((-(d*e) - f)^(5/2)*(d*e - f)^3)
```

fricas [B] time = 1.24, size = 1580, normalized size = 6.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 +
 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 -
(4*A*d^4 + 3*C*d^2)*e^4*f^3 + (5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^
2 - (3*B*d^2*e^5*f - (2*A*d^4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d
^2*e^3*f^3 - (2*A*d^4 + C*d^2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 + 2*(3*
B*d^2*e^4*f^2 - (2*A*d^4 + C*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*sqrt(-d
^2*e^2 + f^2)*log((d^2*e*f*x + f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (
sqrt(-d^2*e^2 + f^2)*sqrt(-d*x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqr
t(d*x + 1))/(f*x + e)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)
*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4*e^7 + B*d
^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^2 + 4*C)*e^
3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d^4*e^6*f - B
*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e^3*f^4 - B*e^
2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4 - e^4*f^6 + (
d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 + 2*(d^6*e^9*f -
 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x), -1/2*(2*B*d^4*e^7 - B*d^2*e^5
*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*
e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 - (4*A*d^4 + 3*C*d^2)*e^4*f^3 +
(5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^2 + 2*(3*B*d^2*e^5*f - (2*A*d^
4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d^2*e^3*f^3 - (2*A*d^4 + C*d^
2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 + 2*(3*B*d^2*e^4*f^2 - (2*A*d^4 + C
*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d
^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e
))/((d^2*e^2 - f^2)*x)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2
)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4*e^7 + B*
d^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^2 + 4*C)*e
^3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d^4*e^6*f -
B*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e^3*f^4 - B*e
^2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4 - e^4*f^6 +
(d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 + 2*(d^6*e^9*f
- 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Undef/Unsigned Inf encountered in limit
```

maple [C] time = 0.05, size = 1449, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(A*f^4*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+2*C*ln(2*(d^2*e*x
+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*e^2*f^2+C*ln(2
*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e
^4+2*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*
x+e))*x^2*f^4+2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/
2)*f+f)/(f*x+e))*d^4*e^4-3*B*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^
2)/f^2)^(1/2)*f+f)/(f*x+e))*d^2*e^3*f+2*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*
(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*x*d^2*e^3*f-4*A*d^2*e^2*f^2*(-(d^2
*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+2*B*d^2*e^3*f*(-(d^2*e^2-f^2)/f^2)^
(1/2)*(-d^2*x^2+1)^(1/2)+2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^
2)/f^2)^(1/2)*f+f)/(f*x+e))*x*d^2*e*f^3-6*B*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2
)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*x*d^2*e^2*f^2+C*ln(2*(d^2*e*x+(-
d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*x^2*d^2*e^2*f^2-3
*B*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e)
)*x^2*d^2*e*f^3+2*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(
1/2)*f+f)/(f*x+e))*x^2*d^4*e^2*f^2+4*A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(
d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*x*d^4*e^3*f-4*C*x*e*f^3*(-(d^2*e^2-f^
2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+4*C*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^
2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*x*e*f^3+2*B*x*f^4*(-(d^2*e^2-f^2)/f^2)^
(1/2)*(-d^2*x^2+1)^(1/2)+B*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1
/2)-3*C*e^2*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+A*ln(2*(d^2*e
*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))*x^2*d^2*f^4+
A*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))
*d^2*e^2*f^2-3*A*x*d^2*e*f^3*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+
B*x*d^2*e^2*f^2*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)+C*x*d^2*e^3*f
*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2))*csgn(d)^2*(d*x+1)^(1/2)*(-d
*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e+f)/(d*e-f)/(d^2*e^2-f^2)/(f*x+e)^2/(-(d
^2*e^2-f^2)/f^2)^(1/2)/f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(f-d*e>0)', see `assume?` for more d
etails)Is f-d*e positive, negative or zero?
```

mupad [B] time = 59.18, size = 9097, normalized size = 36.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^2)/(((d*x + 1)^(1/2) - 1
)^2*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (24*(2*C*f^3 - C*d^2*e^2*f)*((1 - d*
x)^(1/2) - 1)^4)/(((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2))
+ (12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/(((d*x + 1)^(1/2) -
1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*((1 - d*x)^(1/2) - 1)^7*(C*d^3*e
^3 + 2*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)
) - (2*((1 - d*x)^(1/2) - 1)^3*(7*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1
/2) - 1)^3*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*((1 - d*x)^(1/2) - 1)^5*(7
*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2
*e^2*f^2)) + (2*d*e*((1 - d*x)^(1/2) - 1)*(2*C*f^2 + C*d^2*e^2))/(((d*x + 1
)^(1/2) - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2)
 - 1)^2*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) -
```

```
1)^6*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1
)^4*(32*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/
2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*
x + 1)^(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) -
1)^5 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + ((4*((1 - d*x)^(1/2) - 1)^
2*(4*A*d^4*e^4*f - 2*A*f^5 + 7*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^2
*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (8*((1 - d*x)^(1/2) - 1)^4*(2*A*f^5 + 4
*A*d^4*e^4*f - 9*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^
4 - 2*d^2*e^2*f^2)) + (4*((1 - d*x)^(1/2) - 1)^6*(4*A*d^4*e^4*f - 2*A*f^5 +
 7*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*
f^2)) + (2*f*((1 - d*x)^(1/2) - 1)^7*(2*A*d*f^3 - 5*A*d^3*e^2*f))/(e*((d*x
+ 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*f*((1 - d*x)^(1/2)
- 1)^3*(2*A*d*f^3 - 29*A*d^3*e^2*f))/(e*((d*x + 1)^(1/2) - 1)^3*(f^4 + d^4*
e^4 - 2*d^2*e^2*f^2)) + (2*f*((1 - d*x)^(1/2) - 1)^5*(2*A*d*f^3 - 29*A*d^3*
e^2*f))/(e*((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*d*
f*(2*A*f^3 - 5*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1))/(e*((d*x + 1)^(1/2) - 1)
*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2) - 1)^2*(16*
f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)^6*(16*f^
2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1)^4*(32*f^2
- 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/2) - 1)^8)/(
(d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2)
 - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (8*d*
e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f*((1 - d*x)^
(1/2) - 1))/((d*x + 1)^(1/2) - 1)) - ((4*((1 - d*x)^(1/2) - 1)^2*(2*B*f^4 +
 2*B*d^4*e^4 + 5*B*d^2*e^2*f^2))/(e*((d*x + 1)^(1/2) - 1)^2*(f^4 + d^4*e^4
- 2*d^2*e^2*f^2)) - (8*((1 - d*x)^(1/2) - 1)^4*(2*B*f^4 - 2*B*d^4*e^4 + 3*B
*d^2*e^2*f^2))/(e*((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2))
+ (4*((1 - d*x)^(1/2) - 1)^6*(2*B*f^4 + 2*B*d^4*e^4 + 5*B*d^2*e^2*f^2))/(e*
((d*x + 1)^(1/2) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*f*(11*B*d^3*e
^2 + 16*B*d*f^2)*((1 - d*x)^(1/2) - 1)^3)/(((d*x + 1)^(1/2) - 1)^3*(f^4 + d
^4*e^4 - 2*d^2*e^2*f^2)) - (2*f*(11*B*d^3*e^2 + 16*B*d*f^2)*((1 - d*x)^(1/2
) - 1)^5)/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (6*B*
d^3*e^2*f*((1 - d*x)^(1/2) - 1)^7)/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4
- 2*d^2*e^2*f^2)) + (6*B*d^3*e^2*f*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
 - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2) - 1)^2
*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)^6*(
16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1)^4*(32
*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/2) - 1)
^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^
(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 -
(8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f*((1 -
d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + (C*atan(((C*(2*f^2 + d^2*e^2)*((4
*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/
(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4
*d^6*e^6*f^2)) - (4*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(f^8
+ d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (C*(2*f^2 + d^
2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 -
12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^
4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f
^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*
x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*
e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2
*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))
- (C*(2*f^2 + d^2*e^2)*((4*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5)
)/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1
- d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(((d*
x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*
e^6*f^2)) + (C*(2*f^2 + d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*
```

```
e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*
d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(
4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9
*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^
6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/
((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*1i)/(2*(f + d*
e)^(5/2)*(f - d*e)^(5/2)))/((8*(C^2*d^5*e^5 + 4*C^2*d^3*e^3*f^2 + 4*C^2*d*e
*f^4))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (8
*((1 - d*x)^(1/2) - 1)^2*(C^2*d^5*e^5 + 4*C^2*d^3*e^3*f^2 + 4*C^2*d*e*f^4))
/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 -
4*d^6*e^6*f^2)) + (C*(2*f^2 + d^2*e^2)*((4*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e
*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d
^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) - (4*(8*C*d*e*f^7
+ 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4
*e^4*f^4 - 4*d^6*e^6*f^2) + (C*(2*f^2 + d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*
e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^
8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x
)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*
f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^
8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d
*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))
))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)) + (C*(2*f^2 + d^2*e^2)*((4*(8*C*d*e*
f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4
*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 -
 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (C*(2*f^2 + d^2*e^2)*((4
*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9
*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*
e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^
5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/
2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2))
+ (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)
^(5/2)*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))*(2*f^2 + d^
2*e^2)*1i)/((f + d*e)^(5/2)*(f - d*e)^(5/2)) + (A*d^2*atan(((A*d^2*(f^2 + 2
*d^2*e^2)*((4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A
*d^7*e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*
d^4*e^4*f^4 - 4*d^6*e^6*f^2)) - (4*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A*d^
7*e^5*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)
 + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f
^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e
^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^1
1*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2
- 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x
 + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5
/2)*(f - d*e)^(5/2)) - (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*A*d^3*e*f^7 + 8*A*d^
9*e^7*f - 12*A*d^7*e^5*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4
 - 4*d^6*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7
*f - 12*A*d^7*e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2
*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^
11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 +
 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^
2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*
f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1
)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*
d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)
*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))/((8*(4*A^2*d^9*
e^5 + 4*A^2*d^7*e^3*f^2 + A^2*d^5*e*f^4))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 +
6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (8*((1 - d*x)^(1/2) - 1)^2*(4*A^2*d^9*e^5
```

```
+ 4*A^2*d^7*e^3*f^2 + A^2*d^5*e*f^4))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e
^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (A*d^2*(f^2 + 2*d^2*
e^2)*((4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A*d^7*
e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e
^4*f^4 - 4*d^6*e^6*f^2)) - (4*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A*d^7*e^5
*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (A
*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 +
8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^
6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^1
1 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*
d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*
e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)
^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f -
 d*e)^(5/2)) + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f
- 12*A*d^7*e^5*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6
*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*
A*d^7*e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^11*e^11
- 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f
^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*
((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72
*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8
 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*
f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*
e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))*(f^2 + 2*d^2*e^2)*1i)/((f
 + d*e)^(5/2)*(f - d*e)^(5/2)) - (B*d^2*e*f*atan(((B*d^2*e*f*((4*((1 - d*x)
^(1/2) - 1)^2*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(((
d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^
6*e^6*f^2)) - (4*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/
(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (3*B*d^2*
e*f*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*
d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 -
 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8
- 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x +
 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6
*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f
 + d*e)^(5/2)*(f - d*e)^(5/2)))*3i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)) - (
B*d^2*e*f*((4*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(f^
8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1 - d*x
)^(1/2) - 1)^2*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/((
(d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d
^6*e^6*f^2)) + (3*B*d^2*e*f*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f
^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e
^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^1
1*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2
- 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x
 + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*3i)/(2*(f + d*e)^(5
/2)*(f - d*e)^(5/2)))/((72*B^2*d^5*e^3*f^2)/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6
+ 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (3*B*d^2*e*f*((4*((1 - d*x)^(1/2) - 1)^2
*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(((d*x + 1)^(1/2
) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) -
 (4*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(f^8 + d^8*e^
8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (3*B*d^2*e*f*((4*(4*d^
11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 +
 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^
2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*
f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1
)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*
```

```
d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)
*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)) + (3*B*d^2*e*f*((4*
(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(f^8 + d^8*e^8 -
4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2
*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(((d*x + 1)^(1/2
) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) +
 (3*B*d^2*e*f*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7
*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4
*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^
3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10)
)/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 -
 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/
2)) + (72*B^2*d^5*e^3*f^2*((1 - d*x)^(1/2) - 1)^2)/(((d*x + 1)^(1/2) - 1)^2
*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2))))*3i)/((f
 + d*e)^(5/2)*(f - d*e)^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
[Out] Timed out
```

3.8 $\int \frac{(e+fx)^3(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=340

$$-\frac{\sqrt{1-d^2x^2}(e+fx)^2\left(4f^2\left(5Ad^2+4C\right)-3d^2e(Ce-5Bf)\right)}{60d^4f}+\frac{\sqrt{1-d^2x^2}\left(d^2fx\left(-100Ad^2ef^2-30Bd^2e^2f-45\right.\right.}{}$$

Rubi [A] time = 0.63, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {1609, 1654, 833, 780, 216}

$$-\frac{\sqrt{1-d^2x^2}(e+fx)^2\left(5f(4Af+3Be)-C\left(3e^2-\frac{16f^2}{d^2}\right)\right)}{60d^2f}+\frac{\sqrt{1-d^2x^2}\left(d^2fx\left(-100Ad^2ef^2-30Bd^2e^2f-45Bf^3+6Cd^2e^3-71Cef^2\right)+4\left(C\left(-52d^2e^2f^2+3d^4e^4-16f^4\right)-5d^2f\left(4Af\left(4d^2e^2+f^2\right)+3B\left(d^2e^3+4ef^2\right)\right)\right)\right)}{120d^6f}+\frac{\sin^{-1}(dx)\left(8Ad^4e^3+12Ad^2ef^2+12Bd^2e^2f+3Bf^3+4Cd^2e^3+9Cef^2\right)}{8d^5}+\frac{\sqrt{1-d^2x^2}(e+fx)^3(Ce-5Bf)}{20d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^4}{5d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((5*f*(3*B*e + 4*A*f) - C*(3*e^2 - (16*f^2)/d^2))*(e + f*x)^2*Sqrt[1 - d^2
*x^2])/(60*d^2*f) + ((C*e - 5*B*f)*(e + f*x)^3*Sqrt[1 - d^2*x^2])/(20*d^2*f
) - (C*(e + f*x)^4*Sqrt[1 - d^2*x^2])/(5*d^2*f) + ((4*(C*(3*d^4*e^4 - 52*d^
2*e^2*f^2 - 16*f^4) - 5*d^2*f*(4*A*f*(4*d^2*e^2 + f^2) + 3*B*(d^2*e^3 + 4*e
*f^2))) + d^2*f*(6*C*d^2*e^3 - 30*B*d^2*e^2*f - 71*C*e*f^2 - 100*A*d^2*e*f^
2 - 45*B*f^3)*x)*Sqrt[1 - d^2*x^2])/(120*d^6*f) + ((4*C*d^2*e^3 + 8*A*d^4*e
^3 + 12*B*d^2*e^2*f + 9*C*e*f^2 + 12*A*d^2*e*f^2 + 3*B*f^3)*ArcSin[d*x])/(8
*d^5)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
```

```
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx = \int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C(e+fx)^4\sqrt{1-d^2x^2}}{5d^2f} - \frac{\int \frac{(e+fx)^3\left(-\left(\left(4C+5Ad^2\right)f^2\right)+d^2f(Ce-5Bf)x\right)}{\sqrt{1-d^2x^2}}\,dx}{5d^2f^2}$$

$$= \frac{(Ce-5Bf)(e+fx)^3\sqrt{1-d^2x^2}}{20d^2f} - \frac{C(e+fx)^4\sqrt{1-d^2x^2}}{5d^2f} + \frac{\int \frac{(e+fx)^2\left(d^2f^2\left(13Ce+2\right.\right.}{}}{}$$

$$= -\frac{\left(4\left(4C+5Ad^2\right)f^2-3d^2e(Ce-5Bf)\right)(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f} + \frac{(Ce-5Bf)(e+}{20}$$

$$= -\frac{\left(4\left(4C+5Ad^2\right)f^2-3d^2e(Ce-5Bf)\right)(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f} + \frac{(Ce-5Bf)(e+}{20}$$

$$= -\frac{\left(4\left(4C+5Ad^2\right)f^2-3d^2e(Ce-5Bf)\right)(e+fx)^2\sqrt{1-d^2x^2}}{60d^4f} + \frac{(Ce-5Bf)(e+}{20}$$

Mathematica [A] time = 0.39, size = 241, normalized size = 0.71

$$\frac{15d\sin^{-1}(dx)\left(8Ad^4e^3+12Ad^2ef^2+12Bd^2e^2f+3Bf^3+4Cd^2e^3+9Cef^2\right)-\sqrt{1-d^2x^2}\left(20Ad^2f\left(d^2\left(18e^2+9efx+2f^2x^2\right)+4f^2\right)+15B\left(2d^4\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)+d^2f^2(16e+3fx)\right)+C\left(6d^4x\left(10e^3+20e^2fx+15ef^2x^2+4f^3x^3\right)+d^2f\left(240e^2+135efx+32f^2x^2\right)+64f^3\right)\right)}{120d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(Sqrt[1 - d^2*x^2]*(20*A*d^2*f*(4*f^2 + d^2*(18*e^2 + 9*e*f*x + 2*f^2*x^2
)) + 15*B*(d^2*f^2*(16*e + 3*f*x) + 2*d^4*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2
+ f^3*x^3)) + C*(64*f^3 + d^2*f*(240*e^2 + 135*e*f*x + 32*f^2*x^2) + 6*d^4*
x*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3)))) + 15*d*(4*C*d^2*e^3 +
 8*A*d^4*e^3 + 12*B*d^2*e^2*f + 9*C*e*f^2 + 12*A*d^2*e*f^2 + 3*B*f^3)*ArcSi
n[d*x])/(120*d^6)
```

IntegrateAlgebraic [B] time = 0.77, size = 1135, normalized size = 3.34

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 +
d*x]),x]
```

```
[Out] -1/60*(Sqrt[1 - d*x]*(60*C*d^3*e^3 + 120*B*d^4*e^3 + 360*C*d^2*e^2*f + 180*
B*d^3*e^2*f + 360*A*d^4*e^2*f + 225*C*d*e*f^2 + 360*B*d^2*e*f^2 + 180*A*d^3
*e*f^2 + 120*C*f^3 + 75*B*d*f^3 + 120*A*d^2*f^3 - (60*C*d^3*e^3*(1 - d*x)^4
)/(1 + d*x)^4 + (120*B*d^4*e^3*(1 - d*x)^4)/(1 + d*x)^4 + (360*C*d^2*e^2*f*
```

```
(1 - d*x)^4)/(1 + d*x)^4 - (180*B*d^3*e^2*f*(1 - d*x)^4)/(1 + d*x)^4 + (360
*A*d^4*e^2*f*(1 - d*x)^4)/(1 + d*x)^4 - (225*C*d*e*f^2*(1 - d*x)^4)/(1 + d*
x)^4 + (360*B*d^2*e*f^2*(1 - d*x)^4)/(1 + d*x)^4 - (180*A*d^3*e*f^2*(1 - d*
x)^4)/(1 + d*x)^4 + (120*C*f^3*(1 - d*x)^4)/(1 + d*x)^4 - (75*B*d*f^3*(1 -
d*x)^4)/(1 + d*x)^4 + (120*A*d^2*f^3*(1 - d*x)^4)/(1 + d*x)^4 - (120*C*d^3*
e^3*(1 - d*x)^3)/(1 + d*x)^3 + (480*B*d^4*e^3*(1 - d*x)^3)/(1 + d*x)^3 + (9
60*C*d^2*e^2*f*(1 - d*x)^3)/(1 + d*x)^3 - (360*B*d^3*e^2*f*(1 - d*x)^3)/(1
+ d*x)^3 + (1440*A*d^4*e^2*f*(1 - d*x)^3)/(1 + d*x)^3 - (90*C*d*e*f^2*(1 -
d*x)^3)/(1 + d*x)^3 + (960*B*d^2*e*f^2*(1 - d*x)^3)/(1 + d*x)^3 - (360*A*d^
3*e*f^2*(1 - d*x)^3)/(1 + d*x)^3 + (160*C*f^3*(1 - d*x)^3)/(1 + d*x)^3 - (3
0*B*d*f^3*(1 - d*x)^3)/(1 + d*x)^3 + (320*A*d^2*f^3*(1 - d*x)^3)/(1 + d*x)^
3 + (720*B*d^4*e^3*(1 - d*x)^2)/(1 + d*x)^2 + (1200*C*d^2*e^2*f*(1 - d*x)^2
)/(1 + d*x)^2 + (2160*A*d^4*e^2*f*(1 - d*x)^2)/(1 + d*x)^2 + (1200*B*d^2*e*
f^2*(1 - d*x)^2)/(1 + d*x)^2 + (464*C*f^3*(1 - d*x)^2)/(1 + d*x)^2 + (400*A
*d^2*f^3*(1 - d*x)^2)/(1 + d*x)^2 + (120*C*d^3*e^3*(1 - d*x))/(1 + d*x) + (
480*B*d^4*e^3*(1 - d*x))/(1 + d*x) + (960*C*d^2*e^2*f*(1 - d*x))/(1 + d*x)
+ (360*B*d^3*e^2*f*(1 - d*x))/(1 + d*x) + (1440*A*d^4*e^2*f*(1 - d*x))/(1 +
 d*x) + (90*C*d*e*f^2*(1 - d*x))/(1 + d*x) + (960*B*d^2*e*f^2*(1 - d*x))/(1
 + d*x) + (360*A*d^3*e*f^2*(1 - d*x))/(1 + d*x) + (160*C*f^3*(1 - d*x))/(1
+ d*x) + (30*B*d*f^3*(1 - d*x))/(1 + d*x) + (320*A*d^2*f^3*(1 - d*x))/(1 +
d*x)))/(d^6*Sqrt[1 + d*x]*(1 + (1 - d*x)/(1 + d*x))^5) + ((-4*C*d^2*e^3 - 8
*A*d^4*e^3 - 12*B*d^2*e^2*f - 9*C*e*f^2 - 12*A*d^2*e*f^2 - 3*B*f^3)*ArcTan[
Sqrt[1 - d*x]/Sqrt[1 + d*x]])/(4*d^5)
```

fricas [**A**] time = 1.23, size = 286, normalized size = 0.84

$$-\frac{\left(24\,Cd^4f^3x^4 + 120\,Bd^4e^3 + 240\,Bd^2ef^2 + 120\left(3\,Ad^4 + 2\,Cd^2\right)e^2f + 16\left(5\,Ad^2 + 4\,C\right)f^3 + 30\left(3\,Cd^4ef^2 + Bd^4f^3\right)x^3 + 8\left(15\,Cd^4e^2f + 15\,Bd^4ef^2 + \left(5\,Ad^4 + 4\,Cd^2\right)f^3\right)x^2 + 15\left(4\,Cd^4e^3 + 12\,Bd^4e^2f + 3\,Bd^2f^3 + 3\left(4\,Ad^4 + 3\,Cd^2\right)ef^2\right)x\right)\sqrt{dx+1}\,\sqrt{-dx+1} + 30\left(12\,Bd^3e^2f + 3\,Bdf^3 + 4\left(2\,Ad^5 + Cd^3\right)e^3 + 3\left(4\,Ad^3 + 3\,Cd\right)ef^2\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{120\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/120*((24*C*d^4*f^3*x^4 + 120*B*d^4*e^3 + 240*B*d^2*e*f^2 + 120*(3*A*d^4
+ 2*C*d^2)*e^2*f + 16*(5*A*d^2 + 4*C)*f^3 + 30*(3*C*d^4*e*f^2 + B*d^4*f^3)*
x^3 + 8*(15*C*d^4*e^2*f + 15*B*d^4*e*f^2 + (5*A*d^4 + 4*C*d^2)*f^3)*x^2 + 1
5*(4*C*d^4*e^3 + 12*B*d^4*e^2*f + 3*B*d^2*f^3 + 3*(4*A*d^4 + 3*C*d^2)*e*f^2
)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 30*(12*B*d^3*e^2*f + 3*B*d*f^3 + 4*(2*A
*d^5 + C*d^3)*e^3 + 3*(4*A*d^3 + 3*C*d)*e*f^2)*arctan((sqrt(d*x + 1)*sqrt(-
d*x + 1) - 1)/(d*x)))/d^6
```

giac [**A**] time = 1.82, size = 427, normalized size = 1.26

$$-\frac{\left(\left(2\left(dx+1\right)\left(3\left(dx+1\right)\left(\frac{4\left(dx+1\right)Cf^3}{d^5} + \frac{5\,Bd^{26}f^3+15\,Cd^{26}f^2e-16\,Cd^{25}f^3}{d^{30}}\right) + \frac{20\,Ad^{27}f^3+60\,Bd^{27}f^2e-45\,Bd^{26}f^3+60\,Cd^{27}fe^2-135\,Cd^{26}f^2e+88\,Cd^{25}f^3}{d^{30}}\right) + \frac{5\left(36\,Ad^{28}f^2e-16\,Ad^{27}f^3+36\,Bd^{28}fe^2-48\,Bd^{27}f^2e+27\,Bd^{26}f^3+12\,Cd^{28}e^3-48\,Cd^{27}fe^2+81\,Cd^{26}f^2e-32\,Cd^{25}f^3\right)}{d^{30}}\right)\left(dx+1\right) + \frac{15\left(24\,Ad^{29}fe^2-12\,Ad^{28}f^2e+8\,Ad^{27}f^3+8\,Bd^{29}e^3-12\,Bd^{28}fe^2+24\,Bd^{27}f^2e-5\,Bd^{26}f^3-4\,Cd^{28}e^3+24\,Cd^{27}fe^2-15\,Cd^{26}f^2e+8\,Cd^{25}f^3\right)}{d^{30}}\right)\sqrt{dx+1}\,\sqrt{-dx+1} - \frac{30\left(8\,Ad^4e^3+12\,Ad^2f^2e+12\,Bd^2fe^2+3\,Bf^3+4\,Cd^2e^3+9\,Cf^2e\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^4}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] -1/120*(((2*(d*x + 1)*(3*(d*x + 1)*(4*(d*x + 1)*C*f^3/d^5 + (5*B*d^26*f^3 +
 15*C*d^26*f^2*e - 16*C*d^25*f^3)/d^30) + (20*A*d^27*f^3 + 60*B*d^27*f^2*e
- 45*B*d^26*f^3 + 60*C*d^27*f*e^2 - 135*C*d^26*f^2*e + 88*C*d^25*f^3)/d^30)
 + 5*(36*A*d^28*f^2*e - 16*A*d^27*f^3 + 36*B*d^28*f*e^2 - 48*B*d^27*f^2*e +
 27*B*d^26*f^3 + 12*C*d^28*e^3 - 48*C*d^27*f*e^2 + 81*C*d^26*f^2*e - 32*C*d
^25*f^3)/d^30)*(d*x + 1) + 15*(24*A*d^29*f*e^2 - 12*A*d^28*f^2*e + 8*A*d^27
*f^3 + 8*B*d^29*e^3 - 12*B*d^28*f*e^2 + 24*B*d^27*f^2*e - 5*B*d^26*f^3 - 4*
C*d^28*e^3 + 24*C*d^27*f*e^2 - 15*C*d^26*f^2*e + 8*C*d^25*f^3)/d^30)*sqrt(d
*x + 1)*sqrt(-d*x + 1) - 30*(8*A*d^4*e^3 + 12*A*d^2*f^2*e + 12*B*d^2*f*e^2
+ 3*B*f^3 + 4*C*d^2*e^3 + 9*C*f^2*e)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^4)
/d
```

maple [C] time = 0.03, size = 643, normalized size = 1.89

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\left(24\sqrt{-d^2x^2+1}\,C d^4f^3x^4\mathrm{csgn}(d)+30\sqrt{-d^2x^2+1}\,B d^4f^3x^3\mathrm{csgn}(d)+90\sqrt{-d^2x^2+1}\,C d^4e f^2x^3\mathrm{csgn}(d)+40\sqrt{-d^2x^2+1}\,A d^4f^3x^2\mathrm{csgn}(d)+120\sqrt{-d^2x^2+1}\,B d^4e f^2x^2\mathrm{csgn}(d)+120\sqrt{-d^2x^2+1}\,C d^4e^2f x^2\mathrm{csgn}(d)-120A d^5e^3\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+180\sqrt{-d^2x^2+1}\,A d^4e f^2x\,\mathrm{csgn}(d)+180\sqrt{-d^2x^2+1}\,B d^4e^2f x\,\mathrm{csgn}(d)+60\sqrt{-d^2x^2+1}\,C d^4e^3x\,\mathrm{csgn}(d)+360\sqrt{-d^2x^2+1}\,A d^4e^2f\,\mathrm{csgn}(d)+120\sqrt{-d^2x^2+1}\,B d^4e^3\mathrm{csgn}(d)+32\sqrt{-d^2x^2+1}\,C d^2f^3x^2\mathrm{csgn}(d)-180A d^3e f^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)-180B d^3e^2f\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+45\sqrt{-d^2x^2+1}\,B d^2f^3x\,\mathrm{csgn}(d)-60C d^3e^3\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+135\sqrt{-d^2x^2+1}\,C d^2e f^2x\,\mathrm{csgn}(d)+80\sqrt{-d^2x^2+1}\,A d^2f^3\mathrm{csgn}(d)+240\sqrt{-d^2x^2+1}\,B d^2e f^2\mathrm{csgn}(d)+240\sqrt{-d^2x^2+1}\,C d^2e^2f\,\mathrm{csgn}(d)-45B d f^3\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)-135C d e f^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+64\sqrt{-d^2x^2+1}\,C f^3\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{120\sqrt{-d^2x^2+1}\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/120*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(24*(-d^2*x^2+1)^(1/2)*C*d^4*f^3*x^4*cs
gn(d)+30*(-d^2*x^2+1)^(1/2)*B*d^4*f^3*x^3*csgn(d)+90*(-d^2*x^2+1)^(1/2)*C*d
^4*e*f^2*x^3*csgn(d)+40*(-d^2*x^2+1)^(1/2)*A*d^4*f^3*x^2*csgn(d)+120*(-d^2*
x^2+1)^(1/2)*B*d^4*e*f^2*x^2*csgn(d)+120*(-d^2*x^2+1)^(1/2)*C*d^4*e^2*f*x^2
*csgn(d)+180*(-d^2*x^2+1)^(1/2)*A*d^4*e*f^2*x*csgn(d)+180*(-d^2*x^2+1)^(1/2
)*B*d^4*e^2*f*x*csgn(d)+60*(-d^2*x^2+1)^(1/2)*C*d^4*e^3*x*csgn(d)+360*(-d^2
*x^2+1)^(1/2)*A*d^4*e^2*f*csgn(d)-120*A*d^5*e^3*arctan(1/(-d^2*x^2+1)^(1/2)
*d*x*csgn(d))+120*(-d^2*x^2+1)^(1/2)*B*d^4*e^3*csgn(d)+32*(-d^2*x^2+1)^(1/2
)*C*d^2*f^3*x^2*csgn(d)+45*(-d^2*x^2+1)^(1/2)*B*d^2*f^3*x*csgn(d)+135*(-d^2
*x^2+1)^(1/2)*C*d^2*e*f^2*x*csgn(d)+80*(-d^2*x^2+1)^(1/2)*A*d^2*f^3*csgn(d)
-180*A*d^3*e*f^2*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))+240*(-d^2*x^2+1)^
(1/2)*B*d^2*e*f^2*csgn(d)-180*B*d^3*e^2*f*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*c
sgn(d))+240*(-d^2*x^2+1)^(1/2)*C*d^2*e^2*f*csgn(d)-60*C*d^3*e^3*arctan(1/(-
d^2*x^2+1)^(1/2)*d*x*csgn(d))-45*B*d*f^3*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*cs
gn(d))+64*(-d^2*x^2+1)^(1/2)*C*f^3*csgn(d)-135*C*d*e*f^2*arctan(1/(-d^2*x^2
+1)^(1/2)*d*x*csgn(d)))*csgn(d)/d^6/(-d^2*x^2+1)^(1/2)
```

maxima [A] time = 1.05, size = 355, normalized size = 1.04

$$-\frac{\sqrt{-d^2x^2+1}\,Cf^3x^4}{5\,d^2}+\frac{Ae^3\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}\,Be^3}{d^2}-\frac{3\sqrt{-d^2x^2+1}\,Ae^2f}{d^2}-\frac{4\sqrt{-d^2x^2+1}\,Cf^3x^2}{15\,d^4}-\frac{\left(3\,Cef^2+Bf^3\right)\sqrt{-d^2x^2+1}\,x^3}{4\,d^2}-\frac{\left(3\,Ce^2f+3\,Bef^2+Af^3\right)\sqrt{-d^2x^2+1}\,x^2}{3\,d^2}-\frac{\left(Ce^3+3\,Be^2f+3\,Aef^2\right)\sqrt{-d^2x^2+1}\,x}{2\,d^2}+\frac{\left(Ce^3+3\,Be^2f+3\,Aef^2\right)\arcsin(dx)}{2\,d^3}-\frac{8\sqrt{-d^2x^2+1}\,Cf^3}{15\,d^6}-\frac{3\left(3\,Cef^2+Bf^3\right)\sqrt{-d^2x^2+1}\,x}{8\,d^4}-\frac{2\left(3\,Ce^2f+3\,Bef^2+Af^3\right)\sqrt{-d^2x^2+1}}{3\,d^4}+\frac{3\left(3\,Cef^2+Bf^3\right)\arcsin(dx)}{8\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/5*sqrt(-d^2*x^2 + 1)*C*f^3*x^4/d^2 + A*e^3*arcsin(d*x)/d - sqrt(-d^2*x^2
 + 1)*B*e^3/d^2 - 3*sqrt(-d^2*x^2 + 1)*A*e^2*f/d^2 - 4/15*sqrt(-d^2*x^2 + 1
)*C*f^3*x^2/d^4 - 1/4*(3*C*e*f^2 + B*f^3)*sqrt(-d^2*x^2 + 1)*x^3/d^2 - 1/3*
(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*sqrt(-d^2*x^2 + 1)*x^2/d^2 - 1/2*(C*e^3 + 3
*B*e^2*f + 3*A*e*f^2)*sqrt(-d^2*x^2 + 1)*x/d^2 + 1/2*(C*e^3 + 3*B*e^2*f + 3
*A*e*f^2)*arcsin(d*x)/d^3 - 8/15*sqrt(-d^2*x^2 + 1)*C*f^3/d^6 - 3/8*(3*C*e*
f^2 + B*f^3)*sqrt(-d^2*x^2 + 1)*x/d^4 - 2/3*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)
*sqrt(-d^2*x^2 + 1)/d^4 + 3/8*(3*C*e*f^2 + B*f^3)*arcsin(d*x)/d^5
```

mupad [B] time = 35.29, size = 2606, normalized size = 7.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^3*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((((2048*C*f^3)/3 + 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^
(1/2) - 1)^6 + (((2048*C*f^3)/3 + 640*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^14
)/((d*x + 1)^(1/2) - 1)^14 - (((4096*C*f^3)/3 - 832*C*d^2*e^2*f)*((1 - d*x)
^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 - (((4096*C*f^3)/3 - 832*C*d^2*e^2*f
)*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (((12288*C*f^3)/5 +
768*C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (((1
- d*x)^(1/2) - 1)^3*(2*C*d^3*e^3 - (87*C*d*e*f^2)/2))/((d*x + 1)^(1/2) - 1)
^3 - (((1 - d*x)^(1/2) - 1)^17*(2*C*d^3*e^3 - (87*C*d*e*f^2)/2))/((d*x + 1)
^(1/2) - 1)^17 + (((1 - d*x)^(1/2) - 1)^7*(88*C*d^3*e^3 - 42*C*d*e*f^2))/((
d*x + 1)^(1/2) - 1)^7 - (((1 - d*x)^(1/2) - 1)^13*(88*C*d^3*e^3 - 42*C*d*e*
f^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 - d*x)^(1/2) - 1)^5*(40*C*d^3*e^3 + 4
26*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^5 - (((1 - d*x)^(1/2) - 1)^15*(40*C*d^
```

```
3*e^3 + 426*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^15 + (((1 - d*x)^(1/2) - 1)^9
*(52*C*d^3*e^3 - 507*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^9 - (((1 - d*x)^(1/2
) - 1)^11*(52*C*d^3*e^3 - 507*C*d*e*f^2))/((d*x + 1)^(1/2) - 1)^11 - (d*(4*
C*d^2*e^3 + 9*C*e*f^2)*((1 - d*x)^(1/2) - 1))/(2*((d*x + 1)^(1/2) - 1)) + (
d*(4*C*d^2*e^3 + 9*C*e*f^2)*((1 - d*x)^(1/2) - 1)^19)/(2*((d*x + 1)^(1/2) -
 1)^19) + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4
 + (192*C*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16)/(d^
6 + (10*d^6*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (45*d^6*((1
- d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (120*d^6*((1 - d*x)^(1/2) -
1)^6)/((d*x + 1)^(1/2) - 1)^6 + (210*d^6*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1
)^(1/2) - 1)^8 + (252*d^6*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^1
0 + (210*d^6*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (120*d^6*
((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (45*d^6*((1 - d*x)^(1/
2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16 + (10*d^6*((1 - d*x)^(1/2) - 1)^18)/((
d*x + 1)^(1/2) - 1)^18 + (d^6*((1 - d*x)^(1/2) - 1)^20)/((d*x + 1)^(1/2) -
1)^20) - (((64*A*f^3 + 96*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^
(1/2) - 1)^4 + ((64*A*f^3 + 96*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^8)/((d*x
+ 1)^(1/2) - 1)^8 - (((128*A*f^3)/3 - 144*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1
)^6)/((d*x + 1)^(1/2) - 1)^6 + (24*A*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d
*x + 1)^(1/2) - 1)^2 + (24*A*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)
^(1/2) - 1)^10 - (6*A*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)
+ (30*A*d*e*f^2*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (36*A*d*
e*f^2*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (36*A*d*e*f^2*((1
- d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (30*A*d*e*f^2*((1 - d*x)^(1/
2) - 1)^9)/((d*x + 1)^(1/2) - 1)^9 + (6*A*d*e*f^2*((1 - d*x)^(1/2) - 1)^11)
/((d*x + 1)^(1/2) - 1)^11)/(d^4 + (6*d^4*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1
)^(1/2) - 1)^2 + (15*d^4*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 +
 (20*d^4*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (15*d^4*((1 - d
*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (6*d^4*((1 - d*x)^(1/2) - 1)^10
)/((d*x + 1)^(1/2) - 1)^10 + (d^4*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2
) - 1)^12) - ((((3*B*f^3)/2 + 6*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^15)/((d*
x + 1)^(1/2) - 1)^15 - (((23*B*f^3)/2 - 18*B*d^2*e^2*f)*((1 - d*x)^(1/2) -
1)^3)/((d*x + 1)^(1/2) - 1)^3 + (((23*B*f^3)/2 - 18*B*d^2*e^2*f)*((1 - d*x)
^(1/2) - 1)^13)/((d*x + 1)^(1/2) - 1)^13 + (((333*B*f^3)/2 + 90*B*d^2*e^2*f
)*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (((333*B*f^3)/2 + 90*B
*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^11)/((d*x + 1)^(1/2) - 1)^11 - (((671*B*f
^3)/2 - 66*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 +
(((671*B*f^3)/2 - 66*B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^9)/((d*x + 1)^(1/2)
 - 1)^9 + (((1 - d*x)^(1/2) - 1)^4*(48*B*d^3*e^3 + 192*B*d*e*f^2))/((d*x +
1)^(1/2) - 1)^4 + (((1 - d*x)^(1/2) - 1)^12*(48*B*d^3*e^3 + 192*B*d*e*f^2))
/((d*x + 1)^(1/2) - 1)^12 + (((1 - d*x)^(1/2) - 1)^8*(160*B*d^3*e^3 + 128*B
*d*e*f^2))/((d*x + 1)^(1/2) - 1)^8 + (((1 - d*x)^(1/2) - 1)^6*(120*B*d^3*e^
3 + 256*B*d*e*f^2))/((d*x + 1)^(1/2) - 1)^6 + (((1 - d*x)^(1/2) - 1)^10*(12
0*B*d^3*e^3 + 256*B*d*e*f^2))/((d*x + 1)^(1/2) - 1)^10 - (((3*B*f^3)/2 + 6*
B*d^2*e^2*f)*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (8*B*d^3*e^3*((
1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (8*B*d^3*e^3*((1 - d*x)^(1
/2) - 1)^14)/((d*x + 1)^(1/2) - 1)^14)/(d^5 + (8*d^5*((1 - d*x)^(1/2) - 1)^
2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^5*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1
/2) - 1)^4 + (56*d^5*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6 + (70
*d^5*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (56*d^5*((1 - d*x)^
(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (28*d^5*((1 - d*x)^(1/2) - 1)^12)
/((d*x + 1)^(1/2) - 1)^12 + (8*d^5*((1 - d*x)^(1/2) - 1)^14)/((d*x + 1)^(1/
2) - 1)^14 + (d^5*((1 - d*x)^(1/2) - 1)^16)/((d*x + 1)^(1/2) - 1)^16) - (3*
B*f*atan((B*f*(f^2 + 4*d^2*e^2)*((1 - d*x)^(1/2) - 1))/((B*f^3 + 4*B*d^2*e^
2*f)*((d*x + 1)^(1/2) - 1)))*(f^2 + 4*d^2*e^2))/(2*d^5) - (2*A*e*atan((A*e*
((1 - d*x)^(1/2) - 1)*(3*f^2 + 2*d^2*e^2))/((2*A*d^2*e^3 + 3*A*e*f^2)*((d*x
 + 1)^(1/2) - 1)))*(3*f^2 + 2*d^2*e^2))/d^3 - (C*e*atan((C*e*((1 - d*x)^(1/
2) - 1)*(9*f^2 + 4*d^2*e^2))/((4*C*d^2*e^3 + 9*C*e*f^2)*((d*x + 1)^(1/2) -
1)))*(9*f^2 + 4*d^2*e^2))/(2*d^5)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
[Out] Timed out
```

3.9 $\int \frac{(e+fx)^2(A+Bx+Cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=228

$$\frac{\sin^{-1}(dx)\left(4d^2\left(A\left(2d^2e^2+f^2\right)+2Bef\right)+C\left(4d^2e^2+3f^2\right)\right)}{8d^5}+\frac{\sqrt{1-d^2x^2}\left(4\left(C\left(d^2e^3-8ef^2\right)-4f\left(3Ad^2ef+\right.\right.\right.}{}$$

Rubi [A] time = 0.49, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1609, 1654, 833, 780, 216}

$$\frac{\sqrt{1-d^2x^2}\left(4\left(C\left(d^2e^3-8ef^2\right)-4f\left(3Ad^2ef+B\left(d^2e^2+f^2\right)\right)\right)-fx\left(3f^2\left(4Ad^2+3C\right)-2d^2e(Ce-4Bf)\right)\right)}{24d^4f}+\frac{\sin^{-1}(dx)\left(4d^2\left(A\left(2d^2e^2+f^2\right)+2Bef\right)+C\left(4d^2e^2+3f^2\right)\right)}{8d^5}+\frac{\sqrt{1-d^2x^2}(e+fx)^2(Ce-4Bf)}{12d^2f}-\frac{C\sqrt{1-d^2x^2}(e+fx)^3}{4d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((C*e - 4*B*f)*(e + f*x)^2*Sqrt[1 - d^2*x^2])/(12*d^2*f) - (C*(e + f*x)^3*S
qrt[1 - d^2*x^2])/(4*d^2*f) + ((4*(C*(d^2*e^3 - 8*e*f^2) - 4*f*(3*A*d^2*e*f
 + B*(d^2*e^2 + f^2))) - f*(3*(3*C + 4*A*d^2)*f^2 - 2*d^2*e*(C*e - 4*B*f))*
x)*Sqrt[1 - d^2*x^2])/(24*d^4*f) + ((C*(4*d^2*e^2 + 3*f^2) + 4*d^2*(2*B*e*f
 + A*(2*d^2*e^2 + f^2)))*ArcSin[d*x])/(8*d^5)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
```

```
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{1-d^2x^2}}\,dx\\ &= -\frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f}-\frac{\int \frac{(e+fx)^2\left(-\left(\left(3C+4Ad^2\right)f^2\right)+d^2f(Ce-4Bf)x\right)}{\sqrt{1-d^2x^2}}\,dx}{4d^2f^2}\\ &= \frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f}-\frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f}+\frac{\int \frac{(e+fx)\left(d^2f^2\left(7Ce+12A\right.\right.}{}}{}\\ &= \frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f}-\frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f}+\frac{\left(4\left(C\left(d^2e^3-8ef^2\right)\right.\right.}{}\\ &= \frac{(Ce-4Bf)(e+fx)^2\sqrt{1-d^2x^2}}{12d^2f}-\frac{C(e+fx)^3\sqrt{1-d^2x^2}}{4d^2f}+\frac{\left(4\left(C\left(d^2e^3-8ef^2\right)\right.\right.}{}\end{aligned}$$

Mathematica [A] time = 0.22, size = 160, normalized size = 0.70

$$\frac{3\sin^{-1}(dx)\left(4d^2\left(A\left(2d^2e^2+f^2\right)+2Bef\right)+C\left(4d^2e^2+3f^2\right)\right)-d\sqrt{1-d^2x^2}\left(12Ad^2f(4e+fx)+8B\left(d^2\left(3e^2+3efx+f^2x^2\right)+2f^2\right)+C\left(12d^2e^2x+16ef\left(d^2x^2+2\right)+3f^2x\left(2d^2x^2+3\right)\right)\right)}{24d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*Sqrt[1 - d^2*x^2]*(12*A*d^2*f*(4*e + f*x) + C*(12*d^2*e^2*x + 16*e*f*(
2 + d^2*x^2) + 3*f^2*x*(3 + 2*d^2*x^2)) + 8*B*(2*f^2 + d^2*(3*e^2 + 3*e*f*x
 + f^2*x^2)))) + 3*(C*(4*d^2*e^2 + 3*f^2) + 4*d^2*(2*B*e*f + A*(2*d^2*e^2 +
 f^2)))*ArcSin[d*x])/(24*d^5)
```

IntegrateAlgebraic [B] time = 0.47, size = 708, normalized size = 3.11

$$\frac{\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)\left(-8Ad^4e^2-4Ad^2f^2-8Bd^2ef-4Cd^2e^2-3Cf^2\right)}{4d^5}-\frac{\sqrt{1-dx}\left(\frac{144Ad^3ef(1-dx)}{dx+1}+\frac{144Ad^3ef(1-dx)^2}{(dx+1)^2}+\frac{48Ad^3ef(1-dx)^3}{(dx+1)^3}+48Ad^3ef+\frac{12Ad^2f^2(1-dx)}{dx+1}-\frac{12Ad^2f^2(1-dx)^2}{(dx+1)^2}-\frac{12Ad^2f^2(1-dx)^3}{(dx+1)^3}+12Ad^2f^2+\frac{72Bd^3e^2(1-dx)}{dx+1}+\frac{72Bd^3e^2(1-dx)^2}{(dx+1)^2}+\frac{24Bd^3e^2(1-dx)^3}{(dx+1)^3}+24Bd^3e^2+\frac{24Bd^2ef(1-dx)}{dx+1}-\frac{24Bd^2ef(1-dx)^2}{(dx+1)^2}-\frac{24Bd^2ef(1-dx)^3}{(dx+1)^3}+24Bd^2ef+\frac{40Bdf^2(1-dx)}{dx+1}+\frac{40Bdf^2(1-dx)^2}{(dx+1)^2}+\frac{24Bdf^2(1-dx)^3}{(dx+1)^3}+24Bdf^2+\frac{12Cd^2e^2(1-dx)}{dx+1}-\frac{12Cd^2e^2(1-dx)^2}{(dx+1)^2}-\frac{12Cd^2e^2(1-dx)^3}{(dx+1)^3}+12Cd^2e^2+\frac{80Cdef(1-dx)}{dx+1}+\frac{80Cdef(1-dx)^2}{(dx+1)^2}+\frac{48Cdef(1-dx)^3}{(dx+1)^3}+48Cdef-\frac{9Cf^2(1-dx)}{dx+1}+\frac{9Cf^2(1-dx)^2}{(dx+1)^2}-\frac{15Cf^2(1-dx)^3}{(dx+1)^3}+15Cf^2\right)}{12d^5\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 +
d*x]),x]
```

```
[Out] -1/12*(Sqrt[1 - d*x]*(12*C*d^2*e^2 + 24*B*d^3*e^2 + 48*C*d*e*f + 24*B*d^2*e
*f + 48*A*d^3*e*f + 15*C*f^2 + 24*B*d*f^2 + 12*A*d^2*f^2 - (12*C*d^2*e^2*(1
 - d*x)^3)/(1 + d*x)^3 + (24*B*d^3*e^2*(1 - d*x)^3)/(1 + d*x)^3 + (48*C*d*e
*f*(1 - d*x)^3)/(1 + d*x)^3 - (24*B*d^2*e*f*(1 - d*x)^3)/(1 + d*x)^3 + (48*
A*d^3*e*f*(1 - d*x)^3)/(1 + d*x)^3 - (15*C*f^2*(1 - d*x)^3)/(1 + d*x)^3 + (
24*B*d*f^2*(1 - d*x)^3)/(1 + d*x)^3 - (12*A*d^2*f^2*(1 - d*x)^3)/(1 + d*x)^
3 - (12*C*d^2*e^2*(1 - d*x)^2)/(1 + d*x)^2 + (72*B*d^3*e^2*(1 - d*x)^2)/(1
+ d*x)^2 + (80*C*d*e*f*(1 - d*x)^2)/(1 + d*x)^2 - (24*B*d^2*e*f*(1 - d*x)^2
)/(1 + d*x)^2 + (144*A*d^3*e*f*(1 - d*x)^2)/(1 + d*x)^2 + (9*C*f^2*(1 - d*x
)^2)/(1 + d*x)^2 + (40*B*d*f^2*(1 - d*x)^2)/(1 + d*x)^2 - (12*A*d^2*f^2*(1
- d*x)^2)/(1 + d*x)^2 + (12*C*d^2*e^2*(1 - d*x))/(1 + d*x) + (72*B*d^3*e^2*
```

(1 - d*x))/(1 + d*x) + (80*C*d*e*f*(1 - d*x))/(1 + d*x) + (24*B*d^2*e*f*(1
- d*x))/(1 + d*x) + (144*A*d^3*e*f*(1 - d*x))/(1 + d*x) - (9*C*f^2*(1 - d*x
))/(1 + d*x) + (40*B*d*f^2*(1 - d*x))/(1 + d*x) + (12*A*d^2*f^2*(1 - d*x))/
(1 + d*x)))/(d^5*Sqrt[1 + d*x]*(1 + (1 - d*x)/(1 + d*x))^4) + ((-4*C*d^2*e^
2 - 8*A*d^4*e^2 - 8*B*d^2*e*f - 3*C*f^2 - 4*A*d^2*f^2)*ArcTan[Sqrt[1 - d*x]
/Sqrt[1 + d*x]])/(4*d^5)

fricas [A] time = 0.82, size = 192, normalized size = 0.84

$$-\frac{\left(6\,Cd^{3}f^{2}x^{3}+24\,Bd^{3}e^{2}+16\,Bdf^{2}+16\left(3\,Ad^{3}+2\,Cd\right)ef+8\left(2\,Cd^{3}ef+Bd^{3}f^{2}\right)x^{2}+3\left(4\,Cd^{3}e^{2}+8\,Bd^{3}ef+\left(4\,Ad^{3}+3\,Cd\right)f^{2}\right)x\right)\sqrt{dx+1}\,\sqrt{-dx+1}+6\left(8\,Bd^{2}ef+4\left(2\,Ad^{4}+Cd^{2}\right)e^{2}+\left(4\,Ad^{2}+3\,C\right)f^{2}\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{24\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")

[Out] -1/24*((6*C*d^3*f^2*x^3 + 24*B*d^3*e^2 + 16*B*d*f^2 + 16*(3*A*d^3 + 2*C*d)*
e*f + 8*(2*C*d^3*e*f + B*d^3*f^2)*x^2 + 3*(4*C*d^3*e^2 + 8*B*d^3*e*f + (4*A
*d^3 + 3*C*d)*f^2)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(8*B*d^2*e*f + 4*(2*
A*d^4 + C*d^2)*e^2 + (4*A*d^2 + 3*C)*f^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x +
1) - 1)/(d*x)))/d^5

giac [A] time = 1.64, size = 277, normalized size = 1.21

$$-\frac{\left(\left(dx+1\right)\left(2\left(dx+1\right)\left(\frac{3\,(dx+1)Cf^{2}}{d^{4}}+\frac{4\,Bd^{17}f^{2}+8\,Cd^{17}fe-9\,Cd^{16}f^{2}}{d^{20}}\right)+\frac{12\,Ad^{18}f^{2}+24\,Bd^{18}fe-16\,Bd^{17}f^{2}+12\,Cd^{18}e^{2}-32\,Cd^{17}fe+27\,Cd^{16}f^{2}}{d^{20}}\right)+\frac{3\left(16\,Ad^{19}fe-4\,Ad^{18}f^{2}+8\,Bd^{19}e^{2}-8\,Bd^{18}fe+8\,Bd^{17}f^{2}-4\,Cd^{18}e^{2}+16\,Cd^{17}fe-5\,Cd^{16}f^{2}\right)}{d^{20}}\right)\sqrt{dx+1}\,\sqrt{-dx+1}-\frac{6\left(8\,Ad^{4}e^{2}+4\,Ad^{2}f^{2}+8\,Bd^{2}fe+4\,Cd^{2}e^{2}+3\,Cf^{2}\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")

[Out] -1/24*(((d*x + 1)*(2*(d*x + 1)*(3*(d*x + 1)*C*f^2/d^4 + (4*B*d^17*f^2 + 8*C
*d^17*f*e - 9*C*d^16*f^2)/d^20) + (12*A*d^18*f^2 + 24*B*d^18*f*e - 16*B*d^1
7*f^2 + 12*C*d^18*e^2 - 32*C*d^17*f*e + 27*C*d^16*f^2)/d^20) + 3*(16*A*d^19
*f*e - 4*A*d^18*f^2 + 8*B*d^19*e^2 - 8*B*d^18*f*e + 8*B*d^17*f^2 - 4*C*d^18
*e^2 + 16*C*d^17*f*e - 5*C*d^16*f^2)/d^20)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 6
*(8*A*d^4*e^2 + 4*A*d^2*f^2 + 8*B*d^2*f*e + 4*C*d^2*e^2 + 3*C*f^2)*arcsin(1
/2*sqrt(2)*sqrt(d*x + 1))/d^4)/d

maple [C] time = 0.03, size = 423, normalized size = 1.86

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\left(6\sqrt{-d^{2}x^{2}+1}\,Cd^{3}f^{2}x^{3}\operatorname{csgn}(d)+8\sqrt{-d^{2}x^{2}+1}\,Bd^{3}f^{2}x^{2}\operatorname{csgn}(d)+16\sqrt{-d^{2}x^{2}+1}\,Cd^{3}efx^{2}\operatorname{csgn}(d)-24Ad^{4}e^{2}\arctan\left(\frac{dx\operatorname{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)+12\sqrt{-d^{2}x^{2}+1}\,Ad^{3}f^{2}x\operatorname{csgn}(d)+24\sqrt{-d^{2}x^{2}+1}\,Bd^{3}efx\operatorname{csgn}(d)+12\sqrt{-d^{2}x^{2}+1}\,Cd^{3}e^{2}x\operatorname{csgn}(d)+48\sqrt{-d^{2}x^{2}+1}\,Ad^{3}ef\operatorname{csgn}(d)+24\sqrt{-d^{2}x^{2}+1}\,Bd^{3}e^{2}\operatorname{csgn}(d)-12Ad^{2}f^{2}\arctan\left(\frac{dx\operatorname{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)-24Bd^{2}ef\arctan\left(\frac{dx\operatorname{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)-12Cd^{2}e^{2}\arctan\left(\frac{dx\operatorname{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)+9\sqrt{-d^{2}x^{2}+1}\,Cdf^{2}x\operatorname{csgn}(d)+16\sqrt{-d^{2}x^{2}+1}\,Bdf^{2}\operatorname{csgn}(d)+32\sqrt{-d^{2}x^{2}+1}\,Cdef\operatorname{csgn}(d)-9Cf^{2}\arctan\left(\frac{dx\operatorname{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)\right)\operatorname{csgn}(d)}{24\sqrt{-d^{2}x^{2}+1}\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)

[Out] -1/24*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(6*(-d^2*x^2+1)^(1/2)*C*d^3*f^2*x^3*csgn
(d)+8*(-d^2*x^2+1)^(1/2)*B*d^3*f^2*x^2*csgn(d)+16*(-d^2*x^2+1)^(1/2)*C*d^3*
e*f*x^2*csgn(d)+12*(-d^2*x^2+1)^(1/2)*A*d^3*f^2*x*csgn(d)+24*(-d^2*x^2+1)^(
1/2)*B*d^3*e*f*x*csgn(d)+12*(-d^2*x^2+1)^(1/2)*C*d^3*e^2*x*csgn(d)+48*(-d^2
*x^2+1)^(1/2)*A*d^3*e*f*csgn(d)-24*A*d^4*e^2*arctan(1/(-d^2*x^2+1)^(1/2)*d*
x*csgn(d))+24*(-d^2*x^2+1)^(1/2)*B*d^3*e^2*csgn(d)+9*(-d^2*x^2+1)^(1/2)*C*d
*f^2*x*csgn(d)-12*A*d^2*f^2*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))+16*(-d
^2*x^2+1)^(1/2)*B*d*f^2*csgn(d)-24*B*d^2*e*f*arctan(1/(-d^2*x^2+1)^(1/2)*d*
x*csgn(d))+32*(-d^2*x^2+1)^(1/2)*C*d*e*f*csgn(d)-12*C*d^2*e^2*arctan(1/(-d^
2*x^2+1)^(1/2)*d*x*csgn(d))-9*C*f^2*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d)
))*csgn(d)/d^5/(-d^2*x^2+1)^(1/2)

maxima [A] time = 1.27, size = 231, normalized size = 1.01

$$-\frac{\sqrt{-d^{2}x^{2}+1}\,Cf^{2}x^{3}}{4\,d^{2}}+\frac{Ae^{2}\arcsin\left(dx\right)}{d}-\frac{\sqrt{-d^{2}x^{2}+1}\,Be^{2}}{d^{2}}-\frac{2\sqrt{-d^{2}x^{2}+1}\,Aef}{d^{2}}-\frac{\sqrt{-d^{2}x^{2}+1}\left(2\,Cef+Bf^{2}\right)x^{2}}{3\,d^{2}}-\frac{\sqrt{-d^{2}x^{2}+1}\left(Ce^{2}+2\,Bef+Af^{2}\right)x}{2\,d^{2}}-\frac{3\sqrt{-d^{2}x^{2}+1}\,Cf^{2}x}{8\,d^{4}}+\frac{\left(Ce^{2}+2\,Bef+Af^{2}\right)\arcsin\left(dx\right)}{2\,d^{3}}+\frac{3\,Cf^{2}\arcsin\left(dx\right)}{8\,d^{5}}-\frac{2\sqrt{-d^{2}x^{2}+1}\left(2\,Cef+Bf^{2}\right)}{3\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] -1/4*sqrt(-d^2*x^2 + 1)*C*f^2*x^3/d^2 + A*e^2*arcsin(d*x)/d - sqrt(-d^2*x^2
 + 1)*B*e^2/d^2 - 2*sqrt(-d^2*x^2 + 1)*A*e*f/d^2 - 1/3*sqrt(-d^2*x^2 + 1)*(
2*C*e*f + B*f^2)*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(C*e^2 + 2*B*e*f + A*f^2)
*x/d^2 - 3/8*sqrt(-d^2*x^2 + 1)*C*f^2*x/d^4 + 1/2*(C*e^2 + 2*B*e*f + A*f^2)
*arcsin(d*x)/d^3 + 3/8*C*f^2*arcsin(d*x)/d^5 - 2/3*sqrt(-d^2*x^2 + 1)*(2*C*
e*f + B*f^2)/d^4
```

mupad [B] time = 33.64, size = 1732, normalized size = 7.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^2*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((14*A*f^2*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (2*A*f^2*((
1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (14*A*f^2*((1 - d*x)^(1/2) - 1
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*A*f^2*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)
^(1/2) - 1)^7 + (16*A*d*e*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^
2 + (32*A*d*e*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16*A*d*
e*f*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6)/(d^3 + (4*d^3*((1 - d
*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((1 - d*x)^(1/2) - 1)^4)
/((d*x + 1)^(1/2) - 1)^4 + (4*d^3*((1 - d*x)^(1/2) - 1)^6)/((d*x + 1)^(1/2)
 - 1)^6 + (d^3*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8) - ((((1 -
d*x)^(1/2) - 1)^4*(64*B*f^2 + 32*B*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (((1
 - d*x)^(1/2) - 1)^8*(64*B*f^2 + 32*B*d^2*e^2))/((d*x + 1)^(1/2) - 1)^8 - (
((1 - d*x)^(1/2) - 1)^6*((128*B*f^2)/3 - 48*B*d^2*e^2))/((d*x + 1)^(1/2) -
1)^6 + (8*B*d^2*e^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (8*B
*d^2*e^2*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (20*B*d*e*f*(
(1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (24*B*d*e*f*((1 - d*x)^(1
/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (24*B*d*e*f*((1 - d*x)^(1/2) - 1)^7)/
((d*x + 1)^(1/2) - 1)^7 - (20*B*d*e*f*((1 - d*x)^(1/2) - 1)^9)/((d*x + 1)^(
1/2) - 1)^9 + (4*B*d*e*f*((1 - d*x)^(1/2) - 1)^11)/((d*x + 1)^(1/2) - 1)^11
 - (4*B*d*e*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^4 + (6*d^4*(
(1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (15*d^4*((1 - d*x)^(1/2)
- 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (20*d^4*((1 - d*x)^(1/2) - 1)^6)/((d*x +
1)^(1/2) - 1)^6 + (15*d^4*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8
+ (6*d^4*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (d^4*((1 - d*
x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12) - ((((1 - d*x)^(1/2) - 1)^15*((
3*C*f^2)/2 + 2*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^15 - (((1 - d*x)^(1/2) - 1
)^3*((23*C*f^2)/2 - 6*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^3 - (((1 - d*x)^(1/
2) - 1)*((3*C*f^2)/2 + 2*C*d^2*e^2))/((d*x + 1)^(1/2) - 1) + (((1 - d*x)^(1
/2) - 1)^13*((23*C*f^2)/2 - 6*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^13 + (((1 -
 d*x)^(1/2) - 1)^5*((333*C*f^2)/2 + 30*C*d^2*e^2))/((d*x + 1)^(1/2) - 1)^5
- (((1 - d*x)^(1/2) - 1)^11*((333*C*f^2)/2 + 30*C*d^2*e^2))/((d*x + 1)^(1/2
) - 1)^11 - (((1 - d*x)^(1/2) - 1)^7*((671*C*f^2)/2 - 22*C*d^2*e^2))/((d*x
+ 1)^(1/2) - 1)^7 + (((1 - d*x)^(1/2) - 1)^9*((671*C*f^2)/2 - 22*C*d^2*e^2)
)/((d*x + 1)^(1/2) - 1)^9 + (128*C*d*e*f*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1
)^(1/2) - 1)^4 + (512*C*d*e*f*((1 - d*x)^(1/2) - 1)^6)/(3*((d*x + 1)^(1/2)
- 1)^6) + (256*C*d*e*f*((1 - d*x)^(1/2) - 1)^8)/(3*((d*x + 1)^(1/2) - 1)^8)
 + (512*C*d*e*f*((1 - d*x)^(1/2) - 1)^10)/(3*((d*x + 1)^(1/2) - 1)^10) + (1
28*C*d*e*f*((1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12)/(d^5 + (8*d^
5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (28*d^5*((1 - d*x)^(1/
2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (56*d^5*((1 - d*x)^(1/2) - 1)^6)/((d*x
 + 1)^(1/2) - 1)^6 + (70*d^5*((1 - d*x)^(1/2) - 1)^8)/((d*x + 1)^(1/2) - 1)
^8 + (56*d^5*((1 - d*x)^(1/2) - 1)^10)/((d*x + 1)^(1/2) - 1)^10 + (28*d^5*(
(1 - d*x)^(1/2) - 1)^12)/((d*x + 1)^(1/2) - 1)^12 + (8*d^5*((1 - d*x)^(1/2)
```

```
 - 1)^14)/((d*x + 1)^(1/2) - 1)^14 + (d^5*((1 - d*x)^(1/2) - 1)^16)/((d*x +
 1)^(1/2) - 1)^16) - (C*atan((C*((1 - d*x)^(1/2) - 1)*(3*f^2 + 4*d^2*e^2))/
(((d*x + 1)^(1/2) - 1)*(3*C*f^2 + 4*C*d^2*e^2)))*(3*f^2 + 4*d^2*e^2))/(2*d^
5) - (2*A*atan((A*(f^2 + 2*d^2*e^2)*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2
) - 1)*(A*f^2 + 2*A*d^2*e^2)))*(f^2 + 2*d^2*e^2))/d^3 - (4*B*e*f*atan(((1 -
 d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.10 $\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{1-d^2x^2}\left(2\left(3d^2f(Af+Be)-C\left(d^2e^2-2f^2\right)\right)-d^2fx(Ce-3Bf)\right)}{6d^4f}+\frac{\sin^{-1}(dx)\left(2Ad^2e+Bf+Ce\right)}{2d^3}-\frac{C\sqrt{1-d^2}}{3}$$

Rubi [A] time = 0.23, antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1609, 1654, 780, 216}

$$-\frac{\sqrt{1-d^2x^2}\left(2\left(3d^2f(Af+Be)-\frac{1}{2}C\left(2d^2e^2-4f^2\right)\right)-d^2fx(Ce-3Bf)\right)}{6d^4f}+\frac{\sin^{-1}(dx)\left(2Ad^2e+Bf+Ce\right)}{2d^3}-\frac{C\sqrt{1-d^2x^2}\,(e+fx)^2}{3d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(C*(e + f*x)^2*Sqrt[1 - d^2*x^2])/(3*d^2*f) - ((2*(3*d^2*f*(B*e + A*f) - (
C*(2*d^2*e^2 - 4*f^2))/2) - d^2*f*(C*e - 3*B*f)*x)*Sqrt[1 - d^2*x^2])/(6*d^
4*f) + ((C*e + 2*A*d^2*e + B*f)*ArcSin[d*x])/(2*d^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f} - \frac{\int \frac{(e+fx)\left(-\left(\left(2C+3Ad^2\right)f^2\right)+d^2f(Ce-3Bf)x\right)}{\sqrt{1-d^2x^2}}dx}{3d^2f^2}$$

$$= -\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f} - \frac{\left(2\left(3d^2f(Be+Af)-\frac{1}{2}C\left(2d^2e^2-4f^2\right)\right)-d^2f(Ce\right.}{6d^4f}$$

$$= -\frac{C(e+fx)^2\sqrt{1-d^2x^2}}{3d^2f} - \frac{\left(2\left(3d^2f(Be+Af)-\frac{1}{2}C\left(2d^2e^2-4f^2\right)\right)-d^2f(Ce\right.}{6d^4f}$$

Mathematica [A] time = 0.10, size = 88, normalized size = 0.68

$$\frac{3d\sin^{-1}(dx)\left(2Ad^2e+Bf+Ce\right)-\sqrt{1-d^2x^2}\left(6Ad^2f+3Bd^2(2e+fx)+C\left(3d^2ex+2d^2fx^2+4f\right)\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(Sqrt[1 - d^2*x^2]*(6*A*d^2*f + 3*B*d^2*(2*e + f*x) + C*(4*f + 3*d^2*e*x
+ 2*d^2*f*x^2))) + 3*d*(C*e + 2*A*d^2*e + B*f)*ArcSin[d*x])/(6*d^4)
```

IntegrateAlgebraic [B] time = 0.26, size = 275, normalized size = 2.12

$$\frac{\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)\left(-2Ad^2e-Bf-Ce\right)}{d^3}-\frac{\sqrt{1-dx}\left(\frac{12Ad^2f(1-dx)}{dx+1}+\frac{6Ad^2f(1-dx)^2}{(dx+1)^2}+6Ad^2f+\frac{12Bd^2e(1-dx)}{dx+1}+\frac{6Bd^2e(1-dx)^2}{(dx+1)^2}+6Bd^2e-\frac{3Bdf(1-dx)^2}{(dx+1)^2}+3Bdf-\frac{3Cde(1-dx)^2}{(dx+1)^2}+3Cde+\frac{4Cf(1-dx)}{dx+1}+\frac{6Cf(1-dx)^2}{(dx+1)^2}+6Cf\right)}{3d^4\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*
x]),x]
```

```
[Out] -1/3*(Sqrt[1 - d*x]*(3*C*d*e + 6*B*d^2*e + 6*C*f + 3*B*d*f + 6*A*d^2*f - (3
*C*d*e*(1 - d*x)^2)/(1 + d*x)^2 + (6*B*d^2*e*(1 - d*x)^2)/(1 + d*x)^2 + (6*
C*f*(1 - d*x)^2)/(1 + d*x)^2 - (3*B*d*f*(1 - d*x)^2)/(1 + d*x)^2 + (6*A*d^2
*f*(1 - d*x)^2)/(1 + d*x)^2 + (12*B*d^2*e*(1 - d*x))/(1 + d*x) + (4*C*f*(1
- d*x))/(1 + d*x) + (12*A*d^2*f*(1 - d*x))/(1 + d*x)))/(d^4*Sqrt[1 + d*x]*(
1 + (1 - d*x)/(1 + d*x))^3) + ((-(C*e) - 2*A*d^2*e - B*f)*ArcTan[Sqrt[1 - d
*x]/Sqrt[1 + d*x]])/d^3
```

fricas [A] time = 0.96, size = 114, normalized size = 0.88

$$-\frac{\left(2\,Cd^2fx^2+6\,Bd^2e+2\left(3\,Ad^2+2\,C\right)f+3\left(Cd^2e+Bd^2f\right)x\right)\sqrt{dx+1}\,\sqrt{-dx+1}+6\left(Bdf+\left(2\,Ad^3+Cd\right)e\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] -1/6*((2*C*d^2*f*x^2 + 6*B*d^2*e + 2*(3*A*d^2 + 2*C)*f + 3*(C*d^2*e + B*d^2
*f)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 6*(B*d*f + (2*A*d^3 + C*d)*e)*arctan(
(sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^4
```

giac [A] time = 1.31, size = 146, normalized size = 1.12

$$-\frac{\sqrt{dx+1}\,\sqrt{-dx+1}\left((dx+1)\left(\frac{2\,(dx+1)Cf}{d^3}+\frac{3\,Bd^{10}f+3\,Cd^{10}e-4\,Cd^9f}{d^{12}}\right)+\frac{3\left(2\,Ad^{11}f+2\,Bd^{11}e-Bd^{10}f-Cd^{10}e+2\,Cd^9f\right)}{d^{12}}\right)-\frac{6\left(2\,Ad^2e+Bf+Ce\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] -1/6*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)*C*f/d^3 + (3*B*d
^10*f + 3*C*d^10*e - 4*C*d^9*f)/d^12) + 3*(2*A*d^11*f + 2*B*d^11*e - B*d^10
*f - C*d^10*e + 2*C*d^9*f)/d^12) - 6*(2*A*d^2*e + B*f + C*e)*arcsin(1/2*sqr
t(2)*sqrt(d*x + 1))/d^2)/d
```

maple [C] time = 0.02, size = 235, normalized size = 1.81

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(2\sqrt{-d^2x^2+1}\,C\,d^2 f\,x^2\mathrm{csgn}(d)-6A\,d^3 e\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+3\sqrt{-d^2x^2+1}\,B\,d^2 f x\,\mathrm{csgn}(d)+3\sqrt{-d^2x^2+1}\,C\,d^2 e x\,\mathrm{csgn}(d)+6\sqrt{-d^2x^2+1}\,A\,d^2 f\,\mathrm{csgn}(d)+6\sqrt{-d^2x^2+1}\,B\,d^2 e\,\mathrm{csgn}(d)-3Bdf\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)-3Cde\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+4\sqrt{-d^2x^2+1}\,Cf\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{6\sqrt{-d^2x^2+1}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/6*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(2*(-d^2*x^2+1)^(1/2)*C*d^2*f*x^2*csgn(d)
+3*(-d^2*x^2+1)^(1/2)*B*d^2*f*x*csgn(d)+3*(-d^2*x^2+1)^(1/2)*C*d^2*e*x*csgn
(d)+6*(-d^2*x^2+1)^(1/2)*A*d^2*f*csgn(d)-6*A*d^3*e*arctan(1/(-d^2*x^2+1)^(1
/2)*d*x*csgn(d))+6*(-d^2*x^2+1)^(1/2)*B*d^2*e*csgn(d)-3*B*d*f*arctan(1/(-d^
2*x^2+1)^(1/2)*d*x*csgn(d))+4*(-d^2*x^2+1)^(1/2)*C*f*csgn(d)-3*C*d*e*arctan
(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d)))*csgn(d)/d^4/(-d^2*x^2+1)^(1/2)
```

maxima [A] time = 1.31, size = 131, normalized size = 1.01

$$-\frac{\sqrt{-d^2x^2+1}\,Cfx^2}{3\,d^2}+\frac{Ae\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}\,Be}{d^2}-\frac{\sqrt{-d^2x^2+1}\,Af}{d^2}-\frac{\sqrt{-d^2x^2+1}\,(Ce+Bf)x}{2\,d^2}+\frac{(Ce+Bf)\arcsin(dx)}{2\,d^3}-\frac{2\sqrt{-d^2x^2+1}\,Cf}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] -1/3*sqrt(-d^2*x^2 + 1)*C*f*x^2/d^2 + A*e*arcsin(d*x)/d - sqrt(-d^2*x^2 + 1
)*B*e/d^2 - sqrt(-d^2*x^2 + 1)*A*f/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*(C*e + B*f)
*x/d^2 + 1/2*(C*e + B*f)*arcsin(d*x)/d^3 - 2/3*sqrt(-d^2*x^2 + 1)*C*f/d^4
```

mupad [B] time = 12.86, size = 492, normalized size = 3.78

$$\frac{\frac{2Bf\left(\sqrt{1-dx}-1\right)}{\sqrt{dx+1}-1}-\frac{14Bf\left(\sqrt{1-dx}-1\right)^3}{\left(\sqrt{dx+1}-1\right)^3}+\frac{14Bf\left(\sqrt{1-dx}-1\right)^5}{\left(\sqrt{dx+1}-1\right)^5}-\frac{2Bf\left(\sqrt{1-dx}-1\right)^7}{\left(\sqrt{dx+1}-1\right)^7}}{d^3\left(\frac{\left(\sqrt{1-dx}-1\right)^2}{\left(\sqrt{dx+1}-1\right)^2}+1\right)^4}-\frac{\sqrt{1-dx}\left(\frac{2Cf}{3d^4}+\frac{2Cfx}{3d^3}+\frac{Cfx^3}{3d}+\frac{Cfx^2}{3d^2}\right)}{\sqrt{dx+1}}+\frac{\frac{2Ce\left(\sqrt{1-dx}-1\right)}{\sqrt{dx+1}-1}-\frac{14Ce\left(\sqrt{1-dx}-1\right)^3}{\left(\sqrt{dx+1}-1\right)^3}+\frac{14Ce\left(\sqrt{1-dx}-1\right)^5}{\left(\sqrt{dx+1}-1\right)^5}-\frac{2Ce\left(\sqrt{1-dx}-1\right)^7}{\left(\sqrt{dx+1}-1\right)^7}}{d^3\left(\frac{\left(\sqrt{1-dx}-1\right)^2}{\left(\sqrt{dx+1}-1\right)^2}+1\right)^4}-\frac{\left(\frac{Af}{d^2}+\frac{Afx}{d}\right)\sqrt{1-dx}}{\sqrt{dx+1}}-\frac{\left(\frac{Be}{d^2}+\frac{Bex}{d}\right)\sqrt{1-dx}}{\sqrt{dx+1}}-\frac{4Ae\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-dx}-1\right)}{\left(\sqrt{dx+1}-1\right)\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2Bf\,\mathrm{atan}\left(\frac{\sqrt{1-dx}-1}{\sqrt{dx+1}-1}\right)}{d^3}-\frac{2Ce\,\mathrm{atan}\left(\frac{\sqrt{1-dx}-1}{\sqrt{dx+1}-1}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(A + B*x + C*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((2*B*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (14*B*f*((1 - d*x)^(
1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*B*f*((1 - d*x)^(1/2) - 1)^5)/((d
*x + 1)^(1/2) - 1)^5 - (2*B*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1
)^7)/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4) - ((1 -
d*x)^(1/2)*((2*C*f)/(3*d^4) + (2*C*f*x)/(3*d^3) + (C*f*x^3)/(3*d) + (C*f*x^
2)/(3*d^2)))/(d*x + 1)^(1/2) + ((2*C*e*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1
/2) - 1) - (14*C*e*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*C
*e*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (2*C*e*((1 - d*x)^(1/
2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7)/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1
)^(1/2) - 1)^2 + 1)^4) - (((A*f)/d^2 + (A*f*x)/d)*(1 - d*x)^(1/2))/(d*x + 1
)^(1/2) - (((B*e)/d^2 + (B*e*x)/d)*(1 - d*x)^(1/2))/(d*x + 1)^(1/2) - (4*A*
e*atan((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2
)^(1/2) - (2*B*f*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - (
2*C*e*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3
```

sympy [C] time = 158.08, size = 617, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*A*e*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), (
)), 1/(d**2*x**2))/(4*pi**(3/2)*d) + A*e*meijerg(((-1/2, -1/4, 0, 1/4, 1/2,
 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(
4*pi**(3/2)*d) - I*A*f*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4,
 0, 1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - A*f*meijerg(((-1
, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_po
lar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) - I*B*e*meijerg(((-1/4, 1/4),
(0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**
(3/2)*d**2) - B*e*meijerg(((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4)
, (-1, -1/2, -1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
- I*B*f*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4,
 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3) + B*f*meijerg(((-3/2, -5/4,
-1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_polar(-2*I*
pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*C*e*meijerg(((-3/4, -1/4), (-1/2, -
1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2
)*d**3) + C*e*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4),
 (-3/2, -1, -1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) - I
*C*f*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/
2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**4) - C*f*meijerg(((-2, -7/4, -3/
2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(-2*I*p
i)/(d**2*x**2))/(4*pi**(3/2)*d**4)
```

3.11 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=63

$$\frac{\left(2Ad^2+C\right)\sin^{-1}(dx)}{2d^3}-\frac{B\sqrt{1-d^2x^2}}{d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}$$

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {899, 1815, 641, 216}

$$\frac{\left(2Ad^2+C\right)\sin^{-1}(dx)}{2d^3}-\frac{B\sqrt{1-d^2x^2}}{d^2}-\frac{Cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((B*Sqrt[1 - d^2*x^2])/d^2) - (C*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((C + 2*A*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{Cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\int \frac{-C-2Ad^2-2Bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{B\sqrt{1-d^2x^2}}{d^2} - \frac{Cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\left(-C-2Ad^2\right)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{B\sqrt{1-d^2x^2}}{d^2} - \frac{Cx\sqrt{1-d^2x^2}}{2d^2} + \frac{\left(C+2Ad^2\right)\sin^{-1}(dx)}{2d^3}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.71

$$\frac{\left(2Ad^2+C\right)\sin^{-1}(dx) - d\sqrt{1-d^2x^2}\,(2B+Cx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*(2*B + C*x)*Sqrt[1 - d^2*x^2]) + (C + 2*A*d^2)*ArcSin[d*x])/(2*d^3)
```

IntegrateAlgebraic [A] time = 0.14, size = 117, normalized size = 1.86

$$\frac{\left(-2Ad^2-C\right)\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)}{d^3} - \frac{\sqrt{1-dx}\left(\frac{2Bd(1-dx)}{dx+1}+2Bd-\frac{C(1-dx)}{dx+1}+C\right)}{d^3\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((Sqrt[1 - d*x]*(C + 2*B*d - (C*(1 - d*x))/(1 + d*x) + (2*B*d*(1 - d*x))/(
1 + d*x)))/(d^3*Sqrt[1 + d*x]*(1 + (1 - d*x)/(1 + d*x))^2)) + ((-C - 2*A*d^
2)*ArcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/d^3
```

fricas [A] time = 1.45, size = 67, normalized size = 1.06

$$-\frac{(Cdx+2\,Bd)\sqrt{dx+1}\,\sqrt{-dx+1}+2\left(2\,Ad^2+C\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((C*d*x + 2*B*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*A*d^2 + C)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

giac [A] time = 1.29, size = 76, normalized size = 1.21

$$-\frac{\sqrt{dx+1}\,\sqrt{-dx+1}\left(\frac{(dx+1)C}{d^2}+\frac{2\,Bd^5-Cd^4}{d^6}\right)-\frac{2\left(2\,Ad^2+C\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*C/d^2 + (2*B*d^5 - C*d^4)/d^6
) - 2*(2*A*d^2 + C)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)/d
```

maple [C] time = 0.02, size = 117, normalized size = 1.86

$$\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(2Ad^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)-\sqrt{-d^2x^2+1}\,Cdx\,\mathrm{csgn}(d)-2\sqrt{-d^2x^2+1}\,Bd\,\mathrm{csgn}(d)+C\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)\right)\mathrm{csgn}(d)}{2\sqrt{-d^2x^2+1}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] 1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*(2*A*d^2*arctan(1/(-d^2*x^2+1)^(1/2)*d
*x*csgn(d))-(-d^2*x^2+1)^(1/2)*C*d*x*csgn(d)-2*(-d^2*x^2+1)^(1/2)*B*d*csgn(
d)+C*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d)))/(-d^2*x^2+1)^(1/2)*csgn(d)
```

maxima [A] time = 1.42, size = 57, normalized size = 0.90

$$\frac{A\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}\,Cx}{2\,d^2}-\frac{\sqrt{-d^2x^2+1}\,B}{d^2}+\frac{C\arcsin(dx)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] A*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*C*x/d^2 - sqrt(-d^2*x^2 + 1)*B/d^2
 + 1/2*C*arcsin(d*x)/d^3
```

mupad [B] time = 7.53, size = 232, normalized size = 3.68

$$-\frac{\frac{14\,C\,\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}-\frac{14\,C\,\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,C\,\left(\sqrt{1-d\,x}-1\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}-\frac{2\,C\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^4}-\frac{4\,A\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2\,C\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\sqrt{1-d\,x}\,\left(\frac{B}{d^2}+\frac{B\,x}{d}\right)}{\sqrt{d\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((14*C*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*C*((1 - d*x
)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*C*((1 - d*x)^(1/2) - 1)^7)/((d
*x + 1)^(1/2) - 1)^7 - (2*C*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(
d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4) - (4*A*atan((d
*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) -
 (2*C*atan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((1 - d*x)^(
1/2)*(B/d^2 + (B*x)/d))/(d*x + 1)^(1/2)
```

sympy [C] time = 49.74, size = 282, normalized size = 4.48

$$-\frac{iAG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{AG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{iBG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{BG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{iCG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}+\frac{CG_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*A*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) + A*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
 ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi
**(3/2)*d) - I*B*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/
4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - B*meijerg(((-1, -3/4,
-1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) - I*C*meijerg(((-3/4, -1/4), (-1/2, -1
/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)
*d**3) + C*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-
3/2, -1, -1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

3.12 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)}\,dx$

Optimal. Leaf size=122

$$\frac{(Af^2-Bef+Ce^2)\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}-\frac{\sin^{-1}(dx)(Ce-Bf)}{df^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

Rubi [A] time = 0.28, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1609, 1654, 844, 216, 725, 204}

$$\frac{(Af^2-Bef+Ce^2)\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)}{f^2\sqrt{d^2e^2-f^2}}-\frac{\sin^{-1}(dx)(Ce-Bf)}{df^2}-\frac{C\sqrt{1-d^2x^2}}{d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] -((C*Sqrt[1 - d^2*x^2])/(d^2*f)) - ((C*e - B*f)*ArcSin[d*x])/(d*f^2) + ((C*
e^2 - B*e*f + A*f^2)*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2
*x^2])])/(f^2*Sqrt[d^2*e^2 - f^2])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
```

```
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{1-d^2x^2}}\,dx$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{\int \frac{-Ad^2f^2+d^2f(Ce-Bf)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2f^2}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{\left(Ce^2-Bef+Af^2\right)\int \frac{1}{(e+fx)\sqrt{1-d^2x^2}} \cdots}{f^2}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} - \frac{\left(Ce^2-Bef+Af^2\right)\text{Subst}\left(\int \frac{1}{-d^2e^2+f^2-\cdots} \cdots\right.}{f^2}$$

$$= -\frac{C\sqrt{1-d^2x^2}}{d^2f} - \frac{(Ce-Bf)\sin^{-1}(dx)}{df^2} + \frac{\left(Ce^2-Bef+Af^2\right)\tan^{-1}\left(\frac{f+d^2ex}{\sqrt{d^2e^2-f^2}\,\sqrt{1-\cdots}} \cdots\right.}{f^2\sqrt{d^2e^2-f^2}}$$

Mathematica [A] time = 0.13, size = 117, normalized size = 0.96

$$\frac{\frac{\left(f(Af-Be)+Ce^2\right)\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\,\sqrt{d^2e^2-f^2}}\right)}{\sqrt{d^2e^2-f^2}} + \frac{\sin^{-1}(dx)(Bf-Ce)}{d} - \frac{Cf\sqrt{1-d^2x^2}}{d^2}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)),x]
```

```
[Out] (-((C*f*Sqrt[1 - d^2*x^2])/d^2) + ((-(C*e) + B*f)*ArcSin[d*x])/d + ((C*e^2
+ f*(-(B*e) + A*f))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*
x^2])])/Sqrt[d^2*e^2 - f^2])/f^2
```

IntegrateAlgebraic [A] time = 0.00, size = 177, normalized size = 1.45

$$-\frac{2\left(Af^2-Bef+Ce^2\right)\tan^{-1}\left(\frac{\sqrt{1-dx}\,\sqrt{-de-f}\,\sqrt{f-de}}{\sqrt{dx+1}\,(de+f)}\right)}{f^2\sqrt{-de-f}\,\sqrt{f-de}} - \frac{2\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)(Bf-Ce)}{df^2} - \frac{2C\sqrt{1-dx}}{d^2f\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)
),x]
```

```
[Out] (-2*C*Sqrt[1 - d*x])/(d^2*f*Sqrt[1 + d*x]*(1 + (1 - d*x)/(1 + d*x))) - (2*(
-(C*e) + B*f)*ArcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/(d*f^2) - (2*(C*e^2 - B*
e*f + A*f^2)*ArcTan[(Sqrt[-(d*e) - f]*Sqrt[-(d*e) + f]*Sqrt[1 - d*x])/((d*e
 + f)*Sqrt[1 + d*x])])/(Sqrt[-(d*e) - f]*f^2*Sqrt[-(d*e) + f])
```

fricas [**B**] time = 19.36, size = 493, normalized size = 4.04

$$\left[-\frac{\left(Cd^{2}e^{2}-Bd^{2}ef+Ad^{2}f^{2}\right)\sqrt{-d^{2}e^{2}+f^{2}}\log\left(\frac{d^{2}efx+f^{2}-\sqrt{-d^{2}e^{2}+f^{2}}\left(d^{2}ex+f\right)-\left(\sqrt{-d^{2}e^{2}+f^{2}}\sqrt{-dx+1}f+\left(d^{2}e^{2}-f^{2}\right)\sqrt{-dx+1}\right)\sqrt{dx+1}}{fx+e}\right)+\left(Cd^{2}e^{2}f-Cf^{3}\right)\sqrt{dx+1}\sqrt{-dx+1}-2\left(Cd^{3}e^{3}-Bd^{3}e^{2}f-Cdef^{2}+Bdf^{3}\right)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{d^{4}e^{2}f^{2}-d^{2}f^{4}}, \frac{2\left(Cd^{2}e^{2}-Bd^{2}ef+Ad^{2}f^{2}\right)\sqrt{d^{2}e^{2}-f^{2}}\arctan\left(-\frac{\sqrt{d^{2}e^{2}-f^{2}}\sqrt{dx+1}\sqrt{-dx+1}e-\sqrt{d^{2}e^{2}-f^{2}}\left(fx+e\right)}{\left(d^{2}e^{2}-f^{2}\right)x}\right)-\left(Cd^{2}e^{2}f-Cf^{3}\right)\sqrt{dx+1}\sqrt{-dx+1}+2\left(Cd^{3}e^{3}-Bd^{3}e^{2}f-Cdef^{2}+Bdf^{3}\right)\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{d^{4}e^{2}f^{2}-d^{2}f^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] [-((C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
+ f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (sqrt(-d^2*e^2 + f^2)*sqrt(-d*
x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (C*d
^2*e^2*f - C*f^3)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 2*(C*d^3*e^3 - B*d^3*e^2*f
 - C*d*e*f^2 + B*d*f^3)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(
d^4*e^2*f^2 - d^2*f^4), (2*(C*d^2*e^2 - B*d^2*e*f + A*d^2*f^2)*sqrt(d^2*e^2
 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(
d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) - (C*d^2*e^2*f - C*f^3)*sqrt
(d*x + 1)*sqrt(-d*x + 1) + 2*(C*d^3*e^3 - B*d^3*e^2*f - C*d*e*f^2 + B*d*f^3
)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^4*e^2*f^2 - d^2*f^4)
]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Undef/Unsigned Inf encountered in limit
```

maple [**C**] time = 0.00, size = 373, normalized size = 3.06

$$\frac{\left(-A\,d^{2}f^{2}\operatorname{csgn}(d)\ln\left(\frac{2d^{2}ex+2\sqrt{-d^{2}x^{2}+1}\sqrt{-\frac{d^{2}e^{2}-f^{2}}{f^{2}}}f+2f}{fx+e}\right)+B\,d^{2}ef\operatorname{csgn}(d)\ln\left(\frac{2d^{2}ex+2\sqrt{-d^{2}x^{2}+1}\sqrt{-\frac{d^{2}e^{2}-f^{2}}{f^{2}}}f+2f}{fx+e}\right)-C\,d^{2}e^{2}\operatorname{csgn}(d)\ln\left(\frac{2d^{2}ex+2\sqrt{-d^{2}x^{2}+1}\sqrt{-\frac{d^{2}e^{2}-f^{2}}{f^{2}}}f+2f}{fx+e}\right)+\sqrt{-\frac{d^{2}e^{2}-f^{2}}{f^{2}}}\,Bd\,f^{2}\arctan\left(\frac{dx\operatorname{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)-\sqrt{-\frac{d^{2}e^{2}-f^{2}}{f^{2}}}\,Cdef\arctan\left(\frac{dx\operatorname{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)-\sqrt{-d^{2}x^{2}+1}\sqrt{-\frac{d^{2}e^{2}-f^{2}}{f^{2}}}\,C\,f^{2}\operatorname{csgn}(d)\right)\sqrt{-dx+1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)}{\sqrt{-\frac{d^{2}e^{2}-f^{2}}{f^{2}}}\,\sqrt{-d^{2}x^{2}+1}\,d^{2}f^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-A*d^2*f^2*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(
1/2)*f+f)/(f*x+e))+B*d^2*e*f*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^
2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))-C*d^2*e^2*csgn(d)*ln(2*(d^2*e*x+(-d^2*x
^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))+(-(d^2*e^2-f^2)/f^2)^(
1/2)*B*d*f^2*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))-(-(d^2*e^2-f^2)/f^2)^
(1/2)*C*d*e*f*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))-(-d^2*x^2+1)^(1/2)*(
-(d^2*e^2-f^2)/f^2)^(1/2)*C*f^2*csgn(d))*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/(-(d^
2*e^2-f^2)/f^2)^(1/2)/(-d^2*x^2+1)^(1/2)/d^2/f^3*csgn(d)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(f-d*e>0)', see `assume?` for more d
etails)Is f-d*e positive, negative or zero?
```

mupad [B] time = 0.01, size = 5803, normalized size = 47.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (4*C*e*atan((37748736*C^5*d^4*e^10*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
 - 1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8
*f^2)) + (67108864*C^5*e^6*f^4*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1
)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8*f^2
)) - (100663296*C^5*d^2*e^8*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) -
1)*(37748736*C^5*d^4*e^10 + 67108864*C^5*e^6*f^4 - 100663296*C^5*d^2*e^8*f^
2))))/(d*f^2) - (4*B*atan((67108864*B^5*e*f^4*((1 - d*x)^(1/2) - 1))/(((d*x
 + 1)^(1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5
*d^2*e^3*f^2)) + (37748736*B^5*d^4*e^5*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(
1/2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5*d^2*e^
3*f^2)) - (100663296*B^5*d^2*e^3*f^2*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/
2) - 1)*(67108864*B^5*e*f^4 + 37748736*B^5*d^4*e^5 - 100663296*B^5*d^2*e^3*
f^2))))/(d*f) - (8*C*((1 - d*x)^(1/2) - 1)^2)/(f*((d*x + 1)^(1/2) - 1)^2*(d
^2 + (2*d^2*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (d^2*((1 - d
*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4)) - (A*atan((f^2*1i - d^2*e^2*1i
- (f^2*((1 - d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2 + (d^2*e^2*((1 -
 d*x)^(1/2) - 1)^2*1i)/((d*x + 1)^(1/2) - 1)^2)/(f*(f + d*e)^(1/2)*(f - d*e
)^(1/2) - (f*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))/((d*x
 + 1)^(1/2) - 1)^2 + (2*d*e*((1 - d*x)^(1/2) - 1)*(f + d*e)^(1/2)*(f - d*e)
^(1/2))/((d*x + 1)^(1/2) - 1)))*2i)/((f + d*e)^(1/2)*(f - d*e)^(1/2)) - (C*
e^2*atan(((C*e^2*((4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (409
6*((1 - d*x)^(1/2) - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x
 + 1)^(1/2) - 1)^2) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1
)^(1/2) - 1)) + (C*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4)
 + (16384*((1 - d*x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d
*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 -
144*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) - (C*
e^2*((4096*(24*C*d^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*
x)^(1/2) - 1)*(20*C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)
) + (4096*(96*C*d^2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d
*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^
6))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))
/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^
8 - 9*d^6*e^5*f^6))/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*
(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(
1/2)*(f - d*e)^(1/2)))*1i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)) + (C*e^2*(
(4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d*x)^(1/2)
 - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/2) - 1)^2
) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) - 1)) - (C
*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384*((1 - d*
x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(1/2) - 1)
) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e^3*f^6 +
9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(24*C*d
^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(20*
C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*(96*C*d^
2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*x + 1)^(1
/2) - 1)^2) - (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4) + (163
84*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d*x + 1)^(
1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6*e^5*f^6)
)/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))
```

/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)))*1i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)))/((131072*C^4*e^7)/(d*f^4) + (C*e^2*((4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d*x)^(1/2) - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) - 1)) + (C*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) - (C*e^2*((4096*(24*C*d^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(20*C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*(96*C*d^2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)) - (C*e^2*((4096*(32*C^3*e^5*f^3 + 24*C^3*d^2*e^7*f))/(d*f^4) - (4096*((1 - d*x)^(1/2) - 1)^2*(32*C^3*e^5*f^3 - 96*C^3*d^2*e^7*f))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (458752*C^3*e^6*((1 - d*x)^(1/2) - 1))/(f^2*((d*x + 1)^(1/2) - 1)) - (C*e^2*((4096*(16*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(8*C^2*e^4*f^3 + 3*C^2*d^2*e^6*f))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(128*C^2*d^2*e^5*f^4 - 144*C^2*e^3*f^6 + 9*C^2*d^4*e^7*f^2))/(d*f^4*((d*x + 1)^(1/2) - 1)^2) + (C*e^2*((4096*(24*C*d^2*e^3*f^7 - 30*C*d^4*e^5*f^5))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(20*C*e^2*f^6 - 22*C*d^2*e^4*f^4))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*(96*C*d^2*e^3*f^7 - 90*C*d^4*e^5*f^5)*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*x + 1)^(1/2) - 1)^2) - (C*e^2*((4096*(7*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4) + (16384*((1 - d*x)^(1/2) - 1)*(5*d^2*e^2*f^7 - 6*d^4*e^4*f^5))/(f^2*((d*x + 1)^(1/2) - 1)) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^8 - 9*d^6*e^5*f^6))/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)) + (917504*C^4*e^7*((1 - d*x)^(1/2) - 1)^2)/(d*f^4*((d*x + 1)^(1/2) - 1)^2)))*2i)/(f^2*(f + d*e)^(1/2)*(f - d*e)^(1/2)) + (B*e*atan(((B*e*((4096*(24*B^3*d^2*e^4 + 32*B^3*e^2*f^2))/d + (4096*((1 - d*x)^(1/2) - 1)^2*(96*B^3*d^2*e^4 - 32*B^3*e^2*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) + (458752*B^3*e^3*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (B*e*((4096*(16*B^2*e*f^4 + 9*B^2*d^4*e^5))/d + (((1 - d*x)^(1/2) - 1)*(131072*B^2*e^2*f^3 + 49152*B^2*d^2*e^4*f))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(9*B^2*d^4*e^5 - 144*B^2*e*f^4 + 128*B^2*d^2*e^3*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) - (B*e*((4096*(24*B*d^2*e^2*f^4 - 30*B*d^4*e^4*f^2))/d + ((327680*B*e*f^5 - 360448*B*d^2*e^3*f^3)*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (4096*(96*B*d^2*e^2*f^4 - 90*B*d^4*e^4*f^2)*((1 - d*x)^(1/2) - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) + (B*e*((4096*(7*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/d + (((1 - d*x)^(1/2) - 1)*(81920*d^2*e^2*f^5 - 98304*d^4*e^4*f^3))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/(d*((d*x + 1)^(1/2) - 1)^2)))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2)))*1i)/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2)) + (B*e*((4096*(24*B^3*d^2*e^4 + 32*B^3*e^2*f^2))/d + (4096*((1 - d*x)^(1/2) - 1)^2*(96*B^3*d^2*e^4 - 32*B^3*e^2*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) + (458752*B^3*e^3*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (B*e*((4096*(16*B^2*e*f^4 + 9*B^2*d^4*e^5))/d + (((1 - d*x)^(1/2) - 1)*(131072*B^2*e^2*f^3 + 49152*B^2*d^2*e^4*f))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(9*B^2*d^4*e^5 - 144*B^2*e*f^4 + 128*B^2*d^2*e^3*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) + (B*e*((4096*(24*B*d^2*e^2*f^4 - 30*B*d^4*e^4*f^2))/d + ((327680*B*e*f^5 - 360448*B*d^2*e^3*f^3)*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (4096*(96*B*d^2*e^2*f^4 - 90*B*d^4*e^4*f^2)*((1 - d*x)^(1/2) - 1)^2)/(d*((d*x + 1)^(1

```
/2) - 1)^2) - (B*e*((4096*(7*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/d + (((1 - d*x)^
(1/2) - 1)*(81920*d^2*e^2*f^5 - 98304*d^4*e^4*f^3))/((d*x + 1)^(1/2) - 1) +
 (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/(d*((d*x +
 1)^(1/2) - 1)^2)))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2
)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2)))*1i)/(f*(f + d*e)^
(1/2)*(f - d*e)^(1/2)))/((131072*B^4*e^3)/d + (917504*B^4*e^3*((1 - d*x)^(1
/2) - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) + (B*e*((4096*(24*B^3*d^2*e^4 + 32*
B^3*e^2*f^2))/d + (4096*((1 - d*x)^(1/2) - 1)^2*(96*B^3*d^2*e^4 - 32*B^3*e^
2*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) + (458752*B^3*e^3*f*((1 - d*x)^(1/2) -
1))/((d*x + 1)^(1/2) - 1) + (B*e*((4096*(16*B^2*e*f^4 + 9*B^2*d^4*e^5))/d +
 (((1 - d*x)^(1/2) - 1)*(131072*B^2*e^2*f^3 + 49152*B^2*d^2*e^4*f))/((d*x +
 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(9*B^2*d^4*e^5 - 144*B^2*e*f
^4 + 128*B^2*d^2*e^3*f^2))/(d*((d*x + 1)^(1/2) - 1)^2) - (B*e*((4096*(24*B*
d^2*e^2*f^4 - 30*B*d^4*e^4*f^2))/d + ((327680*B*e*f^5 - 360448*B*d^2*e^3*f^
3)*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (4096*(96*B*d^2*e^2*f^4 -
 90*B*d^4*e^4*f^2)*((1 - d*x)^(1/2) - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) + (
B*e*((4096*(7*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/d + (((1 - d*x)^(1/2) - 1)*(819
20*d^2*e^2*f^5 - 98304*d^4*e^4*f^3))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*
x)^(1/2) - 1)^2*(11*d^4*e^3*f^4 - 9*d^6*e^5*f^2))/(d*((d*x + 1)^(1/2) - 1)^
2)))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/
2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1
/2)) - (B*e*((4096*(24*B^3*d^2*e^4 + 32*B^3*e^2*f^2))/d + (4096*((1 - d*x)^
(1/2) - 1)^2*(96*B^3*d^2*e^4 - 32*B^3*e^2*f^2))/(d*((d*x + 1)^(1/2) - 1)^2)
 + (458752*B^3*e^3*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (B*e*((
4096*(16*B^2*e*f^4 + 9*B^2*d^4*e^5))/d + (((1 - d*x)^(1/2) - 1)*(131072*B^2
*e^2*f^3 + 49152*B^2*d^2*e^4*f))/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(
1/2) - 1)^2*(9*B^2*d^4*e^5 - 144*B^2*e*f^4 + 128*B^2*d^2*e^3*f^2))/(d*((d*x
 + 1)^(1/2) - 1)^2) + (B*e*((4096*(24*B*d^2*e^2*f^4 - 30*B*d^4*e^4*f^2))/d
+ ((327680*B*e*f^5 - 360448*B*d^2*e^3*f^3)*((1 - d*x)^(1/2) - 1))/((d*x + 1
)^(1/2) - 1) + (4096*(96*B*d^2*e^2*f^4 - 90*B*d^4*e^4*f^2)*((1 - d*x)^(1/2)
 - 1)^2)/(d*((d*x + 1)^(1/2) - 1)^2) - (B*e*((4096*(7*d^4*e^3*f^4 - 9*d^6*e
^5*f^2))/d + (((1 - d*x)^(1/2) - 1)*(81920*d^2*e^2*f^5 - 98304*d^4*e^4*f^3)
)/((d*x + 1)^(1/2) - 1) + (4096*((1 - d*x)^(1/2) - 1)^2*(11*d^4*e^3*f^4 - 9
*d^6*e^5*f^2))/(d*((d*x + 1)^(1/2) - 1)^2)))/(f*(f + d*e)^(1/2)*(f - d*e)^(
1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^
(1/2))))/(f*(f + d*e)^(1/2)*(f - d*e)^(1/2))))*2i)/(f*(f + d*e)^(1/2)*(f -
d*e)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.13 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx$

Optimal. Leaf size=163

$$\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}-\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{f^2\left(d^2e^2-f^2\right)^{3/2}}+\frac{C\sin^{-1}(dx)}{df^2}$$

Rubi [A] time = 0.30, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1609, 1651, 844, 216, 725, 204}

$$\frac{\sqrt{1-d^2x^2}\left(Af^2-Bef+Ce^2\right)}{f\left(d^2e^2-f^2\right)(e+fx)}-\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\sqrt{d^2e^2-f^2}}\right)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{f^2\left(d^2e^2-f^2\right)^{3/2}}+\frac{C\sin^{-1}(dx)}{df^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(f*(d^2*e^2 - f^2)*(e + f*x)) +
 (C*ArcSin[d*x])/(d*f^2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^3)*A
rcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^2]*Sqrt[1 - d^2*x^2])])/(f^2*(d^2*e^2
 - f^2)^(3/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}(e+fx)^2}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{\int \frac{Ce+Ad^2e-Bf+C\left(\frac{d^2e^2}{f}-f\right)x}{(e+fx)\sqrt{1-d^2x^2}}\,dx}{d^2e^2-f^2}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{C\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{f^2} + \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-B\right.}{d^2e^2-}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} - \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right)S}{d^2}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{f\left(d^2e^2-f^2\right)(e+fx)} + \frac{C\sin^{-1}(dx)}{df^2} + \frac{\left(2Ce+Ad^2e-\frac{Cd^2e^3}{f^2}-Bf\right)t}{\left(d^2e^2-f\right.}$$

Mathematica [A] time = 0.43, size = 211, normalized size = 1.29

$$\frac{-\frac{f\sqrt{1-d^2x^2}\left(f(Af-Be)+Ce^2\right)}{\left(f^2-d^2e^2\right)(e+fx)}-\frac{\log\left(\sqrt{1-d^2x^2}\sqrt{f^2-d^2e^2}+d^2ex+f\right)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{\left(f^2-d^2e^2\right)^{3/2}}+\frac{\log(e+fx)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{\left(f^2-d^2e^2\right)^{3/2}}+\frac{C\sin^{-1}(dx)}{d}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^2),x]
```

```
[Out] (-((f*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[1 - d^2*x^2])/((-(d^2*e^2) + f^2)*(e
+ f*x))) + (C*ArcSin[d*x])/d + ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2*e*f^2 + B*f^
3)*Log[e + f*x])/(-(d^2*e^2) + f^2)^(3/2) - ((C*d^2*e^3 - 2*C*e*f^2 - A*d^2
*e*f^2 + B*f^3)*Log[f + d^2*e*x + Sqrt[-(d^2*e^2) + f^2]*Sqrt[1 - d^2*x^2]]
)/(-(d^2*e^2) + f^2)^(3/2))/f^2
```

IntegrateAlgebraic [A] time = 0.00, size = 235, normalized size = 1.44

$$\frac{2\tan^{-1}\left(\frac{\sqrt{1-dx}\sqrt{-de-f}\sqrt{f-de}}{\sqrt{dx+1}\,(de+f)}\right)\left(-Ad^2ef^2+Bf^3+Cd^2e^3-2Cef^2\right)}{f^2(-de-f)^{3/2}(f-de)^{3/2}}+\frac{2d\sqrt{1-dx}\left(Af^2-Bef+Ce^2\right)}{f\sqrt{dx+1}\,(de-f)(de+f)\left(\frac{de(1-dx)}{dx+1}+de-\frac{f(1-dx)}{dx+1}+f\right)}-\frac{2C\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)}{df^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)
^2),x]
```

```
[Out] (2*d*(C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d*x])/((d*e - f)*f*(d*e + f)*Sqrt[1 +
 d*x]*(d*e + f + (d*e*(1 - d*x))/(1 + d*x) - (f*(1 - d*x))/(1 + d*x))) - (2
```

```
*C*ArcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/(d*f^2) + (2*(C*d^2*e^3 - 2*C*e*f^2
 - A*d^2*e*f^2 + B*f^3)*ArcTan[(Sqrt[-(d*e) - f]*Sqrt[-(d*e) + f]*Sqrt[1 -
d*x])/((d*e + f)*Sqrt[1 + d*x])])/((-(d*e) - f)^(3/2)*f^2*(-(d*e) + f)^(3/2
))
```

fricas [B] time = 76.12, size = 1025, normalized size = 6.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [(C*d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3
*f^3 - (C*d^3*e^5 + B*d*e^2*f^3 - (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f +
B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*f^3)*x)*sqrt(-d^2*e^2 + f^2)*log((d^2*e*f*x
 + f^2 + sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) + (sqrt(-d^2*e^2 + f^2)*sqrt(-d
*x + 1)*f - (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqrt(d*x + 1))/(f*x + e)) + (C*
d^3*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3
)*sqrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*d*e*f^5
 - A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*f^2 + C*
e^2*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqrt(d*x + 1
)*sqrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^2*f^6 + (d
^5*e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x), (C*d^3*e^5*f - B*d^3*e^4*f^2 + B*
d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3 - 2*(C*d^3*e^5 + B*d*e^2*f^3
- (A*d^3 + 2*C*d)*e^3*f^2 + (C*d^3*e^4*f + B*d*e*f^4 - (A*d^3 + 2*C*d)*e^2*
f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt
(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e))/((d^2*e^2 - f^2)*x)) + (C*d^3
*e^5*f - B*d^3*e^4*f^2 + B*d*e^2*f^4 - A*d*e*f^5 + (A*d^3 - C*d)*e^3*f^3)*s
qrt(d*x + 1)*sqrt(-d*x + 1) + (C*d^3*e^4*f^2 - B*d^3*e^3*f^3 + B*d*e*f^5 -
A*d*f^6 + (A*d^3 - C*d)*e^2*f^4)*x - 2*(C*d^4*e^6 - 2*C*d^2*e^4*f^2 + C*e^2
*f^4 + (C*d^4*e^5*f - 2*C*d^2*e^3*f^3 + C*e*f^5)*x)*arctan((sqrt(d*x + 1)*s
qrt(-d*x + 1) - 1)/(d*x)))/(d^5*e^6*f^2 - 2*d^3*e^4*f^4 + d*e^2*f^6 + (d^5*
e^5*f^3 - 2*d^3*e^3*f^5 + d*e*f^7)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Undef/Unsigned Inf encountered in limit
```

maple [C] time = 0.00, size = 899, normalized size = 5.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-A*d^3*e*f^3*x*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^
2)^(1/2)*f+f)/(f*x+e))+C*d^3*e^3*f*x*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/
2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))-A*d^3*e^2*f^2*csgn(d)*ln(2*(d^2
*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))+C*d^3*e^4*
csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f
*x+e))+B*d*f^4*x*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f
```

```
^2)^(1/2)*f+f)/(f*x+e))+(-(d^2*e^2-f^2)/f^2)^(1/2)*C*d^2*e^2*f^2*x*arctan(1
/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))-2*C*d*e*f^3*x*csgn(d)*ln(2*(d^2*e*x+(-d^2*
x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))+B*d*e*f^3*csgn(d)*ln(
2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))+(-(d
^2*e^2-f^2)/f^2)^(1/2)*C*d^2*e^3*f*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))
-2*C*d*e^2*f^2*csgn(d)*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2
)^(1/2)*f+f)/(f*x+e))+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*A*d*f^4
*csgn(d)-(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*B*d*e*f^3*csgn(d)+(-
d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*C*d*e^2*f^2*csgn(d)-(-(d^2*e^2-
f^2)/f^2)^(1/2)*C*f^4*x*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))-(-(d^2*e^2
-f^2)/f^2)^(1/2)*C*e*f^3*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d)))*(d*x+1)^
(1/2)*(-d*x+1)^(1/2)/(-d^2*x^2+1)^(1/2)/(d*e+f)/(d*e-f)/(f*x+e)/(-(d^2*e^2-
f^2)/f^2)^(1/2)/d/f^3*csgn(d)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(f-d*e>0)', see `assume?` for more d
etails)Is f-d*e positive, negative or zero?
```

mupad [**B**] time = 0.01, size = 10198, normalized size = 62.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (A*d^5*e^5*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2
*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2)
 - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2))*2i - A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*
1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1
)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)
^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (
2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d
*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i + (4*A*f^2*((1 - d*x)^(1/2)
- 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (A*d^5*e^5*at
an(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)
^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3
*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x
+ 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 + (A*d^5*e^5*atan(
((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3
/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((
1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2
) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1
)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2)
)*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 - (4*A*f^2*((1 - d*x)
^(1/2) - 1)^3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 - (A
*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) -
 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d
```

```
^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e
^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/
2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
+ 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 + (
A*d^2*e^2*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
 + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 -
(A*d^3*e^3*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
 - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
 d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3
*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*
x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 +
 (A*d^4*e^4*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2)
- 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 -
d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*
e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1
/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
 + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) - 1) - (A*d
^2*e^2*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2
*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2)
 - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) - 1) - (A*d^4*
e^4*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(
f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*
f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1
- d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1)
)/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1
/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 + (8*A*d*e
*f*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2
) - 1)^2)/(d^3*e^4*(f + d*e)^(3/2)*(f - d*e)^(3/2) - d*e^2*f^2*(f + d*e)^(3
/2)*(f - d*e)^(3/2) - (4*e*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d
*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (4*e*f^3*((1 - d*x)^(1/2) - 1)^3*(f + d*
e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 + (2*d^3*e^4*((1 - d*x)^(
1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (d^3
*e^4*((1 - d*x)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1
/2) - 1)^4 - (2*d*e^2*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)
^(3/2))/((d*x + 1)^(1/2) - 1)^2 - (4*d^2*e^3*f*((1 - d*x)^(1/2) - 1)^3*(f +
 d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 - (d*e^2*f^2*((1 - d*x
)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^4 + (
4*d^2*e^3*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x +
1)^(1/2) - 1)) - (B*d^3*e^3*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - ((
(1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2)
 - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2)
- 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f
^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/
2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i - (B*f^4*atan(((f + d*e)^(3/2)*(f -
 d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1
i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2
)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1
/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e
^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) -
1)*8i)/((d*x + 1)^(1/2) - 1) + (B*f^4*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)
*1i - (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x +
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1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1
)^(1/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) +
(2*d*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 -
d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((
d*x + 1)^(1/2) - 1)^3 - B*d*e*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i
- (((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(
1/2) - 1)^2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1
/2) - 1)^2 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d
*e*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)
^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2))*2i - (4*B*f*((1 - d*x)^(1/2) - 1)^
3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 + (4*B*f*((1 - d
*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (B*
d^2*e^2*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) -
1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^
2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^
3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2
) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x +
 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^3*8i)/((d*x + 1)^(1/2) - 1)^3 - (B
*d*e*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^
2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e
^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*(
(1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) -
 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)
^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)^2 - (B*d*
e*f^3*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1/2) - 1)^2*(
f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^3 - d^2*e^2*
f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*d^3*e^3*((1
- d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x)^(1/2) - 1)
)/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1
/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^4 + (8*B*d*e
*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2)
- 1)^2 + (B*d^2*e^2*f^2*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d
*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^
2)/(f^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2
 - (2*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1
 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1
)^2)/((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)*8i)/((d*x + 1)^(1/2) -
 1) + (B*d^3*e^3*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(
1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f
^3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2
*d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*
x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/
((d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^2*4i)/((d*x + 1)^(1/2) - 1)
^2 + (B*d^3*e^3*f*atan(((f + d*e)^(3/2)*(f - d*e)^(3/2)*1i - (((1 - d*x)^(1
/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2)*1i)/((d*x + 1)^(1/2) - 1)^2)/(f^
3 - d^2*e^2*f - (f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (2*
d^3*e^3*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (2*d*e*f^2*((1 - d*x
)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (d^2*e^2*f*((1 - d*x)^(1/2) - 1)^2)/(
(d*x + 1)^(1/2) - 1)^2))*((1 - d*x)^(1/2) - 1)^4*2i)/((d*x + 1)^(1/2) - 1)^
4)/(d^3*e^3*(f + d*e)^(3/2)*(f - d*e)^(3/2) + (4*f^3*((1 - d*x)^(1/2) - 1)^
3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^3 - d*e*f^2*(f + d
*e)^(3/2)*(f - d*e)^(3/2) - (4*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f
 - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (2*d^3*e^3*((1 - d*x)^(1/2) - 1)^2*(
f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (d^3*e^3*((1 - d*
x)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^4 -
(4*d^2*e^2*f*((1 - d*x)^(1/2) - 1)^3*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x
 + 1)^(1/2) - 1)^3 + (4*d^2*e^2*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f
- d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (2*d*e*f^2*((1 - d*x)^(1/2) - 1)^2*(f
 + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 - (d*e*f^2*((1 - d*x
```

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)^(1/2) - 1)^4*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^4) -
((4*C*d*e*((1 - d*x)^(1/2) - 1))/((f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)) -
(4*C*d*e*((1 - d*x)^(1/2) - 1)^3)/((f^2 - d^2*e^2)*((d*x + 1)^(1/2) - 1)^3)
 + (8*C*d^2*e^2*((1 - d*x)^(1/2) - 1)^2)/(f*(f^2 - d^2*e^2)*((d*x + 1)^(1/2
) - 1)^2))/(d^2*e + (4*d*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (
4*d*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 + (2*d^2*e*((1 - d*x
)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (d^2*e*((1 - d*x)^(1/2) - 1)^4)/(
(d*x + 1)^(1/2) - 1)^4) + (4*C*atan((((((1 - d*x)^(1/2) - 1)*((2097152*(288
*e^3*f^11 - 6*d^10*e^13*f - 912*d^2*e^5*f^9 + 1048*d^4*e^7*f^7 - 532*d^6*e^
9*f^5 + 112*d^8*e^11*f^3))/(d*f^2*(d*f^13 - 4*d^3*e^2*f^11 + 6*d^5*e^4*f^9
- 4*d^7*e^6*f^7 + d^9*e^8*f^5)) - (33554432*(20*d^2*e*f^21 - 103*d^4*e^3*f^
19 + 215*d^6*e^5*f^17 - 230*d^8*e^7*f^15 + 130*d^10*e^9*f^13 - 35*d^12*e^11
*f^11 + 3*d^14*e^13*f^9))/(d^5*f^10*(d*f^13 - 4*d^3*e^2*f^11 + 6*d^5*e^4*f^
9 - 4*d^7*e^6*f^7 + d^9*e^8*f^5)) + (8388608*(72*e*f^17 - 452*d^2*e^3*f^15
+ 1024*d^4*e^5*f^13 - 1106*d^6*e^7*f^11 + 597*d^8*e^9*f^9 - 144*d^10*e^11*f
^7 + 9*d^12*e^13*f^5))/(d^3*f^6*(d*f^13 - 4*d^3*e^2*f^11 + 6*d^5*e^4*f^9 -
4*d^7*e^6*f^7 + d^9*e^8*f^5))))/((d*x + 1)^(1/2) - 1) - (33554432*(7*d^2*e^
2*f^19 - 35*d^4*e^4*f^17 + 70*d^6*e^6*f^15 - 70*d^8*e^8*f^13 + 35*d^10*e^10
*f^11 - 7*d^12*e^12*f^9))/(d^5*f^10*(f^12 - 4*d^2*e^2*f^10 + 6*d^4*e^4*f^8
- 4*d^6*e^6*f^6 + d^8*e^8*f^4)) + (2097152*(112*e^4*f^9 + 28*d^8*e^12*f - 3
36*d^2*e^6*f^7 + 364*d^4*e^8*f^5 - 168*d^6*e^10*f^3))/(d*f^2*(f^12 - 4*d^2*
e^2*f^10 + 6*d^4*e^4*f^8 - 4*d^6*e^6*f^6 + d^8*e^8*f^4)) + (8388608*(28*e^2
*f^15 - 168*d^2*e^4*f^13 + 364*d^4*e^6*f^11 - 371*d^6*e^8*f^9 + 182*d^8*e^1
0*f^7 - 35*d^10*e^12*f^5))/(d^3*f^6*(f^12 - 4*d^2*e^2*f^10 + 6*d^4*e^4*f^8
- 4*d^6*e^6*f^6 + d^8*e^8*f^4)))*(d^4*f^14 - 4*d^6*e^2*f^12 + 6*d^8*e^4*f^1
0 - 4*d^10*e^6*f^8 + d^12*e^8*f^6))/(67108864*e*f^12 + 37748736*d^12*e^13 -
 268435456*d^2*e^3*f^10 + 536870912*d^4*e^5*f^8 - 637534208*d^6*e^7*f^6 + 4
69762048*d^8*e^9*f^4 - 201326592*d^10*e^11*f^2)))/(d*f^2) + (log(16*f^15 -
9*d^14*e^14*f - (16*f^15*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 -
 92*d^2*e^2*f^13 + 236*d^4*e^4*f^11 - 352*d^6*e^6*f^9 + 329*d^8*e^8*f^7 - 1
91*d^10*e^10*f^5 + 63*d^12*e^12*f^3 + 16*f^6*(f + d*e)^(9/2)*(f - d*e)^(9/2
) + 12*d^6*e^6*(f + d*e)^(9/2)*(f - d*e)^(9/2) + 15*d^12*e^12*(f + d*e)^(3/
2)*(f - d*e)^(3/2) - (6*d^15*e^15*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
 1) + (16*d*e*f^14*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (92*d^2*e
^2*f^13*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (236*d^4*e^4*f^1
1*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (352*d^6*e^6*f^9*((1 -
 d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (329*d^8*e^8*f^7*((1 - d*x)^(
1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (191*d^10*e^10*f^5*((1 - d*x)^(1/2)
- 1)^2)/((d*x + 1)^(1/2) - 1)^2 - (63*d^12*e^12*f^3*((1 - d*x)^(1/2) - 1)^2
)/((d*x + 1)^(1/2) - 1)^2 - (16*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)
*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 - 24*d^2*e^2*f^10*(f + d*e)^(3/2)
*(f - d*e)^(3/2) + 120*d^4*e^4*f^8*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 228*d^
6*e^6*f^6*(f + d*e)^(3/2)*(f - d*e)^(3/2) + 4*d^2*e^2*f^4*(f + d*e)^(9/2)*(
f - d*e)^(9/2) + 207*d^8*e^8*f^4*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 28*d^4*e
^4*f^2*(f + d*e)^(9/2)*(f - d*e)^(9/2) - 90*d^10*e^10*f^2*(f + d*e)^(3/2)*(
f - d*e)^(3/2) - (88*d^3*e^3*f^12*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
 1) + (216*d^5*e^5*f^10*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (308
*d^7*e^7*f^8*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (274*d^9*e^9*f^
6*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (150*d^11*e^11*f^4*((1 - d
*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (46*d^13*e^13*f^2*((1 - d*x)^(1/2)
- 1))/((d*x + 1)^(1/2) - 1) + (9*d^14*e^14*f*((1 - d*x)^(1/2) - 1)^2)/((d*x
 + 1)^(1/2) - 1)^2 + (48*d^6*e^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f
 - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 + (45*d^12*e^12*((1 - d*x)^(1/2) - 1
)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (376*d^3*e^3
*f^9*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2
) - 1) - (688*d^5*e^5*f^7*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(
3/2))/((d*x + 1)^(1/2) - 1) + (612*d^7*e^7*f^5*((1 - d*x)^(1/2) - 1)*(f + d
*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (152*d^3*e^3*f^3*((1 - d
*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) - (26
```

```
4*d^9*e^9*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x
+ 1)^(1/2) - 1) - (80*d*e*f^11*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d
*e)^(3/2))/((d*x + 1)^(1/2) - 1) + (96*d*e*f^5*((1 - d*x)^(1/2) - 1)*(f + d
*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) - (136*d^2*e^2*f^10*((1 -
d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2
+ (560*d^4*e^4*f^8*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))
/((d*x + 1)^(1/2) - 1)^2 - (912*d^6*e^6*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*
e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (156*d^2*e^2*f^4*((1 -
d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2
+ (733*d^8*e^8*f^4*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))
/((d*x + 1)^(1/2) - 1)^2 - (172*d^4*e^4*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*
e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 - (290*d^10*e^10*f^2*((1
- d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^
2 + (56*d^5*e^5*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((
d*x + 1)^(1/2) - 1) + (44*d^11*e^11*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)
*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1))*(C*d^2*e^3 - 2*C*e*f^2))/(f^2*(f +
 d*e)^(3/2)*(f - d*e)^(3/2)) + (C*e*log(9*d^14*e^14*f - 16*f^15 + (16*f^15*
((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + 92*d^2*e^2*f^13 - 236*d^
4*e^4*f^11 + 352*d^6*e^6*f^9 - 329*d^8*e^8*f^7 + 191*d^10*e^10*f^5 - 63*d^1
2*e^12*f^3 + 16*f^6*(f + d*e)^(9/2)*(f - d*e)^(9/2) + 12*d^6*e^6*(f + d*e)^
(9/2)*(f - d*e)^(9/2) + 15*d^12*e^12*(f + d*e)^(3/2)*(f - d*e)^(3/2) + (6*d
^15*e^15*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (16*d*e*f^14*((1 -
d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (92*d^2*e^2*f^13*((1 - d*x)^(1/2)
- 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (236*d^4*e^4*f^11*((1 - d*x)^(1/2) - 1)^2
)/((d*x + 1)^(1/2) - 1)^2 - (352*d^6*e^6*f^9*((1 - d*x)^(1/2) - 1)^2)/((d*x
 + 1)^(1/2) - 1)^2 + (329*d^8*e^8*f^7*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(
1/2) - 1)^2 - (191*d^10*e^10*f^5*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2)
- 1)^2 + (63*d^12*e^12*f^3*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2
 - (16*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x +
 1)^(1/2) - 1)^2 - 24*d^2*e^2*f^10*(f + d*e)^(3/2)*(f - d*e)^(3/2) + 120*d^
4*e^4*f^8*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 228*d^6*e^6*f^6*(f + d*e)^(3/2)
*(f - d*e)^(3/2) + 4*d^2*e^2*f^4*(f + d*e)^(9/2)*(f - d*e)^(9/2) + 207*d^8*
e^8*f^4*(f + d*e)^(3/2)*(f - d*e)^(3/2) - 28*d^4*e^4*f^2*(f + d*e)^(9/2)*(f
 - d*e)^(9/2) - 90*d^10*e^10*f^2*(f + d*e)^(3/2)*(f - d*e)^(3/2) + (88*d^3*
e^3*f^12*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) - (216*d^5*e^5*f^10*(
(1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1) + (308*d^7*e^7*f^8*((1 - d*x)^(
1/2) - 1))/((d*x + 1)^(1/2) - 1) - (274*d^9*e^9*f^6*((1 - d*x)^(1/2) - 1))/
((d*x + 1)^(1/2) - 1) + (150*d^11*e^11*f^4*((1 - d*x)^(1/2) - 1))/((d*x + 1
)^(1/2) - 1) - (46*d^13*e^13*f^2*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
1) - (9*d^14*e^14*f*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)^2 + (48*
d^6*e^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)
^(1/2) - 1)^2 + (45*d^12*e^12*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f -
d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (376*d^3*e^3*f^9*((1 - d*x)^(1/2) - 1
)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (688*d^5*e^5*f^7
*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) -
1) + (612*d^7*e^7*f^5*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2)
)/((d*x + 1)^(1/2) - 1) - (152*d^3*e^3*f^3*((1 - d*x)^(1/2) - 1)*(f + d*e)^
(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) - (264*d^9*e^9*f^3*((1 - d*x)^
(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1) - (80*d*e
*f^11*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/
2) - 1) + (96*d*e*f^5*((1 - d*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2)
)/((d*x + 1)^(1/2) - 1) - (136*d^2*e^2*f^10*((1 - d*x)^(1/2) - 1)^2*(f + d*
e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (560*d^4*e^4*f^8*((1 -
d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2
- (912*d^6*e^6*f^6*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))
/((d*x + 1)^(1/2) - 1)^2 + (156*d^2*e^2*f^4*((1 - d*x)^(1/2) - 1)^2*(f + d*
e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1)^2 + (733*d^8*e^8*f^4*((1 -
d*x)^(1/2) - 1)^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2
- (172*d^4*e^4*f^2*((1 - d*x)^(1/2) - 1)^2*(f + d*e)^(9/2)*(f - d*e)^(9/2))
```

```
/((d*x + 1)^(1/2) - 1)^2 - (290*d^10*e^10*f^2*((1 - d*x)^(1/2) - 1)^2*(f +
d*e)^(3/2)*(f - d*e)^(3/2))/((d*x + 1)^(1/2) - 1)^2 + (56*d^5*e^5*f*((1 - d
*x)^(1/2) - 1)*(f + d*e)^(9/2)*(f - d*e)^(9/2))/((d*x + 1)^(1/2) - 1) + (44
*d^11*e^11*f*((1 - d*x)^(1/2) - 1)*(f + d*e)^(3/2)*(f - d*e)^(3/2))/((d*x +
 1)^(1/2) - 1))*(2*f^2 - d^2*e^2))/(f^2*(f + d*e)^(3/2)*(f - d*e)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
[Out] Timed out
```

3.14 $\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}\,(e+fx)^3}\,dx$

Optimal. Leaf size=248

$$\frac{\sqrt{1-d^2x^2}\,\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}+\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\,\sqrt{d^2e^2-f^2}}\right)\left(C\left(d^2e^2+2f^2\right)-d^2\left(3Bef-A\left(2d^2e^2+f^2\right)\right)\right)}{2\left(d^2e^2-f^2\right)^{5/2}}-\frac{\sqrt{1-d}}{}$$

Rubi [A] time = 0.33, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {1609, 1651, 807, 725, 204}

$$\frac{\sqrt{1-d^2x^2}\,\left(Af^2-Bef+Ce^2\right)}{2f\left(d^2e^2-f^2\right)(e+fx)^2}-\frac{\sqrt{1-d^2x^2}\,\left(-3Ad^2ef^2+Bd^2e^2f+2Bf^3+Cd^2e^3-4Cef^2\right)}{2f\left(d^2e^2-f^2\right)^2(e+fx)}+\frac{\tan^{-1}\left(\frac{d^2ex+f}{\sqrt{1-d^2x^2}\,\sqrt{d^2e^2-f^2}}\right)\left(C\left(d^2e^2+2f^2\right)-d^2\left(3Bef-A\left(2d^2e^2+f^2\right)\right)\right)}{2\left(d^2e^2-f^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] ((C*e^2 - B*e*f + A*f^2)*Sqrt[1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)*(e + f*x)^
2) - ((C*d^2*e^3 + B*d^2*e^2*f - 4*C*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3)*Sqrt[
1 - d^2*x^2])/(2*f*(d^2*e^2 - f^2)^2*(e + f*x)) + ((C*(d^2*e^2 + 2*f^2) - d
^2*(3*B*e*f - A*(2*d^2*e^2 + f^2)))*ArcTan[(f + d^2*e*x)/(Sqrt[d^2*e^2 - f^
2]*Sqrt[1 - d^2*x^2])])/(2*(d^2*e^2 - f^2)^(5/2))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
```

```
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{1-dx}\sqrt{1+dx}\,(e+fx)^3}\,dx = \int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{1-d^2x^2}}\,dx$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} + \frac{\int \frac{2\left(Ce+Ad^2e-Bf\right)+\left(Bd^2e+\frac{Cd^2e^2}{f}-2Cf-Ad^2f\right)x}{(e+fx)^2\sqrt{1-d^2x^2}}\,dx}{2\left(d^2e^2-f^2\right)}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+\right.}{2f\left(d^2e^2-f^2\right)^2(e+fx)}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+\right.}{2f\left(d^2e^2-f^2\right)^2(e+fx)}$$

$$= \frac{\left(Ce^2-Bef+Af^2\right)\sqrt{1-d^2x^2}}{2f\left(d^2e^2-f^2\right)(e+fx)^2} - \frac{\left(Cd^2e^3+Bd^2e^2f-4Cef^2-3Ad^2ef^2+\right.}{2f\left(d^2e^2-f^2\right)^2(e+fx)}$$

Mathematica [A] time = 0.38, size = 273, normalized size = 1.10

$$\frac{1}{2}\left(-\frac{\log\left(\sqrt{1-d^2x^2}\sqrt{f^2-d^2e^2}+d^2ex+f\right)\left(d^2\left(A\left(2d^2e^2+f^2\right)-3Bef\right)+C\left(d^2e^2+2f^2\right)\right)}{\left(f^2-d^2e^2\right)^{5/2}}+\frac{\log(e+fx)\left(d^2\left(A\left(2d^2e^2+f^2\right)-3Bef\right)+C\left(d^2e^2+2f^2\right)\right)}{\left(f^2-d^2e^2\right)^{5/2}}-\frac{\sqrt{1-d^2x^2}\left(-Ad^2ef(4e+3fx)+Af^3+Bd^2e^2(2e+fx)+Bf^2(e+2fx)+Ce\left(d^2e^2x-3ef-4f^2x\right)\right)}{\left(f^2-d^2e^2\right)^2(e+fx)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)^3),x]
```

```
[Out] (-((Sqrt[1 - d^2*x^2]*(A*f^3 + B*d^2*e^2*(2*e + f*x) + B*f^2*(e + 2*f*x) -
A*d^2*e*f*(4*e + 3*f*x) + C*e*(-3*e*f + d^2*e^2*x - 4*f^2*x)))/((-(d^2*e^2)
 + f^2)^2*(e + f*x)^2)) + ((C*(d^2*e^2 + 2*f^2) + d^2*(-3*B*e*f + A*(2*d^2*
e^2 + f^2)))*Log[e + f*x])/(-(d^2*e^2) + f^2)^(5/2) - ((C*(d^2*e^2 + 2*f^2)
 + d^2*(-3*B*e*f + A*(2*d^2*e^2 + f^2)))*Log[f + d^2*e*x + Sqrt[-(d^2*e^2)
+ f^2]*Sqrt[1 - d^2*x^2]])/(-(d^2*e^2) + f^2)^(5/2))/2
```

IntegrateAlgebraic [B] time = 0.00, size = 533, normalized size = 2.15

$$\frac{\tan^{-1}\left(\frac{\sqrt{1-dx}\sqrt{-de-f}\sqrt{f-de}}{\sqrt{dx+1}(de+f)}\right)\left(2Ad^4e^2\sqrt{f-de}+Ad^2f^2\sqrt{f-de}-3Bd^2ef\sqrt{f-de}+Cd^2e^2\sqrt{f-de}+2Cf^2\sqrt{f-de}\right)}{(-de-f)^{5/2}(de-f)^3}-\frac{d\sqrt{1-dx}\left(-\frac{4Ad^3e^2f(1-dx)}{dx+1}-4Ad^3e^2f+\frac{3Ad^2ef^2(1-dx)}{dx+1}-3Ad^2ef^2+\frac{Adf^3(1-dx)}{dx+1}+Adf^3+\frac{2Bd^3e^3(1-dx)}{dx+1}+2Bd^3e^3-\frac{Bd^2e^2f(1-dx)}{dx+1}+Bd^2e^2f+\frac{Bdef^2(1-dx)}{dx+1}+Bdef^2-\frac{2Bf^3(1-dx)}{dx+1}+2Bf^3-\frac{Cd^2e^3(1-dx)}{dx+1}+Cd^2e^3-\frac{3Cde^2f(1-dx)}{dx+1}-3Cde^2f+\frac{4Cef^2(1-dx)}{dx+1}-4Cef^2\right)}{\sqrt{dx+1}(de-f)^2(de+f)^2\left(\frac{de(1-dx)}{dx+1}+de-\frac{f(1-dx)}{dx+1}+f\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]*(e + f*x)
^3),x]
```

```
[Out] -((d*Sqrt[1 - d*x]*(C*d^2*e^3 + 2*B*d^3*e^3 - 3*C*d*e^2*f + B*d^2*e^2*f - 4
*A*d^3*e^2*f - 4*C*e*f^2 + B*d*e*f^2 - 3*A*d^2*e*f^2 + 2*B*f^3 + A*d*f^3 -
(C*d^2*e^3*(1 - d*x))/(1 + d*x) + (2*B*d^3*e^3*(1 - d*x))/(1 + d*x) - (3*C*
d*e^2*f*(1 - d*x))/(1 + d*x) - (B*d^2*e^2*f*(1 - d*x))/(1 + d*x) - (4*A*d^3
*e^2*f*(1 - d*x))/(1 + d*x) + (4*C*e*f^2*(1 - d*x))/(1 + d*x) + (B*d*e*f^2*
(1 - d*x))/(1 + d*x) + (3*A*d^2*e*f^2*(1 - d*x))/(1 + d*x) - (2*B*f^3*(1 -
d*x))/(1 + d*x) + (A*d*f^3*(1 - d*x))/(1 + d*x)))/((d*e - f)^2*(d*e + f)^2*
Sqrt[1 + d*x]*(d*e + f + (d*e*(1 - d*x))/(1 + d*x) - (f*(1 - d*x))/(1 + d*x
))^2)) + ((C*d^2*e^2*Sqrt[-(d*e) + f] + 2*A*d^4*e^2*Sqrt[-(d*e) + f] - 3*B*
```

```
d^2*e*f*Sqrt[-(d*e) + f] + 2*C*f^2*Sqrt[-(d*e) + f] + A*d^2*f^2*Sqrt[-(d*e)
 + f])*ArcTan[(Sqrt[-(d*e) - f]*Sqrt[-(d*e) + f]*Sqrt[1 - d*x])/((d*e + f)*
Sqrt[1 + d*x])])/((-(d*e) - f)^(5/2)*(d*e - f)^3)
```

fricas [B] time = 0.85, size = 1580, normalized size = 6.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/2*(2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 +
 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 -
(4*A*d^4 + 3*C*d^2)*e^4*f^3 + (5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^
2 - (3*B*d^2*e^5*f - (2*A*d^4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d
^2*e^3*f^3 - (2*A*d^4 + C*d^2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 + 2*(3*
B*d^2*e^4*f^2 - (2*A*d^4 + C*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*sqrt(-d
^2*e^2 + f^2)*log((d^2*e*f*x + f^2 - sqrt(-d^2*e^2 + f^2)*(d^2*e*x + f) - (
sqrt(-d^2*e^2 + f^2)*sqrt(-d*x + 1)*f + (d^2*e^2 - f^2)*sqrt(-d*x + 1))*sqr
t(d*x + 1))/(f*x + e)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2)
*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4*e^7 + B*d
^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^2 + 4*C)*e^
3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d^4*e^6*f - B
*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e^3*f^4 - B*e^
2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4 - e^4*f^6 + (
d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 + 2*(d^6*e^9*f -
 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x), -1/2*(2*B*d^4*e^7 - B*d^2*e^5
*f^2 - (4*A*d^4 + 3*C*d^2)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*
e^2*f^5 + (2*B*d^4*e^5*f^2 - B*d^2*e^3*f^4 - (4*A*d^4 + 3*C*d^2)*e^4*f^3 +
(5*A*d^2 + 3*C)*e^2*f^5 - B*e*f^6 - A*f^7)*x^2 + 2*(3*B*d^2*e^5*f - (2*A*d^
4 + C*d^2)*e^6 - (A*d^2 + 2*C)*e^4*f^2 + (3*B*d^2*e^3*f^3 - (2*A*d^4 + C*d^
2)*e^4*f^2 - (A*d^2 + 2*C)*e^2*f^4)*x^2 + 2*(3*B*d^2*e^4*f^2 - (2*A*d^4 + C
*d^2)*e^5*f - (A*d^2 + 2*C)*e^3*f^3)*x)*sqrt(d^2*e^2 - f^2)*arctan(-(sqrt(d
^2*e^2 - f^2)*sqrt(d*x + 1)*sqrt(-d*x + 1)*e - sqrt(d^2*e^2 - f^2)*(f*x + e
))/((d^2*e^2 - f^2)*x)) + (2*B*d^4*e^7 - B*d^2*e^5*f^2 - (4*A*d^4 + 3*C*d^2
)*e^6*f + (5*A*d^2 + 3*C)*e^4*f^3 - B*e^3*f^4 - A*e^2*f^5 + (C*d^4*e^7 + B*
d^4*e^6*f + B*d^2*e^4*f^3 - (3*A*d^4 + 5*C*d^2)*e^5*f^2 + (3*A*d^2 + 4*C)*e
^3*f^4 - 2*B*e^2*f^5)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*B*d^4*e^6*f -
B*d^2*e^4*f^3 - (4*A*d^4 + 3*C*d^2)*e^5*f^2 + (5*A*d^2 + 3*C)*e^3*f^4 - B*e
^2*f^5 - A*e*f^6)*x)/(d^6*e^10 - 3*d^4*e^8*f^2 + 3*d^2*e^6*f^4 - e^4*f^6 +
(d^6*e^8*f^2 - 3*d^4*e^6*f^4 + 3*d^2*e^4*f^6 - e^2*f^8)*x^2 + 2*(d^6*e^9*f
- 3*d^4*e^7*f^3 + 3*d^2*e^5*f^5 - e^3*f^7)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Undef/Unsigned Inf encountered in limit
```

maple [C] time = 0.00, size = 1449, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(2*A*d^4*e^2*f^2*x^2*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/
f^2)^(1/2)*f+f)/(f*x+e))+4*A*d^4*e^3*f*x*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(
-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))+2*A*d^4*e^4*ln(2*(d^2*e*x+(-d^2*x^2
+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))+A*d^2*f^4*x^2*ln(2*(d^2*
e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))-3*B*d^2*e*f
^3*x^2*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*
x+e))+C*d^2*e^2*f^2*x^2*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^
2)^(1/2)*f+f)/(f*x+e))+2*A*d^2*e*f^3*x*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(
d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))-6*B*d^2*e^2*f^2*x*ln(2*(d^2*e*x+(-d^2
*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))+2*C*d^2*e^3*f*x*ln(2
*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))+A*d^2
*e^2*f^2*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(
f*x+e))-3*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*A*d^2*e*f^3*x-3*B*d
^2*e^3*f*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(
f*x+e))+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*B*d^2*e^2*f^2*x+C*d^2
*e^4*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+
e))+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*C*d^2*e^3*f*x+2*C*f^4*x^2
*ln(2*(d^2*e*x+(-d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))-
4*(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*A*d^2*e^2*f^2+2*(-(d^2*e^2-
f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*B*d^2*e^3*f+4*C*e*f^3*x*ln(2*(d^2*e*x+(-
d^2*x^2+1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))+2*(-(d^2*e^2-f^2)
/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*B*f^4*x+2*C*e^2*f^2*ln(2*(d^2*e*x+(-d^2*x^2+
1)^(1/2)*(-(d^2*e^2-f^2)/f^2)^(1/2)*f+f)/(f*x+e))-4*(-(d^2*e^2-f^2)/f^2)^(1
/2)*(-d^2*x^2+1)^(1/2)*C*e*f^3*x+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1
/2)*A*f^4+(-(d^2*e^2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*B*e*f^3-3*(-(d^2*e^
2-f^2)/f^2)^(1/2)*(-d^2*x^2+1)^(1/2)*C*e^2*f^2)*(d*x+1)^(1/2)*(-d*x+1)^(1/2
)/(-d^2*x^2+1)^(1/2)/(d*e+f)/(d*e-f)/(d^2*e^2-f^2)/(f*x+e)^2/(-(d^2*e^2-f^2
)/f^2)^(1/2)/f*csgn(d)^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(f-d*e>0)', see `assume?` for more d
etails)Is f-d*e positive, negative or zero?
```

mupad [B] time = 0.01, size = 9097, normalized size = 36.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^2)/(((d*x + 1)^(1/2) - 1
)^2*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (24*(2*C*f^3 - C*d^2*e^2*f)*((1 - d*
x)^(1/2) - 1)^4)/(((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2))
 + (12*(2*C*f^3 + C*d^2*e^2*f)*((1 - d*x)^(1/2) - 1)^6)/(((d*x + 1)^(1/2) -
1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*((1 - d*x)^(1/2) - 1)^7*(C*d^3*e
^3 + 2*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)
) - (2*((1 - d*x)^(1/2) - 1)^3*(7*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1
/2) - 1)^3*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*((1 - d*x)^(1/2) - 1)^5*(7
*C*d^3*e^3 - 34*C*d*e*f^2))/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2
*e^2*f^2)) + (2*d*e*((1 - d*x)^(1/2) - 1)*(2*C*f^2 + C*d^2*e^2))/(((d*x + 1
)^(1/2) - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2)
 - 1)^2*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) -
```

```
 1)^6*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1
)^4*(32*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/
2) - 1)^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*
x + 1)^(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) -
1)^5 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f
*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + ((4*((1 - d*x)^(1/2) - 1)^
2*(4*A*d^4*e^4*f - 2*A*f^5 + 7*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^2
*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (8*((1 - d*x)^(1/2) - 1)^4*(2*A*f^5 + 4
*A*d^4*e^4*f - 9*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^
4 - 2*d^2*e^2*f^2)) + (4*((1 - d*x)^(1/2) - 1)^6*(4*A*d^4*e^4*f - 2*A*f^5 +
 7*A*d^2*e^2*f^3))/(e^2*((d*x + 1)^(1/2) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*
f^2)) + (2*f*((1 - d*x)^(1/2) - 1)^7*(2*A*d*f^3 - 5*A*d^3*e^2*f))/(e*((d*x
+ 1)^(1/2) - 1)^7*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*f*((1 - d*x)^(1/2)
- 1)^3*(2*A*d*f^3 - 29*A*d^3*e^2*f))/(e*((d*x + 1)^(1/2) - 1)^3*(f^4 + d^4*
e^4 - 2*d^2*e^2*f^2)) + (2*f*((1 - d*x)^(1/2) - 1)^5*(2*A*d*f^3 - 29*A*d^3*
e^2*f))/(e*((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (2*d*
f*(2*A*f^3 - 5*A*d^2*e^2*f)*((1 - d*x)^(1/2) - 1))/(e*((d*x + 1)^(1/2) - 1)
*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2) - 1)^2*(16*
f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)^6*(16*f^
2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1)^4*(32*f^2
- 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/2) - 1)^8)/(
(d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^(1/2)
 - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 - (8*d*
e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f*((1 - d*x)^
(1/2) - 1))/((d*x + 1)^(1/2) - 1)) - ((4*((1 - d*x)^(1/2) - 1)^2*(2*B*f^4 +
 2*B*d^4*e^4 + 5*B*d^2*e^2*f^2))/(e*((d*x + 1)^(1/2) - 1)^2*(f^4 + d^4*e^4
- 2*d^2*e^2*f^2)) - (8*((1 - d*x)^(1/2) - 1)^4*(2*B*f^4 - 2*B*d^4*e^4 + 3*B
*d^2*e^2*f^2))/(e*((d*x + 1)^(1/2) - 1)^4*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2))
+ (4*((1 - d*x)^(1/2) - 1)^6*(2*B*f^4 + 2*B*d^4*e^4 + 5*B*d^2*e^2*f^2))/(e*
((d*x + 1)^(1/2) - 1)^6*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) + (2*f*(11*B*d^3*e
^2 + 16*B*d*f^2)*((1 - d*x)^(1/2) - 1)^3)/(((d*x + 1)^(1/2) - 1)^3*(f^4 + d
^4*e^4 - 2*d^2*e^2*f^2)) - (2*f*(11*B*d^3*e^2 + 16*B*d*f^2)*((1 - d*x)^(1/2
) - 1)^5)/(((d*x + 1)^(1/2) - 1)^5*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)) - (6*B*
d^3*e^2*f*((1 - d*x)^(1/2) - 1)^7)/(((d*x + 1)^(1/2) - 1)^7*(f^4 + d^4*e^4
- 2*d^2*e^2*f^2)) + (6*B*d^3*e^2*f*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2)
 - 1)*(f^4 + d^4*e^4 - 2*d^2*e^2*f^2)))/(d^2*e^2 + (((1 - d*x)^(1/2) - 1)^2
*(16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^2 + (((1 - d*x)^(1/2) - 1)^6*(
16*f^2 + 4*d^2*e^2))/((d*x + 1)^(1/2) - 1)^6 - (((1 - d*x)^(1/2) - 1)^4*(32
*f^2 - 6*d^2*e^2))/((d*x + 1)^(1/2) - 1)^4 + (d^2*e^2*((1 - d*x)^(1/2) - 1)
^8)/((d*x + 1)^(1/2) - 1)^8 + (8*d*e*f*((1 - d*x)^(1/2) - 1)^3)/((d*x + 1)^
(1/2) - 1)^3 - (8*d*e*f*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) - 1)^5 -
(8*d*e*f*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 + (8*d*e*f*((1 -
d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)) + (C*atan(((C*(2*f^2 + d^2*e^2)*((4
*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/
(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4
*d^6*e^6*f^2)) - (4*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(f^8
+ d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (C*(2*f^2 + d^
2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 -
12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^
4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f
^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*
x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*
e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2
*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))
- (C*(2*f^2 + d^2*e^2)*((4*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5)
)/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1
- d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(((d*
x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*
e^6*f^2)) + (C*(2*f^2 + d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*
```

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e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*
d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(
4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9
*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^
6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/
((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*1i)/(2*(f + d*
e)^(5/2)*(f - d*e)^(5/2)))/((8*(C^2*d^5*e^5 + 4*C^2*d^3*e^3*f^2 + 4*C^2*d*e
*f^4))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (8
*((1 - d*x)^(1/2) - 1)^2*(C^2*d^5*e^5 + 4*C^2*d^3*e^3*f^2 + 4*C^2*d*e*f^4))
/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 -
4*d^6*e^6*f^2)) + (C*(2*f^2 + d^2*e^2)*((4*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e
*f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d
^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) - (4*(8*C*d*e*f^7
+ 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4
*e^4*f^4 - 4*d^6*e^6*f^2) + (C*(2*f^2 + d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*
e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^
8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x
)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*
f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^
8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d
*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))
))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)) + (C*(2*f^2 + d^2*e^2)*((4*(8*C*d*e*
f^7 + 4*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2*(8*C*d*e*f^7 + 4
*C*d^7*e^7*f - 12*C*d^3*e^3*f^5))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 -
 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (C*(2*f^2 + d^2*e^2)*((4
*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9
*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*
e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^
5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/
2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2))
+ (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)
^(5/2)*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))*(2*f^2 + d^
2*e^2)*1i)/((f + d*e)^(5/2)*(f - d*e)^(5/2)) + (A*d^2*atan(((A*d^2*(f^2 + 2
*d^2*e^2)*((4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A
*d^7*e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*
d^4*e^4*f^4 - 4*d^6*e^6*f^2)) - (4*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A*d^
7*e^5*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)
 + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f
^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e
^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^1
1*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2
- 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x
 + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5
/2)*(f - d*e)^(5/2)) - (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*A*d^3*e*f^7 + 8*A*d^
9*e^7*f - 12*A*d^7*e^5*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4
 - 4*d^6*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7
*f - 12*A*d^7*e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2
*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^
11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 +
 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^
2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*
f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1
)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*
d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)
*(f - d*e)^(5/2)))*1i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))/((8*(4*A^2*d^9*
e^5 + 4*A^2*d^7*e^3*f^2 + A^2*d^5*e*f^4))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 +
6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (8*((1 - d*x)^(1/2) - 1)^2*(4*A^2*d^9*e^5
```

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+ 4*A^2*d^7*e^3*f^2 + A^2*d^5*e*f^4))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e
^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (A*d^2*(f^2 + 2*d^2*
e^2)*((4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A*d^7*
e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e
^4*f^4 - 4*d^6*e^6*f^2)) - (4*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*A*d^7*e^5
*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (A
*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 +
8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^
6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^1
1 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*
d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*
e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)
^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f -
 d*e)^(5/2)) + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f
- 12*A*d^7*e^5*f^3))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6
*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2*(4*A*d^3*e*f^7 + 8*A*d^9*e^7*f - 12*
A*d^7*e^5*f^3))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (A*d^2*(f^2 + 2*d^2*e^2)*((4*(4*d^11*e^11
- 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f
^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*
((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72
*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8
 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*
f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*
e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))*(f^2 + 2*d^2*e^2)*1i)/((f
 + d*e)^(5/2)*(f - d*e)^(5/2)) - (B*d^2*e*f*atan(((B*d^2*e*f*((4*((1 - d*x)
^(1/2) - 1)^2*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(((
d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^
6*e^6*f^2)) - (4*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/
(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (3*B*d^2*
e*f*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*
d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 -
 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8
- 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x +
 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6
*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f
 + d*e)^(5/2)*(f - d*e)^(5/2)))*3i)/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)) - (
B*d^2*e*f*((4*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(f^
8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1 - d*x
)^(1/2) - 1)^2*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/((
(d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d
^6*e^6*f^2)) + (3*B*d^2*e*f*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f
^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e
^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^1
1*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2
- 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6
*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x
 + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)))*3i)/(2*(f + d*e)^(5
/2)*(f - d*e)^(5/2)))/((72*B^2*d^5*e^3*f^2)/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6
+ 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (3*B*d^2*e*f*((4*((1 - d*x)^(1/2) - 1)^2
*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(((d*x + 1)^(1/2
) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) -
 (4*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(f^8 + d^8*e^
8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) + (3*B*d^2*e*f*((4*(4*d^
11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7*f^4 - 12*d^9*e^9*f^2 +
 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^
2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^3*e^3*f^8 - 88*d^5*e^5*
f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10))/(((d*x + 1)^(1/2) - 1
)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) + (64*
```

```
d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1)))/(2*(f + d*e)^(5/2)
*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2)) + (3*B*d^2*e*f*((4*
(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(f^8 + d^8*e^8 -
4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2) - (4*((1 - d*x)^(1/2) - 1)^2
*(12*B*d^3*e^2*f^6 - 24*B*d^5*e^4*f^4 + 12*B*d^7*e^6*f^2))/(((d*x + 1)^(1/2
) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2)) +
 (3*B*d^2*e*f*((4*(4*d^11*e^11 - 12*d^3*e^3*f^8 + 8*d^5*e^5*f^6 + 8*d^7*e^7
*f^4 - 12*d^9*e^9*f^2 + 4*d*e*f^10))/(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4
*e^4*f^4 - 4*d^6*e^6*f^2) + (4*((1 - d*x)^(1/2) - 1)^2*(4*d^11*e^11 + 52*d^
3*e^3*f^8 - 88*d^5*e^5*f^6 + 72*d^7*e^7*f^4 - 28*d^9*e^9*f^2 - 12*d*e*f^10)
)/(((d*x + 1)^(1/2) - 1)^2*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 -
 4*d^6*e^6*f^2)) + (64*d^2*e^2*f*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) -
1)))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/2))))/(2*(f + d*e)^(5/2)*(f - d*e)^(5/
2)) + (72*B^2*d^5*e^3*f^2*((1 - d*x)^(1/2) - 1)^2)/(((d*x + 1)^(1/2) - 1)^2
*(f^8 + d^8*e^8 - 4*d^2*e^2*f^6 + 6*d^4*e^4*f^4 - 4*d^6*e^6*f^2))))*3i)/((f
 + d*e)^(5/2)*(f - d*e)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] Timed out
```

3.15 $\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{1-d^2x^2}\,\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\sin^{-1}(dx)}{2d^3}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

Rubi [A] time = 0.14, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1609, 1809, 780, 216}

$$-\frac{\sqrt{1-d^2x^2}\,\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\sin^{-1}(dx)}{2d^3}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(c*x^2*Sqrt[1 - d^2*x^2])/(3*d^2) - ((2*(2*c + 3*a*d^2) + 3*b*d^2*x)*Sqrt[
1 - d^2*x^2])/(6*d^4) + (b*ArcSin[d*x])/(2*d^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{x(a+bx+cx^2)}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\int \frac{x(-2c-3ad^2-3bd^2x)}{\sqrt{1-d^2x^2}}\,dx}{3d^2}$$
$$= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\sin^{-1}(dx)}{2d^3}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 0.72

$$\frac{3bd\sin^{-1}(dx) - \sqrt{1-d^2x^2}\left(3d^2(2a+bx)+2c\left(d^2x^2+2\right)\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(Sqrt[1 - d^2*x^2]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2))) + 3*b*d*ArcSi
n[d*x])/(6*d^4)
```

IntegrateAlgebraic [B] time = 0.00, size = 179, normalized size = 2.27

$$-\frac{\sqrt{1-dx}\left(\frac{12ad^2(1-dx)}{dx+1}+\frac{6ad^2(1-dx)^2}{(dx+1)^2}+6ad^2-\frac{3bd(1-dx)^2}{(dx+1)^2}+3bd+\frac{4c(1-dx)}{dx+1}+\frac{6c(1-dx)^2}{(dx+1)^2}+6c\right)}{3d^4\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)^3} - \frac{b\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(Sqrt[1 - d*x]*(6*c + 3*b*d + 6*a*d^2 + (6*c*(1 - d*x)^2)/(1 + d*x)^2
- (3*b*d*(1 - d*x)^2)/(1 + d*x)^2 + (6*a*d^2*(1 - d*x)^2)/(1 + d*x)^2 + (4*
c*(1 - d*x))/(1 + d*x) + (12*a*d^2*(1 - d*x))/(1 + d*x)))/(d^4*Sqrt[1 + d*x
]*(1 + (1 - d*x)/(1 + d*x))^3) - (b*ArcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/d^
3
```

fricas [A] time = 1.14, size = 78, normalized size = 0.99

$$-\frac{6\,bd\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)+\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\,\sqrt{-dx+1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*(6*b*d*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)) + (2*c*d^2*x^2
 + 3*b*d^2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

giac [A] time = 1.31, size = 101, normalized size = 1.28

$$-\frac{\sqrt{dx+1}\,\sqrt{-dx+1}\left((dx+1)\left(\frac{2\,(dx+1)c}{d^3}+\frac{3\,bd^{10}-4\,cd^9}{d^{12}}\right)+\frac{3\left(2\,ad^{11}-bd^{10}+2\,cd^9\right)}{d^{12}}\right)-\frac{6\,b\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)*c/d^3 + (3*b*d^1
0 - 4*c*d^9)/d^12) + 3*(2*a*d^11 - b*d^10 + 2*c*d^9)/d^12) - 6*b*arcsin(1/2
*sqrt(2)*sqrt(d*x + 1))/d^2)/d
```

maple [C] time = 0.00, size = 139, normalized size = 1.76

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(2\sqrt{-d^2x^2+1}\,c\,d^2x^2\mathrm{csgn}(d)+3\sqrt{-d^2x^2+1}\,b\,d^2x\,\mathrm{csgn}(d)+6\sqrt{-d^2x^2+1}\,a\,d^2\mathrm{csgn}(d)-3bd\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+4\sqrt{-d^2x^2+1}\,c\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{6\sqrt{-d^2x^2+1}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/6*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(2*(-d^2*x^2+1)^(1/2)*c*d^2*x^2*csgn(d)+3
*(-d^2*x^2+1)^(1/2)*b*d^2*x*csgn(d)+6*(-d^2*x^2+1)^(1/2)*a*d^2*csgn(d)-3*b*
d*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))+4*(-d^2*x^2+1)^(1/2)*c*csgn(d))/
(-d^2*x^2+1)^(1/2)/d^4*csgn(d)
```

maxima [A] time = 1.27, size = 87, normalized size = 1.10

$$-\frac{\sqrt{-d^2x^2+1}\,cx^2}{3\,d^2}-\frac{\sqrt{-d^2x^2+1}\,bx}{2\,d^2}-\frac{\sqrt{-d^2x^2+1}\,a}{d^2}+\frac{b\arcsin(dx)}{2\,d^3}-\frac{2\,\sqrt{-d^2x^2+1}\,c}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*sqrt(-d^2*x^2 + 1)*c*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*b*x/d^2 - sqrt(-
d^2*x^2 + 1)*a/d^2 + 1/2*b*arcsin(d*x)/d^3 - 2/3*sqrt(-d^2*x^2 + 1)*c/d^4
```

mupad [B] time = 7.61, size = 244, normalized size = 3.09

$$-\frac{\sqrt{1-d\,x}\,\left(\frac{a}{d^2}+\frac{a\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{2\,b\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,b\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{14\,b\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,b\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{2\,b\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}-\frac{\sqrt{1-d\,x}\,\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(a/d^2 + (a*x)/d))/(d*x + 1)^(1/2) - (2*b*atan(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((1 - d*x)^(1/2) - 1)^3)/
((d*x + 1)^(1/2) - 1)^3 - (14*b*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) -
 1)^5 + (2*b*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (2*b*((1 -
d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x
 + 1)^(1/2) - 1)^2 + 1)^4) - ((1 - d*x)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d
) + (c*x^2)/(3*d^2) + (2*c*x)/(3*d^3)))/(d*x + 1)^(1/2)
```

sympy [C] time = 82.52, size = 313, normalized size = 3.96

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{aG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}+\frac{bG_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}-\frac{icG_{6,6}^{6,2}\left(\begin{matrix}-\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1\\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}-\frac{cG_{6,6}^{2,6}\left(\begin{matrix}-2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0),
()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - a*meijerg(((-1, -3/4, -1/2, -1/4,
```

```
0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I*pi)/(d**2*x
**2))/(4*pi**(3/2)*d**2) - I*b*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)),
((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3) + b*m
eijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1,
 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*c*meijerg(((-5
/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(d*
*2*x**2))/(4*pi**(3/2)*d**4) - c*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()
), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d**4)
```

3.16 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=63

$$\frac{(2ad^2+c)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {899, 1815, 641, 216}

$$\frac{(2ad^2+c)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\int \frac{-c-2ad^2-2bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\left(-c-2ad^2\right)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} + \frac{\left(c+2ad^2\right)\sin^{-1}(dx)}{2d^3}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.71

$$\frac{\left(2ad^2+c\right)\sin^{-1}(dx) - d\sqrt{1-d^2x^2}\,(2b+cx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] (-(d*(2*b + c*x)*Sqrt[1 - d^2*x^2]) + (c + 2*a*d^2)*ArcSin[d*x])/(2*d^3)
```

IntegrateAlgebraic [A] time = 0.00, size = 117, normalized size = 1.86

$$\frac{\left(-2ad^2-c\right)\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)}{d^3} - \frac{\sqrt{1-dx}\left(\frac{2bd(1-dx)}{dx+1}+2bd-\frac{c(1-dx)}{dx+1}+c\right)}{d^3\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] -((Sqrt[1 - d*x]*(c + 2*b*d - (c*(1 - d*x))/(1 + d*x) + (2*b*d*(1 - d*x))/(
1 + d*x)))/(d^3*Sqrt[1 + d*x]*(1 + (1 - d*x)/(1 + d*x))^2)) + ((-c - 2*a*d^
2)*ArcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/d^3
```

fricas [A] time = 0.97, size = 67, normalized size = 1.06

$$-\frac{(cdx+2\,bd)\sqrt{dx+1}\,\sqrt{-dx+1}\, + 2\left(2\,ad^2+c\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

giac [A] time = 1.32, size = 76, normalized size = 1.21

$$-\frac{\sqrt{dx+1}\,\sqrt{-dx+1}\left(\frac{(dx+1)c}{d^2}+\frac{2\,bd^5-cd^4}{d^6}\right) - \frac{2\left(2\,ad^2+c\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*c/d^2 + (2*b*d^5 - c*d^4)/d^6
) - 2*(2*a*d^2 + c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)/d
```

maple [C] time = 0.00, size = 117, normalized size = 1.86

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(-2a\,d^2\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+\sqrt{-d^2x^2+1}\,cdx\,\mathrm{csgn}(d)+2\sqrt{-d^2x^2+1}\,bd\,\mathrm{csgn}(d)-c\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)\right)\mathrm{csgn}(d)}{2\sqrt{-d^2x^2+1}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(-2*a*d^2*arctan(1/(-d^2*x^2+1)^(1/2)*d*x
*csgn(d))+(-d^2*x^2+1)^(1/2)*c*d*x*csgn(d)+2*(-d^2*x^2+1)^(1/2)*b*d*csgn(d)
-c*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d)))/(-d^2*x^2+1)^(1/2)/d^3*csgn(d)
```

maxima [A] time = 1.28, size = 57, normalized size = 0.90

$$\frac{a\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}\,cx}{2\,d^2}-\frac{\sqrt{-d^2x^2+1}\,b}{d^2}+\frac{c\arcsin(dx)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt(-d^2*x^2 + 1)*b/d^2
 + 1/2*c*arcsin(d*x)/d^3
```

mupad [B] time = 7.41, size = 232, normalized size = 3.68

$$-\frac{\sqrt{1-d\,x}\,\left(\frac{b}{d^2}+\frac{b\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,c\left(\sqrt{1-d\,x}-1\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}-\frac{14\,c\left(\sqrt{1-d\,x}-1\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,c\left(\sqrt{1-d\,x}-1\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}-\frac{2\,c\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(b/d^2 + (b*x)/d))/(d*x + 1)^(1/2) - (4*a*atan((d*((1 -
d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (2*c*a
tan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((1 - d*x)^(
1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*c*((1 - d*x)^(1/2) - 1)^5)/((d*x
 + 1)^(1/2) - 1)^5 + (2*c*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7
- (2*c*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2)
 - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4)
```

sympy [C] time = 49.68, size = 282, normalized size = 4.48

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{aG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{bG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{icG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}+\frac{cG_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) + a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
 ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi
**(3/2)*d) - I*b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/
4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - b*meijerg(((-1, -3/4,
-1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijerg(((-3/4, -1/4), (-1/2, -1
/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)
*d**3) + c*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-
3/2, -1, -1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

3.17 $\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=48

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\sin^{-1}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

Rubi [A] time = 0.18, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {1609, 1809, 844, 216, 266, 63, 208}

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\sin^{-1}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\,\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x\sqrt{1-d^2x^2}}\,dx\\ &= -\frac{c\sqrt{1-d^2x^2}}{d^2}-\frac{\int \frac{-ad^2-bd^2x}{x\sqrt{1-d^2x^2}}\,dx}{d^2}\\ &= -\frac{c\sqrt{1-d^2x^2}}{d^2}+a\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx+b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx\\ &= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx,x,x^2\right)\\ &= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}-\frac{a\,\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx,x,\sqrt{1-d^2x^2}\right)}{d^2}\\ &= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 1.00

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\sin^{-1}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

IntegrateAlgebraic [A] time = 0.00, size = 95, normalized size = 1.98

$$-2a\tanh^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)-\frac{2b\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)}{d}-\frac{2c\sqrt{1-dx}}{d^2\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-2*c*Sqrt[1 - d*x])/(d^2*Sqrt[1 + d*x]*(1 + (1 - d*x)/(1 + d*x))) - (2*b*A
rcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/d - 2*a*ArcTanh[Sqrt[1 - d*x]/Sqrt[1 +
d*x]]
```

fricas [A] time = 1.00, size = 81, normalized size = 1.69

$$\frac{ad^2\log\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{x}\right)-2\,bd\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)-\sqrt{dx+1}\,\sqrt{-dx+1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] (a*d^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - 2*b*d*arctan((sqrt(d*x +
 1)*sqrt(-d*x + 1) - 1)/(d*x)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [70,22
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[42,56]-a*ln(abs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-
d*x+1)+2*sqrt(2))/sqrt(d*x+1)))+a*ln(abs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*s
qrt(2))-2-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1)))-2*b*(-1/2*pi-atan(s
qrt(d*x+1)*((-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^2-1)/(-2*sqrt(-d
*x+1)+2*sqrt(2))))/d-2*c*d^2/2/d^4*sqrt(d*x+1)*sqrt(-d*x+1)
```

maple [C] time = 0.00, size = 96, normalized size = 2.00

$$\frac{\left(-a\,d^{2}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^{2}x^{2}+1}}\right)\mathrm{csgn}(d)+bd\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-(dx+1)(dx-1)}}\right)-\sqrt{-d^{2}x^{2}+1}\;c\,\mathrm{csgn}(d)\right)\sqrt{-dx+1}\;\sqrt{dx+1}\;\mathrm{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}\;d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-csgn(d)*arctanh(1/(-d^2*x^2+1)^(1/2))*a*d^2-(-d^2*x^2+1)^(1/2)*c*csgn(d)+
b*d*arctan(1/(-(d*x+1)*(d*x-1))^(1/2)*d*x*csgn(d)))*(-d*x+1)^(1/2)*(d*x+1)^
(1/2)/d^2*csgn(d)/(-d^2*x^2+1)^(1/2)
```

maxima [A] time = 1.27, size = 57, normalized size = 1.19

$$-a\log\left(\frac{2\sqrt{-d^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)+\frac{b\arcsin\left(dx\right)}{d}-\frac{\sqrt{-d^{2}x^{2}+1}\,c}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -a*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + b*arcsin(d*x)/d - sqrt(-d^
2*x^2 + 1)*c/d^2
```

mupad [B] time = 4.33, size = 122, normalized size = 2.54

$$a\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^{2}}{\left(\sqrt{d\,x+1}-1\right)^{2}}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{\sqrt{1-d\,x}\;\left(\frac{c}{d^{2}}+\frac{c\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{4\,b\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^{2}}}\right)}{\sqrt{d^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] a*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((1 - d*x)^(1/2)*(c/d^2 + (c*x)/d))/
(d*x + 1)^(1/2) - (4*b*atan((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1
)*(d^2)^(1/2))))/(d^2)^(1/2)
```

sympy [C] time = 55.72, size = 245, normalized size = 5.10

$$\frac{iaG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{aG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{ibG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{bG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} - \frac{cG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
1/(d**2*x**2))/(4*pi**(3/2)) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((
1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 - I*b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), (
)), 1/(d**2*x**2))/(4*pi**(3/2)*d) + b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d) - I*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0,
1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - c*meijerg(((-1, -3/4
, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2
*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

3.18 $\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=48

$$-\frac{a\sqrt{1-d^2x^2}}{x}-b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{c\sin^{-1}(dx)}{d}$$

Rubi [A] time = 0.18, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1609, 1807, 844, 216, 266, 63, 208}

$$-\frac{a\sqrt{1-d^2x^2}}{x}-b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{c\sin^{-1}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x^2\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} - \int \frac{-b-cx}{x\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + b\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx + c\int \frac{1}{\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} + \frac{1}{2}b\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, x^2\right)\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)}{d^2}\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 1.00

$$-\frac{a\sqrt{1-d^2x^2}}{x} - b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right) + \frac{c\sin^{-1}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

IntegrateAlgebraic [A] time = 0.00, size = 93, normalized size = 1.94

$$\frac{2ad\sqrt{1-dx}}{\sqrt{dx+1}\left(\frac{1-dx}{dx+1}-1\right)} - 2b\tanh^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right) - \frac{2c\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] (2*a*d*Sqrt[1 - d*x])/(Sqrt[1 + d*x]*(-1 + (1 - d*x)/(1 + d*x))) - (2*c*Arc
Tan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/d - 2*b*ArcTanh[Sqrt[1 - d*x]/Sqrt[1 + d*
x]]
```

fricas [A] time = 1.31, size = 84, normalized size = 1.75

$$\frac{bdx\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right) - \sqrt{dx+1}\sqrt{-dx+1}\,ad - 2\,cx\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] (b*d*x*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - sqrt(d*x + 1)*sqrt(-d*x
+ 1)*a*d - 2*c*x*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [70,22
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[42,56]1/d*(-2*c*(-1/2*pi-atan(sqrt(d*x+1)*((-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2
))/sqrt(d*x+1))^2-1)/(-2*sqrt(-d*x+1)+2*sqrt(2))))-b*d*ln(abs(2*sqrt(d*x+1)
/(-2*sqrt(-d*x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))
)+b*d*ln(abs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-d*x+
1)+2*sqrt(2))/sqrt(d*x+1)))-4*a*d^2*(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(
2))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))/(-(2*sqrt(d*x+1)/(-2*sqrt(
-d*x+1)+2*sqrt(2))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^2+4))
```

maple [C] time = 0.00, size = 97, normalized size = 2.02

$$\frac{\left(-bdx\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\mathrm{csgn}(d)-\sqrt{-d^2x^2+1}\;ad\,\mathrm{csgn}(d)+cx\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)\right)\sqrt{-dx+1}\;\sqrt{dx+1}\;\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}\;dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-csgn(d)*d*arctanh(1/(-d^2*x^2+1)^(1/2))*x*b-(-d^2*x^2+1)^(1/2)*a*d*csgn(d
)+c*x*arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d)))*(-d*x+1)^(1/2)*(d*x+1)^(1/2
)*csgn(d)/(-d^2*x^2+1)^(1/2)/x/d
```

maxima [A] time = 1.32, size = 57, normalized size = 1.19

$$-b\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{c\arcsin\left(dx\right)}{d}-\frac{\sqrt{-d^2x^2+1}\;a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -b*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + c*arcsin(d*x)/d - sqrt(-d^
2*x^2 + 1)*a/x
```

mupad [B] time = 4.27, size = 114, normalized size = 2.38

$$b\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{4\,c\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{a\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] b*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - (4*c*atan((d*((1 - d*x)^(1/2) - 1))/
(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (a*(1 - d*x)^(1/2)*(d*x
 + 1)^(1/2))/x
```

sympy [C] time = 50.05, size = 221, normalized size = 4.60

$$\frac{iadG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{adG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{bG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{cG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, 1/(d**2*x**2))/(4*pi**(3/2)) + a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), (
)), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) + I*b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2),
(0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi*
*(3/2)) - I*c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1
, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) + c*meijerg(((-1/2, -1/4, 0, 1/4,
 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*d)
```

3.19 $\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}$$

Rubi [A] time = 0.18, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1609, 1807, 807, 266, 63, 208}

$$-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*Sqrt[1 - d^2*x^2])/(2*x^2) - (b*Sqrt[1 - d^2*x^2])/x - ((2*c + a*d^2)*A
rcTanh[Sqrt[1 - d^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
```

```
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x^3\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{1}{2}\int \frac{-2b-\left(2c+ad^2\right)x}{x^2\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(-2c-ad^2\right)\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{4}\left(-2c-ad^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(a+\frac{2c}{d^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx,x,\sqrt{1-d^2x^2}\right)\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(2c+ad^2\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 56, normalized size = 0.79

$$-\frac{\sqrt{1-d^2x^2}\,(a+2bx)}{2x^2}-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/2*((a + 2*b*x)*Sqrt[1 - d^2*x^2])/x^2 - ((2*c + a*d^2)*ArcTanh[Sqrt[1 -
d^2*x^2]])/2
```

IntegrateAlgebraic [A] time = 0.00, size = 112, normalized size = 1.58

$$\left(-ad^2-2c\right)\tanh^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)-\frac{d\sqrt{1-dx}\left(\frac{ad(1-dx)}{dx+1}+ad-\frac{2b(1-dx)}{dx+1}+2b\right)}{\sqrt{dx+1}\left(\frac{1-dx}{dx+1}-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((d*Sqrt[1 - d*x]*(2*b + a*d - (2*b*(1 - d*x))/(1 + d*x) + (a*d*(1 - d*x))
/(1 + d*x)))/(Sqrt[1 + d*x]*(-1 + (1 - d*x)/(1 + d*x))^2)) + (-2*c - a*d^2)
*ArcTanh[Sqrt[1 - d*x]/Sqrt[1 + d*x]]
```

fricas [A] time = 0.88, size = 65, normalized size = 0.92

$$\frac{\left(ad^2+2\,c\right)x^2\log\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{x}\right)-(2\,bx+a)\sqrt{dx+1}\,\sqrt{-dx+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*((a*d^2 + 2*c)*x^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - (2*b*x +
 a)*sqrt(d*x + 1)*sqrt(-d*x + 1))/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [70,22
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[42,56]1/d*(-1/2*(a*d^3+2*c*d)*ln(abs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt
(2))+2-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1)))+1/2*(a*d^3+2*c*d)*ln(a
bs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-d*x+1)+2*sqrt(
2))/sqrt(d*x+1)))-(2*a*d^3*(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-1/2*(
-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^3-4*b*d^2*(2*sqrt(d*x+1)/(-2*sqrt(-
d*x+1)+2*sqrt(2))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^3+8*a*d^3*(2
*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sq
rt(d*x+1))+16*b*d^2*(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-1/2*(-2*sqrt
(-d*x+1)+2*sqrt(2))/sqrt(d*x+1)))/((2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2
))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^2-4)^2)
```

maple [C] time = 0.00, size = 108, normalized size = 1.52

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(a\,d^{2}x^{2}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^{2}x^{2}+1}}\right)+2c\,x^{2}\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^{2}x^{2}+1}}\right)+2\sqrt{-d^{2}x^{2}+1}\,bx+\sqrt{-d^{2}x^{2}+1}\,a\right)\mathrm{csgn}(d)^{2}}{2\sqrt{-d^{2}x^{2}+1}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctanh(1/(-d^2*x^2+1)^(1/2))*
x^2*a*d^2+2*arctanh(1/(-d^2*x^2+1)^(1/2))*x^2*c+2*(-d^2*x^2+1)^(1/2)*b*x+(-
d^2*x^2+1)^(1/2)*a)/(-d^2*x^2+1)^(1/2)/x^2
```

maxima [A] time = 1.28, size = 98, normalized size = 1.38

$$-\frac{1}{2}\,ad^{2}\log\left(\frac{2\sqrt{-d^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)-c\log\left(\frac{2\sqrt{-d^{2}x^{2}+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-d^{2}x^{2}+1}\,b}{x}-\frac{\sqrt{-d^{2}x^{2}+1}\,a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/2*a*d^2*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) - c*log(2*sqrt(-d^2*
x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-d^2*x^2 + 1)*b/x - 1/2*sqrt(-d^2*x^2 +
1)*a/x^2
```

mupad [B] time = 6.30, size = 312, normalized size = 4.39

$$c\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^{2}}{\left(\sqrt{d\,x+1}-1\right)^{2}}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{\frac{a\,d^{2}\left(\sqrt{1-d\,x}-1\right)^{2}}{\left(\sqrt{d\,x+1}-1\right)^{2}}-\frac{a\,d^{2}}{2}+\frac{15\,a\,d^{2}\left(\sqrt{1-d\,x}-1\right)^{4}}{2\left(\sqrt{d\,x+1}-1\right)^{4}}}{\frac{16\left(\sqrt{1-d\,x}-1\right)^{2}}{\left(\sqrt{d\,x+1}-1\right)^{2}}-\frac{32\left(\sqrt{1-d\,x}-1\right)^{4}}{\left(\sqrt{d\,x+1}-1\right)^{4}}+\frac{16\left(\sqrt{1-d\,x}-1\right)^{6}}{\left(\sqrt{d\,x+1}-1\right)^{6}}}+\frac{a\,d^{2}\,\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^{2}}{\left(\sqrt{d\,x+1}-1\right)^{2}}-1\right)}{2}-\frac{a\,d^{2}\,\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2}-\frac{b\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}+\frac{a\,d^{2}\left(\sqrt{1-d\,x}-1\right)^{2}}{32\left(\sqrt{d\,x+1}-1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] c*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((a*d^2*((1 - d*x)^(1/2) - 1)^2)/((d
*x + 1)^(1/2) - 1)^2 - (a*d^2)/2 + (15*a*d^2*((1 - d*x)^(1/2) - 1)^4)/(2*((
d*x + 1)^(1/2) - 1)^4))/((16*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
^2 - (32*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16*((1 - d*x)^
(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6) + (a*d^2*log(((1 - d*x)^(1/2) - 1)^2
/((d*x + 1)^(1/2) - 1)^2 - 1))/2 - (a*d^2*log(((1 - d*x)^(1/2) - 1)/((d*x +
 1)^(1/2) - 1)))/2 - (b*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/x + (a*d^2*((1 - d
*x)^(1/2) - 1)^2)/(32*((d*x + 1)^(1/2) - 1)^2)
```

sympy [C] time = 80.63, size = 218, normalized size = 3.07

$$\frac{iad^2 G_{6,6}^{5,3}\left(\begin{matrix} \frac{7}{4}, \frac{9}{4}, 1 & 2, 2, \frac{5}{2} \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} & 0 \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{ad^2 G_{6,6}^{2,6}\left(\begin{matrix} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 & \\ \frac{5}{4}, \frac{7}{4} & 1, \frac{3}{2}, \frac{3}{2}, 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibdG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 & \frac{3}{2}, \frac{3}{2}, 2 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 & 0 \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{bdG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 & \\ \frac{3}{4}, \frac{5}{4} & \frac{1}{2}, 1, 1, 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{icG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{cG_{6,6}^{2,6}\left(\begin{matrix} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 & \\ \frac{1}{4}, \frac{3}{4} & 0, \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0
,)), 1/(d**2*x**2))/(4*pi**(3/2)) - a*d**2*meijerg(((1, 5/4, 3/2, 7/4, 2, 1
), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)) + I*b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7
/4, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + b*d*meijerg(((1/2, 3/4, 1, 5/
4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)) + I*c*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1
, 5/4, 3/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - c*meijerg(((0, 1/4, 1/2,
 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*
x**2))/(4*pi**(3/2))
```

3.20 $\int \sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=591

$$-\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)(e+fx)^2\sqrt{ac-bcx}\left(8a^2Cf^2-b^2\left(3Ce^2-7f(2Af+Be)\right)\right)}{70b^4f}+\frac{x\sqrt{a+bx}\sqrt{ac-bcx}\left(A\left(6\right.\right.}{}$$

Rubi [A] time = 1.52, antiderivative size = 584, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1610, 1654, 833, 780, 195, 217, 203}

$$\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)(e+fx)^2\sqrt{ac-bcx}\left(-\frac{8a^2Cf^2}{b^2}-7f(2Af+Be)+3Ce^2\right)}{70b^2f}-\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)\sqrt{ac-bcx}\left(3b^2fx\left(a^2f^2(35Bf+41Ce)-b^2\left(6Ce^3-14ef(7Af+Be)\right)\right)+8\left(2a^2b^2f^2\left(7f(Af+3Be)+15Ce^2\right)+8a^4Cf^4+b^4\left(-\left(3Ce^4-7e^2f(12Af+Be)\right)\right)\right)\right)}{840b^6f}+\frac{a^2\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)\left(A\left(6a^2b^2ef^2+8b^4e^3\right)+2a^2b^2e^2(3Bf+Ce)+a^4f^2(Bf+3Ce)\right)}{16b^5\sqrt{a^2c-b^2cx^2}}+\frac{x\sqrt{a+bx}\sqrt{ac-bcx}\left(A\left(6a^2b^2ef^2+8b^4e^3\right)+2a^2b^2e^2(3Bf+Ce)+a^4f^2(Bf+3Ce)\right)}{16b^4}+\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)(e+fx)^3\sqrt{ac-bcx}(3Ce-7Bf)}{42b^2f}-\frac{C\sqrt{a+bx}\left(a^2-b^2x^2\right)(e+fx)^4\sqrt{ac-bcx}}{7b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] ((a^4*f^2*(3*C*e + B*f) + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b^4*e^3 + 6*a^
2*b^2*e*f^2))*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/(16*b^4) + ((3*C*e^2 - (8*
a^2*C*f^2)/b^2 - 7*f*(B*e + 2*A*f))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*
x)^2*(a^2 - b^2*x^2))/(70*b^2*f) + ((3*C*e - 7*B*f)*Sqrt[a + b*x]*Sqrt[a*c
- b*c*x]*(e + f*x)^3*(a^2 - b^2*x^2))/(42*b^2*f) - (C*Sqrt[a + b*x]*Sqrt[a*
c - b*c*x]*(e + f*x)^4*(a^2 - b^2*x^2))/(7*b^2*f) - (Sqrt[a + b*x]*Sqrt[a*c
 - b*c*x]*(8*(8*a^4*C*f^4 + 2*a^2*b^2*f^2*(15*C*e^2 + 7*f*(3*B*e + A*f)) -
b^4*(3*C*e^4 - 7*e^2*f*(B*e + 12*A*f))) + 3*b^2*f*(a^2*f^2*(41*C*e + 35*B*f
) - b^2*(6*C*e^3 - 14*e*f*(B*e + 7*A*f)))*x)*(a^2 - b^2*x^2))/(840*b^6*f) +
 (a^2*Sqrt[c]*(a^4*f^2*(3*C*e + B*f) + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b
^4*e^3 + 6*a^2*b^2*e*f^2))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*ArcTan[(b*Sqrt[c
]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(16*b^5*Sqrt[a^2*c - b^2*c*x^2])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^3\left(A+Bx+Cx^2\right)dx = \frac{\left(\sqrt{a+bx}\,\sqrt{ac-bcx}\right)\int (e+fx)^3\sqrt{a^2c-b^2cx^2}\,\left(A+Bx+C\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{C\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^4\left(a^2-b^2x^2\right)}{7b^2f} - \frac{\left(\sqrt{a+bx}\,\sqrt{\right.}{}$$

$$= \frac{(3Ce-7Bf)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^3\left(a^2-b^2x^2\right)}{42b^2f} - \frac{C\sqrt{}{}$$

$$= -\frac{\left(8a^2Cf^2-b^2\left(3Ce^2-7f(Be+2Af)\right)\right)\sqrt{a+bx}\,\sqrt{ac-bc}{70b^4f}$$

$$= -\frac{\left(8a^2Cf^2-b^2\left(3Ce^2-7f(Be+2Af)\right)\right)\sqrt{a+bx}\,\sqrt{ac-bc}{70b^4f}$$

$$= \frac{\left(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A\left(8b^4e^3+6a^2b^2\right.\right.}{16b^4}$$

$$= \frac{\left(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A\left(8b^4e^3+6a^2b^2\right.\right.}{16b^4}$$

$$= \frac{\left(a^4f^2(3Ce+Bf)+2a^2b^2e^2(Ce+3Bf)+A\left(8b^4e^3+6a^2b^2\right.\right.}{16b^4}$$

Mathematica [A] time = 1.46, size = 427, normalized size = 0.72

$$\frac{\sqrt{c(a-bx)}\left(210a^{5/2}b\sqrt{a-bx}\sqrt{\frac{bx}{a}+1}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)\left(a^4f^2(Bf+3Ce)+A\left(6a^2b^2ef^2+8b^4e^3\right)+2a^2b^2e^2(3Bf+Ce)\right)+\left(a^2-b^2x^2\right)\left(128a^6Cf^3+a^4b^2f\left(7f(32Af+96Be+15Bfx)+C\left(672e^2+315efx+64f^2x^2\right)\right)+2a^2b^4\left(7Af\left(120e^2+45efx+8f^2x^2\right)+7B\left(40e^3+45e^2fx+24ef^2x^2+5f^3x^3\right)+3Cx\left(35e^3+56e^2fx+35ef^2x^2+8f^3x^3\right)\right)-4b^6x\left(21A\left(10e^3+20e^2fx+15ef^2x^2+4f^3x^3\right)+x\left(7B\left(20e^3+45e^2fx+36ef^2x^2+10f^3x^3\right)+3Cx\left(35e^3+84e^2fx+70ef^2x^2+20f^3x^3\right)\right)\right)\right)\right)}{1680b^6(bx-a)\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c*(a - b*x)]*((a^2 - b^2*x^2)*(128*a^6*C*f^3 + a^4*b^2*f*(7*f*(96*B*e
 + 32*A*f + 15*B*f*x) + C*(672*e^2 + 315*e*f*x + 64*f^2*x^2)) + 2*a^2*b^4*(
7*A*f*(120*e^2 + 45*e*f*x + 8*f^2*x^2) + 7*B*(40*e^3 + 45*e^2*f*x + 24*e*f^
2*x^2 + 5*f^3*x^3) + 3*C*x*(35*e^3 + 56*e^2*f*x + 35*e*f^2*x^2 + 8*f^3*x^3)
) - 4*b^6*x*(21*A*(10*e^3 + 20*e^2*f*x + 15*e*f^2*x^2 + 4*f^3*x^3) + x*(7*B
*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3*x^3) + 3*C*x*(35*e^3 + 84*e^2
*f*x + 70*e*f^2*x^2 + 20*f^3*x^3)))) + 210*a^(5/2)*b*(a^4*f^2*(3*C*e + B*f)
 + 2*a^2*b^2*e^2*(C*e + 3*B*f) + A*(8*b^4*e^3 + 6*a^2*b^2*e*f^2))*Sqrt[a -
b*x]*Sqrt[1 + (b*x)/a]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]))/(1680*b^6*
(-a + b*x)*Sqrt[a + b*x])
```

IntegrateAlgebraic [B] time = 2.00, size = 2590, normalized size = 4.38

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3*(A + B*x + C
*x^2),x]
```

```
[Out] ((840*a^2*A*b^5*c^7*e^3*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] + (210*a^4*b^3*c^7
*C*e^3*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] + (630*a^4*b^3*B*c^7*e^2*f*Sqrt[a*c
 - b*c*x])/Sqrt[a + b*x] + (630*a^4*A*b^3*c^7*e*f^2*Sqrt[a*c - b*c*x])/Sqrt
[a + b*x] + (315*a^6*b*c^7*C*e*f^2*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] + (105*
a^6*b*B*c^7*f^3*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] + (3360*a^2*A*b^5*c^6*e^3*
(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (2240*a^3*b^4*B*c^6*e^3*(a*c - b*c*x
)^(3/2))/(a + b*x)^(3/2) - (840*a^4*b^3*c^6*C*e^3*(a*c - b*c*x)^(3/2))/(a +
 b*x)^(3/2) - (6720*a^3*A*b^4*c^6*e^2*f*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2
) - (2520*a^4*b^3*B*c^6*e^2*f*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (6720*
a^5*b^2*c^6*C*e^2*f*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (2520*a^4*A*b^3*
c^6*e*f^2*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (6720*a^5*b^2*B*c^6*e*f^2*
(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (4620*a^6*b*c^6*C*e*f^2*(a*c - b*c*x
)^(3/2))/(a + b*x)^(3/2) - (2240*a^5*A*b^2*c^6*f^3*(a*c - b*c*x)^(3/2))/(a
+ b*x)^(3/2) - (1540*a^6*b*B*c^6*f^3*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) -
 (2240*a^7*c^6*C*f^3*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) + (4200*a^2*A*b^5
*c^5*e^3*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (8960*a^3*b^4*B*c^5*e^3*(a*
c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (2310*a^4*b^3*c^5*C*e^3*(a*c - b*c*x)^(
5/2))/(a + b*x)^(5/2) - (26880*a^3*A*b^4*c^5*e^2*f*(a*c - b*c*x)^(5/2))/(a
+ b*x)^(5/2) - (6930*a^4*b^3*B*c^5*e^2*f*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/
2) - (10752*a^5*b^2*c^5*C*e^2*f*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (693
0*a^4*A*b^3*c^5*e*f^2*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (10752*a^5*b^2
*B*c^5*e*f^2*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) + (3255*a^6*b*c^5*C*e*f^2
*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (3584*a^5*A*b^2*c^5*f^3*(a*c - b*c*
x)^(5/2))/(a + b*x)^(5/2) + (1085*a^6*b*B*c^5*f^3*(a*c - b*c*x)^(5/2))/(a +
 b*x)^(5/2) + (1792*a^7*c^5*C*f^3*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (1
3440*a^3*b^4*B*c^4*e^3*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) - (40320*a^3*A*
b^4*c^4*e^2*f*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) - (8064*a^5*b^2*c^4*C*e^
2*f*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) - (8064*a^5*b^2*B*c^4*e*f^2*(a*c -
 b*c*x)^(7/2))/(a + b*x)^(7/2) - (2688*a^5*A*b^2*c^4*f^3*(a*c - b*c*x)^(7/2
))/(a + b*x)^(7/2) - (7296*a^7*c^4*C*f^3*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/
2) - (4200*a^2*A*b^5*c^3*e^3*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2) - (8960*a
^3*b^4*B*c^3*e^3*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2) + (2310*a^4*b^3*c^3*C
*e^3*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2) - (26880*a^3*A*b^4*c^3*e^2*f*(a*c
```

```
 - b*c*x)^(9/2))/(a + b*x)^(9/2) + (6930*a^4*b^3*B*c^3*e^2*f*(a*c - b*c*x)^
(9/2))/(a + b*x)^(9/2) - (10752*a^5*b^2*c^3*C*e^2*f*(a*c - b*c*x)^(9/2))/(a
 + b*x)^(9/2) + (6930*a^4*A*b^3*c^3*e*f^2*(a*c - b*c*x)^(9/2))/(a + b*x)^(9
/2) - (10752*a^5*b^2*B*c^3*e*f^2*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2) - (32
55*a^6*b*c^3*C*e*f^2*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2) - (3584*a^5*A*b^2
*c^3*f^3*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2) - (1085*a^6*b*B*c^3*f^3*(a*c
 - b*c*x)^(9/2))/(a + b*x)^(9/2) + (1792*a^7*c^3*C*f^3*(a*c - b*c*x)^(9/2))/
(a + b*x)^(9/2) - (3360*a^2*A*b^5*c^2*e^3*(a*c - b*c*x)^(11/2))/(a + b*x)^(
11/2) - (2240*a^3*b^4*B*c^2*e^3*(a*c - b*c*x)^(11/2))/(a + b*x)^(11/2) + (8
40*a^4*b^3*c^2*C*e^3*(a*c - b*c*x)^(11/2))/(a + b*x)^(11/2) - (6720*a^3*A*b
^4*c^2*e^2*f*(a*c - b*c*x)^(11/2))/(a + b*x)^(11/2) + (2520*a^4*b^3*B*c^2*e
^2*f*(a*c - b*c*x)^(11/2))/(a + b*x)^(11/2) - (6720*a^5*b^2*c^2*C*e^2*f*(a*
c - b*c*x)^(11/2))/(a + b*x)^(11/2) + (2520*a^4*A*b^3*c^2*e*f^2*(a*c - b*c*
x)^(11/2))/(a + b*x)^(11/2) - (6720*a^5*b^2*B*c^2*e*f^2*(a*c - b*c*x)^(11/2
))/(a + b*x)^(11/2) + (4620*a^6*b*c^2*C*e*f^2*(a*c - b*c*x)^(11/2))/(a + b*
x)^(11/2) - (2240*a^5*A*b^2*c^2*f^3*(a*c - b*c*x)^(11/2))/(a + b*x)^(11/2)
 + (1540*a^6*b*B*c^2*f^3*(a*c - b*c*x)^(11/2))/(a + b*x)^(11/2) - (2240*a^7*
c^2*C*f^3*(a*c - b*c*x)^(11/2))/(a + b*x)^(11/2) - (840*a^2*A*b^5*c*e^3*(a*
c - b*c*x)^(13/2))/(a + b*x)^(13/2) - (210*a^4*b^3*c*C*e^3*(a*c - b*c*x)^(1
3/2))/(a + b*x)^(13/2) - (630*a^4*b^3*B*c*e^2*f*(a*c - b*c*x)^(13/2))/(a +
b*x)^(13/2) - (630*a^4*A*b^3*c*e*f^2*(a*c - b*c*x)^(13/2))/(a + b*x)^(13/2)
 - (315*a^6*b*c*C*e*f^2*(a*c - b*c*x)^(13/2))/(a + b*x)^(13/2) - (105*a^6*b
*B*c*f^3*(a*c - b*c*x)^(13/2))/(a + b*x)^(13/2))/(840*b^6*(c + (a*c - b*c*x
)/(a + b*x))^7) + ((-8*a^2*A*b^4*Sqrt[c]*e^3 - 2*a^4*b^2*Sqrt[c]*C*e^3 - 6*
a^4*b^2*B*Sqrt[c]*e^2*f - 6*a^4*A*b^2*Sqrt[c]*e*f^2 - 3*a^6*Sqrt[c]*C*e*f^2
 - a^6*B*Sqrt[c]*f^3)*ArcTan[Sqrt[a*c - b*c*x]/(Sqrt[c]*Sqrt[a + b*x])])/(8
*b^5)
```

fricas [A] time = 0.89, size = 1001, normalized size = 1.69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/3360*(105*(6*B*a^4*b^3*e^2*f + B*a^6*b*f^3 + 2*(C*a^4*b^3 + 4*A*a^2*b^5)
*e^3 + 3*(C*a^6*b + 2*A*a^4*b^3)*e*f^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-
b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(240*C*b^6*f^3*x^6 - 5
60*B*a^2*b^4*e^3 - 672*B*a^4*b^2*e*f^2 + 280*(3*C*b^6*e*f^2 + B*b^6*f^3)*x^
5 + 48*(21*C*b^6*e^2*f + 21*B*b^6*e*f^2 - (C*a^2*b^4 - 7*A*b^6)*f^3)*x^4 -
336*(2*C*a^4*b^2 + 5*A*a^2*b^4)*e^2*f - 32*(4*C*a^6 + 7*A*a^4*b^2)*f^3 + 70
*(6*C*b^6*e^3 + 18*B*b^6*e^2*f - B*a^2*b^4*f^3 - 3*(C*a^2*b^4 - 6*A*b^6)*e*
f^2)*x^3 + 16*(35*B*b^6*e^3 - 21*B*a^2*b^4*e*f^2 - 21*(C*a^2*b^4 - 5*A*b^6)
*e^2*f - (4*C*a^4*b^2 + 7*A*a^2*b^4)*f^3)*x^2 - 105*(6*B*a^2*b^4*e^2*f + B*
a^4*b^2*f^3 + 2*(C*a^2*b^4 - 4*A*b^6)*e^3 + 3*(C*a^4*b^2 + 2*A*a^2*b^4)*e*f
^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^6, -1/1680*(105*(6*B*a^4*b^3*e^2
*f + B*a^6*b*f^3 + 2*(C*a^4*b^3 + 4*A*a^2*b^5)*e^3 + 3*(C*a^6*b + 2*A*a^4*b
^3)*e*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2
*c*x^2 - a^2*c)) - (240*C*b^6*f^3*x^6 - 560*B*a^2*b^4*e^3 - 672*B*a^4*b^2*e
*f^2 + 280*(3*C*b^6*e*f^2 + B*b^6*f^3)*x^5 + 48*(21*C*b^6*e^2*f + 21*B*b^6*
e*f^2 - (C*a^2*b^4 - 7*A*b^6)*f^3)*x^4 - 336*(2*C*a^4*b^2 + 5*A*a^2*b^4)*e^
2*f - 32*(4*C*a^6 + 7*A*a^4*b^2)*f^3 + 70*(6*C*b^6*e^3 + 18*B*b^6*e^2*f - B
*a^2*b^4*f^3 - 3*(C*a^2*b^4 - 6*A*b^6)*e*f^2)*x^3 + 16*(35*B*b^6*e^3 - 21*B
*a^2*b^4*e*f^2 - 21*(C*a^2*b^4 - 5*A*b^6)*e^2*f - (4*C*a^4*b^2 + 7*A*a^2*b^
4)*f^3)*x^2 - 105*(6*B*a^2*b^4*e^2*f + B*a^4*b^2*f^3 + 2*(C*a^2*b^4 - 4*A*b
^6)*e^3 + 3*(C*a^4*b^2 + 2*A*a^2*b^4)*e*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x
 + a))/b^6]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 1446, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/1680*(b*x+a)^(1/2)*(-c*(b*x-a))^(1/2)*(-630*B*(b^2*c)^(1/2)*(-(b^2*x^2-a^
2)*c)^(1/2)*x*a^2*b^4*e^2*f+105*B*arctan((b^2*c)^(1/2)*x/(-(b^2*x^2-a^2)*c)
^(1/2))*a^6*b^2*c*f^3+240*C*x^6*b^6*f^3*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1
/2)+280*B*x^5*b^6*f^3*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+336*A*x^4*b^6*
f^3*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+420*C*x^3*b^6*e^3*(b^2*c)^(1/2)*
(-(b^2*x^2-a^2)*c)^(1/2)+560*B*x^2*b^6*e^3*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)
^(1/2)-224*A*a^4*b^2*f^3*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)-560*B*a^2*b
^4*e^3*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+210*C*arctan((b^2*c)^(1/2)*x/
(-(b^2*x^2-a^2)*c)^(1/2))*a^4*b^4*c*e^3+840*A*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)
*c)^(1/2)*x*b^6*e^3+840*A*arctan((b^2*c)^(1/2)*x/(-(b^2*x^2-a^2)*c)^(1/2))*
a^2*b^6*c*e^3-128*C*a^6*f^3*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)-112*A*x^
2*a^2*b^4*f^3*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+1680*A*x^2*b^6*e^2*f*(
b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)-64*C*x^2*a^4*b^2*f^3*(b^2*c)^(1/2)*(-
(b^2*x^2-a^2)*c)^(1/2)-1680*A*a^2*b^4*e^2*f*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c
)^(1/2)-672*B*a^4*b^2*e*f^2*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)-672*C*a^
4*b^2*e^2*f*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+630*A*arctan((b^2*c)^(1/
2)*x/(-(b^2*x^2-a^2)*c)^(1/2))*a^4*b^4*c*e*f^2+630*B*arctan((b^2*c)^(1/2)*x
/(-(b^2*x^2-a^2)*c)^(1/2))*a^4*b^4*c*e^2*f-105*B*(b^2*c)^(1/2)*(-(b^2*x^2-a
^2)*c)^(1/2)*x*a^4*b^2*f^3-210*C*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*a
^2*b^4*e^3+1008*C*x^4*b^6*e^2*f*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+1260
*A*x^3*b^6*e*f^2*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)-70*B*x^3*a^2*b^4*f^
3*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+1260*B*x^3*b^6*e^2*f*(b^2*c)^(1/2)
*(-(b^2*x^2-a^2)*c)^(1/2)-315*C*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*a^
4*b^2*e*f^2-630*A*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*a^2*b^4*e*f^2-21
0*C*x^3*a^2*b^4*e*f^2*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)-336*B*x^2*a^2*
b^4*e*f^2*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)-336*C*x^2*a^2*b^4*e^2*f*(b
^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+315*C*arctan((b^2*c)^(1/2)*x/(-(b^2*x^
2-a^2)*c)^(1/2))*a^6*b^2*c*e*f^2+840*C*x^5*b^6*e*f^2*(b^2*c)^(1/2)*(-(b^2*x
^2-a^2)*c)^(1/2)+1008*B*x^4*b^6*e*f^2*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2
)-48*C*x^4*a^2*b^4*f^3*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2))/(-(b^2*x^2-a
^2)*c)^(1/2)/b^6/(b^2*c)^(1/2)
```

maxima [A] time = 1.46, size = 584, normalized size = 0.99

$$-\frac{\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}Cf^{3}x^{4}}{7\,b^{2}c}+\frac{Aa^{2}\sqrt{c}e^{3}\arcsin\left(\frac{bx}{a}\right)}{2\,b}+\frac{1}{2}\sqrt{-b^{2}cx^{2}+a^{2}c}\,Ae^{3}x-\frac{4\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}Ca^{2}f^{3}x^{2}}{35\,b^{4}c}+\frac{\left(3\,Cef^{2}+Bf^{3}\right)a^{6}\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{16\,b^{5}}+\frac{\left(Ce^{3}+3\,Be^{2}f+3\,Aef^{2}\right)a^{4}\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8\,b^{3}}-\frac{\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}Be^{3}}{3\,b^{2}c}-\frac{\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}Ae^{2}f}{b^{2}c}-\frac{8\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}Ca^{4}f^{3}}{105\,b^{6}c}+\frac{\sqrt{-b^{2}cx^{2}+a^{2}c}\left(3\,Cef^{2}+Bf^{3}\right)a^{4}x}{16\,b^{4}}+\frac{\sqrt{-b^{2}cx^{2}+a^{2}c}\left(Ce^{3}+3\,Be^{2}f+3\,Aef^{2}\right)a^{2}x}{8\,b^{2}}-\frac{\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}\left(3\,Cef^{2}+Bf^{3}\right)x^{3}}{6\,b^{2}c}-\frac{\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}\left(3\,Ce^{2}f+3\,Bef^{2}+Af^{3}\right)x^{2}}{5\,b^{2}c}-\frac{\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}\left(3\,Cef^{2}+Bf^{3}\right)a^{2}x}{8\,b^{4}c}-\frac{\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}\left(Ce^{3}+3\,Be^{2}f+3\,Aef^{2}\right)x}{4\,b^{2}c}-\frac{2\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}\left(3\,Ce^{2}f+3\,Bef^{2}+Af^{3}\right)a^{2}}{15\,b^{4}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/7*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f^3*x^4/(b^2*c) + 1/2*A*a^2*sqrt(c)*e^3*a
rcsin(b*x/a)/b + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*e^3*x - 4/35*(-b^2*c*x^2 +
a^2*c)^(3/2)*C*a^2*f^3*x^2/(b^4*c) + 1/16*(3*C*e*f^2 + B*f^3)*a^6*sqrt(c)*a
```

```
rcsin(b*x/a)/b^5 + 1/8*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*a^4*sqrt(c)*arcsin(b
*x/a)/b^3 - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*B*e^3/(b^2*c) - (-b^2*c*x^2 + a^
2*c)^(3/2)*A*e^2*f/(b^2*c) - 8/105*(-b^2*c*x^2 + a^2*c)^(3/2)*C*a^4*f^3/(b^
6*c) + 1/16*sqrt(-b^2*c*x^2 + a^2*c)*(3*C*e*f^2 + B*f^3)*a^4*x/b^4 + 1/8*sq
rt(-b^2*c*x^2 + a^2*c)*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*a^2*x/b^2 - 1/6*(-b^
2*c*x^2 + a^2*c)^(3/2)*(3*C*e*f^2 + B*f^3)*x^3/(b^2*c) - 1/5*(-b^2*c*x^2 +
a^2*c)^(3/2)*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*x^2/(b^2*c) - 1/8*(-b^2*c*x^2
+ a^2*c)^(3/2)*(3*C*e*f^2 + B*f^3)*a^2*x/(b^4*c) - 1/4*(-b^2*c*x^2 + a^2*c)
^(3/2)*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*x/(b^2*c) - 2/15*(-b^2*c*x^2 + a^2*c
)^(3/2)*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*a^2/(b^4*c)
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^3*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

3.21 $\int \sqrt{a+bx}\sqrt{ac-bcx}\,(e+fx)^2\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=451

$$-\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)\sqrt{ac-bcx}\left(3fx\left(5a^2Cf^2-b^2\left(2Ce^2-2f(5Af+2Be)\right)\right)+8\left(2a^2f^2(Bf+2Ce)-b^2e\left(C\right.\right.\right.}{120b^4f}$$

Rubi [A] time = 1.01, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1610, 1654, 833, 780, 195, 217, 203}

$$-\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)\sqrt{ac-bcx}\left(3fx\left(5a^2Cf^2-b^2\left(2Ce^2-2f(5Af+2Be)\right)\right)+8\left(2a^2f^2(Bf+2Ce)-\frac{1}{8}b^2\left(8Ce^3-16ef(5Af+Be)\right)\right)\right)}{120b^4f}+\frac{a^2\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)\left(2A\left(a^2b^2f^2+4b^4e^2\right)+2a^2b^2e(2Bf+Ce)+a^4Cf^2\right)}{16b^5\sqrt{a^2c-b^2cx^2}}+\frac{x\sqrt{a+bx}\sqrt{ac-bcx}\left(2A\left(a^2b^2f^2+4b^4e^2\right)+2a^2b^2e(2Bf+Ce)+a^4Cf^2\right)}{16b^4}+\frac{\sqrt{a+bx}\left(a^2-b^2x^2\right)(e+fx)^2\sqrt{ac-bcx}(Ce-2Bf)}{10b^2f}-\frac{C\sqrt{a+bx}\left(a^2-b^2x^2\right)(e+fx)^3\sqrt{ac-bcx}}{6b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] ((a^4*C*f^2 + 2*a^2*b^2*e*(C*e + 2*B*f) + 2*A*(4*b^4*e^2 + a^2*b^2*f^2))*x*
Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/(16*b^4) + ((C*e - 2*B*f)*Sqrt[a + b*x]*Sq
rt[a*c - b*c*x]*(e + f*x)^2*(a^2 - b^2*x^2))/(10*b^2*f) - (C*Sqrt[a + b*x]*
Sqrt[a*c - b*c*x]*(e + f*x)^3*(a^2 - b^2*x^2))/(6*b^2*f) - (Sqrt[a + b*x]*S
qrt[a*c - b*c*x]*(8*(2*a^2*f^2*(2*C*e + B*f) - (b^2*(8*C*e^3 - 16*e*f*(B*e
+ 5*A*f)))/8) + 3*f*(5*a^2*C*f^2 - b^2*(2*C*e^2 - 2*f*(2*B*e + 5*A*f)))*x)*
(a^2 - b^2*x^2))/(120*b^4*f) + (a^2*Sqrt[c]*(a^4*C*f^2 + 2*a^2*b^2*e*(C*e +
 2*B*f) + 2*A*(4*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*Ar
cTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(16*b^5*Sqrt[a^2*c - b^2*c*x^2
])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
```

```
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2\left(A+Bx+Cx^2\right)dx = \frac{\left(\sqrt{a+bx}\,\sqrt{ac-bcx}\right)\int (e+fx)^2\sqrt{a^2c-b^2cx^2}\,\left(A+Bx+C \cdots\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{C\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^3\left(a^2-b^2x^2\right)}{6b^2f} - \frac{\left(\sqrt{a+bx}\,\sqrt{a \cdots\right.}{\cdots}$$

$$= \frac{(Ce-2Bf)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2\left(a^2-b^2x^2\right)}{10b^2f} - \frac{C\sqrt{a \cdots}{\cdots}$$

$$= \frac{(Ce-2Bf)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2\left(a^2-b^2x^2\right)}{10b^2f} - \frac{C\sqrt{a \cdots}{\cdots}$$

$$= \frac{\left(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A\left(4b^4e^2+a^2b^2f^2\right)\right)x\sqrt{a - \cdots}}{16b^4}$$

$$= \frac{\left(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A\left(4b^4e^2+a^2b^2f^2\right)\right)x\sqrt{a - \cdots}}{16b^4}$$

$$= \frac{\left(a^4Cf^2+2a^2b^2e(Ce+2Bf)+2A\left(4b^4e^2+a^2b^2f^2\right)\right)x\sqrt{a - \cdots}}{16b^4}$$

Mathematica [A] time = 1.02, size = 311, normalized size = 0.69

$$\frac{\sqrt{c(a-bx)}\left(b\left(a^2-b^2x^2\right)\left(a^4f(32Bf+64Ce+15Cfx)+2a^2b^2\left(5Af(16e+3fx)+B\left(40e^2+30efx+8f^2x^2\right)+Cx\left(15e^2+16efx+5f^2x^2\right)\right)-4b^4x\left(5A\left(6e^2+8efx+3f^2x^2\right)+x\left(2B\left(10e^2+15efx+6f^2x^2\right)+Cx\left(15e^2+24efx+10f^2x^2\right)\right)\right)\right)+30a^{5/2}\sqrt{a-bx}\sqrt{\frac{bx}{a}+1}\,\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)\left(a^4Cf^2+2A\left(a^2b^2f^2+4b^4e^2\right)+2a^2b^2e(2Bf+Ce)\right)\right)}{240b^5(bx-a)\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(A + B*x + C*x^2),x]
```

```
[Out] (Sqrt[c*(a - b*x)]*(b*(a^2 - b^2*x^2)*(a^4*f*(64*C*e + 32*B*f + 15*C*f*x) +
 2*a^2*b^2*(5*A*f*(16*e + 3*f*x) + C*x*(15*e^2 + 16*e*f*x + 5*f^2*x^2) + B*
(40*e^2 + 30*e*f*x + 8*f^2*x^2)) - 4*b^4*x*(5*A*(6*e^2 + 8*e*f*x + 3*f^2*x^
2) + x*(2*B*(10*e^2 + 15*e*f*x + 6*f^2*x^2) + C*x*(15*e^2 + 24*e*f*x + 10*f
^2*x^2)))) + 30*a^(5/2)*(a^4*C*f^2 + 2*a^2*b^2*e*(C*e + 2*B*f) + 2*A*(4*b^4
*e^2 + a^2*b^2*f^2))*Sqrt[a - b*x]*Sqrt[1 + (b*x)/a]*ArcSin[Sqrt[a - b*x]/(
Sqrt[2]*Sqrt[a])]))/(240*b^5*(-a + b*x)*Sqrt[a + b*x])
```

IntegrateAlgebraic [B] time = 1.29, size = 1792, normalized size = 3.97

result too large to display

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(A + B*x + C
*x^2),x]
```

```
[Out] ((120*a^2*A*b^4*c^6*e^2*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] + (30*a^4*b^2*c^6*
C*e^2*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] + (60*a^4*b^2*B*c^6*e*f*Sqrt[a*c - b
*c*x])/Sqrt[a + b*x] + (30*a^4*A*b^2*c^6*f^2*Sqrt[a*c - b*c*x])/Sqrt[a + b*
x] + (15*a^6*c^6*C*f^2*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] + (360*a^2*A*b^4*c^
5*e^2*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (320*a^3*b^3*B*c^5*e^2*(a*c -
b*c*x)^(3/2))/(a + b*x)^(3/2) - (150*a^4*b^2*c^5*C*e^2*(a*c - b*c*x)^(3/2))
/(a + b*x)^(3/2) - (640*a^3*A*b^3*c^5*e*f*(a*c - b*c*x)^(3/2))/(a + b*x)^(3
/2) - (300*a^4*b^2*B*c^5*e*f*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (640*a^
5*b*c^5*C*e*f*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (150*a^4*A*b^2*c^5*f^2
*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (320*a^5*b*B*c^5*f^2*(a*c - b*c*x)^
(3/2))/(a + b*x)^(3/2) - (235*a^6*c^5*C*f^2*(a*c - b*c*x)^(3/2))/(a + b*x)^
(3/2) + (240*a^2*A*b^4*c^4*e^2*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (960*
a^3*b^3*B*c^4*e^2*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (180*a^4*b^2*c^4*C
*e^2*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (1920*a^3*A*b^3*c^4*e*f*(a*c -
b*c*x)^(5/2))/(a + b*x)^(5/2) - (360*a^4*b^2*B*c^4*e*f*(a*c - b*c*x)^(5/2))
/(a + b*x)^(5/2) - (384*a^5*b*c^4*C*e*f*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2
) - (180*a^4*A*b^2*c^4*f^2*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (192*a^5*
b*B*c^4*f^2*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) + (390*a^6*c^4*C*f^2*(a*c
- b*c*x)^(5/2))/(a + b*x)^(5/2) - (240*a^2*A*b^4*c^3*e^2*(a*c - b*c*x)^(7/2
))/(a + b*x)^(7/2) - (960*a^3*b^3*B*c^3*e^2*(a*c - b*c*x)^(7/2))/(a + b*x)^
(7/2) + (180*a^4*b^2*c^3*C*e^2*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) - (1920
*a^3*A*b^3*c^3*e*f*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) + (360*a^4*b^2*B*c^
3*e*f*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) - (384*a^5*b*c^3*C*e*f*(a*c - b*
c*x)^(7/2))/(a + b*x)^(7/2) + (180*a^4*A*b^2*c^3*f^2*(a*c - b*c*x)^(7/2))/(
a + b*x)^(7/2) - (192*a^5*b*B*c^3*f^2*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2)
- (390*a^6*c^3*C*f^2*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) - (360*a^2*A*b^4*
c^2*e^2*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2) - (320*a^3*b^3*B*c^2*e^2*(a*c
- b*c*x)^(9/2))/(a + b*x)^(9/2) + (150*a^4*b^2*c^2*C*e^2*(a*c - b*c*x)^(9/2
))/(a + b*x)^(9/2) - (640*a^3*A*b^3*c^2*e*f*(a*c - b*c*x)^(9/2))/(a + b*x)^
(9/2) + (300*a^4*b^2*B*c^2*e*f*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2) - (640*
a^5*b*c^2*C*e*f*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2) + (150*a^4*A*b^2*c^2*f
^2*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2) - (320*a^5*b*B*c^2*f^2*(a*c - b*c*x
)^(9/2))/(a + b*x)^(9/2) + (235*a^6*c^2*C*f^2*(a*c - b*c*x)^(9/2))/(a + b*x
)^(9/2) - (120*a^2*A*b^4*c*e^2*(a*c - b*c*x)^(11/2))/(a + b*x)^(11/2) - (30
*a^4*b^2*c*C*e^2*(a*c - b*c*x)^(11/2))/(a + b*x)^(11/2) - (60*a^4*b^2*B*c*e
*f*(a*c - b*c*x)^(11/2))/(a + b*x)^(11/2) - (30*a^4*A*b^2*c*f^2*(a*c - b*c*
x)^(11/2))/(a + b*x)^(11/2) - (15*a^6*c*C*f^2*(a*c - b*c*x)^(11/2))/(a + b*
x)^(11/2))/(120*b^5*(c + (a*c - b*c*x)/(a + b*x))^6) + ((-8*a^2*A*b^4*Sqrt[
c]*e^2 - 2*a^4*b^2*Sqrt[c]*C*e^2 - 4*a^4*b^2*B*Sqrt[c]*e*f - 2*a^4*A*b^2*Sq
rt[c]*f^2 - a^6*Sqrt[c]*C*f^2)*ArcTan[Sqrt[a*c - b*c*x]/(Sqrt[c]*Sqrt[a + b
*x])])/(8*b^5)
```

fricas [A] time = 0.98, size = 703, normalized size = 1.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/480*(15*(4*B*a^4*b^2*e*f + 2*(C*a^4*b^2 + 4*A*a^2*b^4)*e^2 + (C*a^6 + 2*
A*a^4*b^2)*f^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x +
a)*b*sqrt(-c)*x - a^2*c) + 2*(40*C*b^5*f^2*x^5 - 80*B*a^2*b^3*e^2 - 32*B*a^
4*b*f^2 + 48*(2*C*b^5*e*f + B*b^5*f^2)*x^4 + 10*(6*C*b^5*e^2 + 12*B*b^5*e*f
 - (C*a^2*b^3 - 6*A*b^5)*f^2)*x^3 - 32*(2*C*a^4*b + 5*A*a^2*b^3)*e*f + 16*(
5*B*b^5*e^2 - B*a^2*b^3*f^2 - 2*(C*a^2*b^3 - 5*A*b^5)*e*f)*x^2 - 15*(4*B*a^
2*b^3*e*f + 2*(C*a^2*b^3 - 4*A*b^5)*e^2 + (C*a^4*b + 2*A*a^2*b^3)*f^2)*x)*s
qrt(-b*c*x + a*c)*sqrt(b*x + a))/b^5, -1/240*(15*(4*B*a^4*b^2*e*f + 2*(C*a^
4*b^2 + 4*A*a^2*b^4)*e^2 + (C*a^6 + 2*A*a^4*b^2)*f^2)*sqrt(c)*arctan(sqrt(-
b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (40*C*b^5*f^2
*x^5 - 80*B*a^2*b^3*e^2 - 32*B*a^4*b*f^2 + 48*(2*C*b^5*e*f + B*b^5*f^2)*x^4
 + 10*(6*C*b^5*e^2 + 12*B*b^5*e*f - (C*a^2*b^3 - 6*A*b^5)*f^2)*x^3 - 32*(2*
C*a^4*b + 5*A*a^2*b^3)*e*f + 16*(5*B*b^5*e^2 - B*a^2*b^3*f^2 - 2*(C*a^2*b^3
 - 5*A*b^5)*e*f)*x^2 - 15*(4*B*a^2*b^3*e*f + 2*(C*a^2*b^3 - 4*A*b^5)*e^2 +
(C*a^4*b + 2*A*a^2*b^3)*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^5]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.02, size = 987, normalized size = 2.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/240*(b*x+a)^(1/2)*(-(b*x-a)*c)^(1/2)*(40*C*x^5*b^4*f^2*(-(b^2*x^2-a^2)*c)
^(1/2)*(b^2*c)^(1/2)+48*B*x^4*b^4*f^2*(-(b^2*x^2-a^2)*c)^(1/2)*(b^2*c)^(1/2
)+60*A*x^3*b^4*f^2*(-(b^2*x^2-a^2)*c)^(1/2)*(b^2*c)^(1/2)+60*C*x^3*b^4*e^2*
(-(b^2*x^2-a^2)*c)^(1/2)*(b^2*c)^(1/2)+80*B*x^2*b^4*e^2*(-(b^2*x^2-a^2)*c)^
(1/2)*(b^2*c)^(1/2)-80*B*a^2*b^2*e^2*(-(b^2*x^2-a^2)*c)^(1/2)*(b^2*c)^(1/2)
-60*B*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*a^2*b^2*e*f+15*C*arctan((b^2
*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^6*c*f^2-32*B*a^4*f^2*(-(b^2*x^2-a^2
)*c)^(1/2)*(b^2*c)^(1/2)-64*C*a^4*e*f*(-(b^2*x^2-a^2)*c)^(1/2)*(b^2*c)^(1/2
)+30*A*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^4*b^2*c*f^2+120*A
*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^2*b^4*c*e^2+30*C*arctan
((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^4*b^2*c*e^2+120*A*(b^2*c)^(1/2
)*(-(b^2*x^2-a^2)*c)^(1/2)*x*b^4*e^2-15*C*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^
(1/2)*x*a^4*f^2-32*C*x^2*a^2*b^2*e*f*(-(b^2*x^2-a^2)*c)^(1/2)*(b^2*c)^(1/2)
-30*A*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*a^2*b^2*f^2-30*C*(b^2*c)^(1/
2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*a^2*b^2*e^2-10*C*x^3*a^2*b^2*f^2*(-(b^2*x^2-a
^2)*c)^(1/2)*(b^2*c)^(1/2)+160*A*x^2*b^4*e*f*(-(b^2*x^2-a^2)*c)^(1/2)*(b^2*
c)^(1/2)-16*B*x^2*a^2*b^2*f^2*(-(b^2*x^2-a^2)*c)^(1/2)*(b^2*c)^(1/2)-160*A*
a^2*b^2*e*f*(-(b^2*x^2-a^2)*c)^(1/2)*(b^2*c)^(1/2)+60*B*arctan((b^2*c)^(1/2
```

```
)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^4*b^2*c*e*f+96*C*x^4*b^4*e*f*(-(b^2*x^2-a^2
)*c)^(1/2)*(b^2*c)^(1/2)+120*B*x^3*b^4*e*f*(-(b^2*x^2-a^2)*c)^(1/2)*(b^2*c)
^(1/2))/(-(b^2*x^2-a^2)*c)^(1/2)/b^4/(b^2*c)^(1/2)
```

maxima [A] time = 2.07, size = 417, normalized size = 0.92

$$\frac{Aa^2\sqrt{c}e^2\arcsin\left(\frac{bx}{a}\right)}{2b}+\frac{Ca^6\sqrt{c}f^2\arcsin\left(\frac{bx}{a}\right)}{16b^5}+\frac{1}{2}\sqrt{-b^2cx^2+a^2c}Ae^2x+\frac{\sqrt{-b^2cx^2+a^2c}Ca^4f^2x}{16b^4}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}Cf^2x^3}{6b^2c}+\frac{\left(Ce^2+2Bef+Af^2\right)a^4\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8b^3}+\frac{\sqrt{-b^2cx^2+a^2c}\left(Ce^2+2Bef+Af^2\right)a^2x}{8b^2}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}Ca^2f^2x}{8b^4c}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}Be^2}{3b^2c}-\frac{2\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}Aef}{3b^2c}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}\left(2Cef+Bf^2\right)x^2}{5b^2c}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}\left(Ce^2+2Bef+Af^2\right)x}{4b^2c}-\frac{2\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}\left(2Cef+Bf^2\right)a^2}{15b^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*A*a^2*sqrt(c)*e^2*arcsin(b*x/a)/b + 1/16*C*a^6*sqrt(c)*f^2*arcsin(b*x/a
)/b^5 + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*e^2*x + 1/16*sqrt(-b^2*c*x^2 + a^2*c
)*C*a^4*f^2*x/b^4 - 1/6*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f^2*x^3/(b^2*c) + 1/8*
(C*e^2 + 2*B*e*f + A*f^2)*a^4*sqrt(c)*arcsin(b*x/a)/b^3 + 1/8*sqrt(-b^2*c*x
^2 + a^2*c)*(C*e^2 + 2*B*e*f + A*f^2)*a^2*x/b^2 - 1/8*(-b^2*c*x^2 + a^2*c)^
(3/2)*C*a^2*f^2*x/(b^4*c) - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*B*e^2/(b^2*c) -
2/3*(-b^2*c*x^2 + a^2*c)^(3/2)*A*e*f/(b^2*c) - 1/5*(-b^2*c*x^2 + a^2*c)^(3/
2)*(2*C*e*f + B*f^2)*x^2/(b^2*c) - 1/4*(-b^2*c*x^2 + a^2*c)^(3/2)*(C*e^2 +
2*B*e*f + A*f^2)*x/(b^2*c) - 2/15*(-b^2*c*x^2 + a^2*c)^(3/2)*(2*C*e*f + B*f
^2)*a^2/(b^4*c)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^2*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

3.22 $\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=300

$$\frac{x\sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(a^2(Bf+Ce)+4Ab^2e\right)}{8b^2}-\frac{\sqrt{a+bx}\,\left(a^2-b^2x^2\right)\sqrt{ac-bcx}\,\left(4\left(2a^2Cf^2-b^2\left(3Ce^2-5f(Af+\right.\right.\right.}{60b^4f}$$

Rubi [A] time = 0.45, antiderivative size = 297, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1610, 1654, 780, 195, 217, 203}

$$-\frac{\sqrt{a+bx}\,\left(a^2-b^2x^2\right)\sqrt{ac-bcx}\,\left(4\left(2a^2Cf^2-b^2\left(3Ce^2-5f(Af+Be)\right)\right)-3b^2fx(3Ce-5Bf)\right)}{60b^4f}+\frac{a^2\sqrt{c}\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)\left(a^2(Bf+Ce)+4Ab^2e\right)}{8b^3\sqrt{a^2c-b^2cx^2}}+\frac{1}{8}x\sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(\frac{a^2(Bf+Ce)}{b^2}+4Ae\right)-\frac{C\sqrt{a+bx}\,\left(a^2-b^2x^2\right)(e+fx)^2\sqrt{ac-bcx}}{5b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] ((4*A*e + (a^2*(C*e + B*f))/b^2)*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 - (C*
Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2*(a^2 - b^2*x^2))/(5*b^2*f) - (S
qrt[a + b*x]*Sqrt[a*c - b*c*x]*(4*(2*a^2*C*f^2 - b^2*(3*C*e^2 - 5*f*(B*e +
A*f))) - 3*b^2*f*(3*C*e - 5*B*f)*x)*(a^2 - b^2*x^2))/(60*b^4*f) + (a^2*Sqrt
[c]*(4*A*b^2*e + a^2*(C*e + B*f))*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*ArcTan[(b
*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^3*Sqrt[a^2*c - b^2*c*x^2])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)\left(A+Bx+Cx^2\right)dx = \frac{\left(\sqrt{a+bx}\,\sqrt{ac-bcx}\right)\int(e+fx)\sqrt{a^2c-b^2cx^2}\,\left(A+Bx+\right.}{\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{C\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2\left(a^2-b^2x^2\right)}{5b^2f} - \frac{\left(\sqrt{a+bx}\right.}{}$$

$$= -\frac{C\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2\left(a^2-b^2x^2\right)}{5b^2f} - \frac{\sqrt{a+bx}\,\sqrt{}}{}$$

$$= \frac{1}{8}\left(4Ae+\frac{a^2(Ce+Bf)}{b^2}\right)x\sqrt{a+bx}\,\sqrt{ac-bcx} - \frac{C\sqrt{a+b}}{}$$

$$= \frac{1}{8}\left(4Ae+\frac{a^2(Ce+Bf)}{b^2}\right)x\sqrt{a+bx}\,\sqrt{ac-bcx} - \frac{C\sqrt{a+b}}{}$$

$$= \frac{1}{8}\left(4Ae+\frac{a^2(Ce+Bf)}{b^2}\right)x\sqrt{a+bx}\,\sqrt{ac-bcx} - \frac{C\sqrt{a+b}}{}$$

Mathematica [A] time = 0.68, size = 200, normalized size = 0.67

$$-\frac{c\left(30a^{5/2}b\sqrt{a-bx}\sqrt{\frac{bx}{a}+1}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)\left(a^2(Bf+Ce)+4Ab^2e\right)+\left(a^2-b^2x^2\right)\left(16a^4Cf+a^2b^2(40Af+5B(8e+3fx)+Cx(15e+8fx))-2b^4x(10A(3e+2fx)+x(5B(4e+3fx)+3Cx(5e+4fx)))\right)\right)}{120b^4\sqrt{a+bx}\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)*(A + B*x + C*x^2),x]
```

```
[Out] -1/120*(c*((a^2 - b^2*x^2)*(16*a^4*C*f + a^2*b^2*(40*A*f + 5*B*(8*e + 3*f*x
) + C*x*(15*e + 8*f*x)) - 2*b^4*x*(10*A*(3*e + 2*f*x) + x*(5*B*(4*e + 3*f*x
) + 3*C*x*(5*e + 4*f*x)))) + 30*a^(5/2)*b*(4*A*b^2*e + a^2*(C*e + B*f))*Sqr
t[a - b*x]*Sqrt[1 + (b*x)/a]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]))/(b^4
*Sqrt[c*(a - b*x)]*Sqrt[a + b*x])
```

IntegrateAlgebraic [B] time = 0.64, size = 647, normalized size = 2.16

$$\frac{\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\sqrt{a+bx}}\right)\left(-a^4B\sqrt{c}f+a^4\left(-\sqrt{c}\right)Ce-4a^2Ab^2\sqrt{c}e\right)}{4b^3}-\frac{a^2c\sqrt{ac-bcx}\left(\frac{160a^3c^3Cf(ac-bcx)}{a+bx}-\frac{64a^3c^2Cf(ac-bcx)^2}{(a+bx)^2}+\frac{160a^3cCf(ac-bcx)^3}{(a+bx)^3}-15a^2bBc^4f+\frac{90a^2bBc^3f(ac-bcx)}{a+bx}-\frac{90a^2bBcf(ac-bcx)^3}{(a+bx)^3}+\frac{15a^2bBf(ac-bcx)^4}{(a+bx)^4}-15a^2bc^4Ce+\frac{90a^2bc^3Ce(ac-bcx)}{a+bx}-\frac{90a^2bcCe(ac-bcx)^3}{(a+bx)^3}+\frac{15a^2bCe(ac-bcx)^4}{(a+bx)^4}-\frac{120Ab^3c^3e(ac-bcx)}{a+bx}+\frac{120Ab^3ce(ac-bcx)^3}{(a+bx)^3}+\frac{60Ab^3e(ac-bcx)^4}{(a+bx)^4}+\frac{160aAb^2c^3f(ac-bcx)}{a+bx}+\frac{320aAb^2c^2f(ac-bcx)^2}{(a+bx)^2}+\frac{160aAb^2cf(ac-bcx)^3}{(a+bx)^3}+\frac{160ab^2Bc^3e(ac-bcx)}{a+bx}+\frac{320ab^2Bc^2e(ac-bcx)^2}{(a+bx)^2}+\frac{160ab^2Bce(ac-bcx)^3}{(a+bx)^3}-60Ab^3c^4e\right)}{60b^4\sqrt{a+bx}\left(\frac{ac-bcx}{a+bx}+c\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)*(A + B*x + C*x
^2),x]
```

```
[Out] -1/60*(a^2*c*Sqrt[a*c - b*c*x]*(-60*A*b^3*c^4*e - 15*a^2*b*c^4*C*e - 15*a^2
*b*B*c^4*f - (120*A*b^3*c^3*e*(a*c - b*c*x))/(a + b*x) + (160*a*b^2*B*c^3*e
```

```
*(a*c - b*c*x))/(a + b*x) + (90*a^2*b*c^3*C*e*(a*c - b*c*x))/(a + b*x) + (1
60*a*A*b^2*c^3*f*(a*c - b*c*x))/(a + b*x) + (90*a^2*b*B*c^3*f*(a*c - b*c*x)
)/(a + b*x) + (160*a^3*c^3*C*f*(a*c - b*c*x))/(a + b*x) + (320*a*b^2*B*c^2*
e*(a*c - b*c*x)^2)/(a + b*x)^2 + (320*a*A*b^2*c^2*f*(a*c - b*c*x)^2)/(a + b
*x)^2 - (64*a^3*c^2*C*f*(a*c - b*c*x)^2)/(a + b*x)^2 + (120*A*b^3*c*e*(a*c
- b*c*x)^3)/(a + b*x)^3 + (160*a*b^2*B*c*e*(a*c - b*c*x)^3)/(a + b*x)^3 - (
90*a^2*b*c*C*e*(a*c - b*c*x)^3)/(a + b*x)^3 + (160*a*A*b^2*c*f*(a*c - b*c*x
)^3)/(a + b*x)^3 - (90*a^2*b*B*c*f*(a*c - b*c*x)^3)/(a + b*x)^3 + (160*a^3*
c*C*f*(a*c - b*c*x)^3)/(a + b*x)^3 + (60*A*b^3*e*(a*c - b*c*x)^4)/(a + b*x)
^4 + (15*a^2*b*C*e*(a*c - b*c*x)^4)/(a + b*x)^4 + (15*a^2*b*B*f*(a*c - b*c*
x)^4)/(a + b*x)^4))/(b^4*Sqrt[a + b*x]*(c + (a*c - b*c*x)/(a + b*x))^5) + (
(-4*a^2*A*b^2*Sqrt[c]*e - a^4*Sqrt[c]*C*e - a^4*B*Sqrt[c]*f)*ArcTan[Sqrt[a*
c - b*c*x]/(Sqrt[c]*Sqrt[a + b*x])])/(4*b^3)
```

fricas [A] time = 1.20, size = 441, normalized size = 1.47

$$\left[\frac{15\left(Ba^4bf+\left(Ca^4b+4\,Aa^2b^3\right)e\right)\sqrt{-c}\,\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\left(24\,Cb^4fx^4-40\,Ba^2b^2e+30\left(Cb^4e+Bb^4f\right)x^3+8\left(5\,Bb^4e-\left(Ca^2b^2-5\,Ab^4\right)f\right)x^2-8\left(2\,Ca^4+5\,Aa^2b^2\right)f-15\left(Ba^2b^2f+\left(Ca^2b^2-4\,Ab^4\right)e\right)x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{240\,b^4},\ -\frac{15\left(Ba^4bf+\left(Ca^4b+4\,Aa^2b^3\right)e\right)\sqrt{c}\,\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)-\left(24\,Cb^4fx^4-40\,Ba^2b^2e+30\left(Cb^4e+Bb^4f\right)x^3+8\left(5\,Bb^4e-\left(Ca^2b^2-5\,Ab^4\right)f\right)x^2-8\left(2\,Ca^4+5\,Aa^2b^2\right)f-15\left(Ba^2b^2f+\left(Ca^2b^2-4\,Ab^4\right)e\right)x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{120\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/240*(15*(B*a^4*b*f + (C*a^4*b + 4*A*a^2*b^3)*e)*sqrt(-c)*log(2*b^2*c*x^2
 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(24*C*b^4*f
*x^4 - 40*B*a^2*b^2*e + 30*(C*b^4*e + B*b^4*f)*x^3 + 8*(5*B*b^4*e - (C*a^2*
b^2 - 5*A*b^4)*f)*x^2 - 8*(2*C*a^4 + 5*A*a^2*b^2)*f - 15*(B*a^2*b^2*f + (C*
a^2*b^2 - 4*A*b^4)*e)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^4, -1/120*(15*
(B*a^4*b*f + (C*a^4*b + 4*A*a^2*b^3)*e)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*s
qrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (24*C*b^4*f*x^4 - 40*B*a^2*
b^2*e + 30*(C*b^4*e + B*b^4*f)*x^3 + 8*(5*B*b^4*e - (C*a^2*b^2 - 5*A*b^4)*f
)*x^2 - 8*(2*C*a^4 + 5*A*a^2*b^2)*f - 15*(B*a^2*b^2*f + (C*a^2*b^2 - 4*A*b^
4)*e)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^4]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 588, normalized size = 1.96

$$\frac{\sqrt{bx+a}\,\sqrt{-(bx-a)\,c}\,\left(60A\,a^2b^4ce\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-\left(b^2x^2-a^2\right)c}}\right)+15B\,a^4b^2cf\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-\left(b^2x^2-a^2\right)c}}\right)+15C\,a^4b^2ce\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-\left(b^2x^2-a^2\right)c}}\right)+24\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,C\,b^4fx^4+30\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,B\,b^4fx^3+30\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,C\,b^4ex^3+40\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,A\,b^4fx^2+40\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,B\,b^4ex^2-8\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,C\,a^2b^2fx^2+60\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,A\,b^4ex-15\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,B\,a^2b^2fx-15\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,C\,a^2b^2ex-40\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,A\,a^2b^2f-40\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,B\,a^2b^2e-16\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,C\,a^4f\right)}{120\sqrt{-\left(b^2x^2-a^2\right)c}\,\sqrt{b^2c}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/120*(b*x+a)^(1/2)*(-(b*x-a)*c)^(1/2)*(24*C*x^4*b^4*f*(b^2*c)^(1/2)*(-(b^2
*x^2-a^2)*c)^(1/2)+30*B*x^3*b^4*f*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+30
*C*x^3*b^4*e*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+60*A*arctan((b^2*c)^(1/
2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^2*b^4*c*e+40*A*x^2*b^4*f*(b^2*c)^(1/2)*(-(
b^2*x^2-a^2)*c)^(1/2)+15*B*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)
*a^4*b^2*c*f+40*B*x^2*b^4*e*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+15*C*arc
tan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^4*b^2*c*e-8*C*x^2*a^2*b^2*f
*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+60*A*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*
c)^(1/2)*x*b^4*e-15*B*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*a^2*b^2*f-15
```

```
*C*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*a^2*b^2*e-40*A*a^2*b^2*f*(b^2*c
)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)-40*B*a^2*b^2*e*(b^2*c)^(1/2)*(-(b^2*x^2-a^
2)*c)^(1/2)-16*C*a^4*f*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2))/(-(b^2*x^2-a
^2)*c)^(1/2)/b^4/(b^2*c)^(1/2)
```

maxima [A] time = 2.25, size = 248, normalized size = 0.83

$$\frac{Aa^2\sqrt{c}\,e\arcsin\left(\frac{bx}{a}\right)}{2b}+\frac{1}{2}\sqrt{-b^2cx^2+a^2c}\,Aex+\frac{(Ce+Bf)a^4\sqrt{c}\arcsin\left(\frac{bx}{a}\right)}{8b^3}+\frac{\sqrt{-b^2cx^2+a^2c}\,(Ce+Bf)a^2x}{8b^2}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}Cfx^2}{5b^2c}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}Be}{3b^2c}-\frac{2\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}Ca^2f}{15b^4c}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}Af}{3b^2c}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}(Ce+Bf)x}{4b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] 1/2*A*a^2*sqrt(c)*e*arcsin(b*x/a)/b + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*A*e*x +
1/8*(C*e + B*f)*a^4*sqrt(c)*arcsin(b*x/a)/b^3 + 1/8*sqrt(-b^2*c*x^2 + a^2*c
)*(C*e + B*f)*a^2*x/b^2 - 1/5*(-b^2*c*x^2 + a^2*c)^(3/2)*C*f*x^2/(b^2*c) -
1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*B*e/(b^2*c) - 2/15*(-b^2*c*x^2 + a^2*c)^(3/2
)*C*a^2*f/(b^4*c) - 1/3*(-b^2*c*x^2 + a^2*c)^(3/2)*A*f/(b^2*c) - 1/4*(-b^2*
c*x^2 + a^2*c)^(3/2)*(C*e + B*f)*x/(b^2*c)
```

mupad [B] time = 30.58, size = 1765, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] ((B*a^4*c^8*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(2*((a + b*x)^(1/2) - a^
(1/2))) - (B*a^4*c*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^15)/(2*((a + b*x)^
(1/2) - a^(1/2))^15) - (35*B*a^4*c^7*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/(2*((a + b*x)^(1/2) - a^(1/2))^3) + (273*B*a^4*c^6*f*((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))^5)/(2*((a + b*x)^(1/2) - a^(1/2))^5) - (715*B*a^4*c^5*f*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(2*((a + b*x)^(1/2) - a^(1/2))^7) + (7
15*B*a^4*c^4*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(2*((a + b*x)^(1/2) -
 a^(1/2))^9) - (273*B*a^4*c^3*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^11)/(2*
((a + b*x)^(1/2) - a^(1/2))^11) + (35*B*a^4*c^2*f*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^13)/(2*((a + b*x)^(1/2) - a^(1/2))^13))/(b^3*c^8 + (b^3*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^16)/((a + b*x)^(1/2) - a^(1/2))^16 + (8*b^3*c^7
*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (28
*b^3*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))
^4 + (56*b^3*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) -
a^(1/2))^6 + (70*b^3*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^
(1/2) - a^(1/2))^8 + (56*b^3*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((
a + b*x)^(1/2) - a^(1/2))^10 + (28*b^3*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^12)/((a + b*x)^(1/2) - a^(1/2))^12 + (8*b^3*c*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^14)/((a + b*x)^(1/2) - a^(1/2))^14) - (a*c - b*c*x)^(1/2)*((2*C*
a^4*f*(a + b*x)^(1/2))/(15*b^4) - (C*f*x^4*(a + b*x)^(1/2))/5 + (C*a^2*f*x^
2*(a + b*x)^(1/2))/(15*b^2)) + ((C*a^4*c^8*e*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2)))/(2*((a + b*x)^(1/2) - a^(1/2))) - (C*a^4*c*e*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))^15)/(2*((a + b*x)^(1/2) - a^(1/2))^15) - (35*C*a^4*c^7*e*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(2*((a + b*x)^(1/2) - a^(1/2))^3) + (273*
C*a^4*c^6*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(2*((a + b*x)^(1/2) - a^
(1/2))^5) - (715*C*a^4*c^5*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(2*((a
+ b*x)^(1/2) - a^(1/2))^7) + (715*C*a^4*c^4*e*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^9)/(2*((a + b*x)^(1/2) - a^(1/2))^9) - (273*C*a^4*c^3*e*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^11)/(2*((a + b*x)^(1/2) - a^(1/2))^11) + (35*C*a^4*
c^2*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^13)/(2*((a + b*x)^(1/2) - a^(1/2)
)^13))/(b^3*c^8 + (b^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^16)/((a + b*x)^(
1/2) - a^(1/2))^16 + (8*b^3*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a
+ b*x)^(1/2) - a^(1/2))^2 + (28*b^3*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
```

```
^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (56*b^3*c^5*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6 + (70*b^3*c^4*((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (56*b^3*c^3*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (28*b^3*c^2*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12)/((a + b*x)^(1/2) - a^(1/2))^12 + (8*
b^3*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^14)/((a + b*x)^(1/2) - a^(1/2))^1
4) + (A*e*x*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/2 - (A*f*(a^2 - b^2*x^2)*(
a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/(3*b^2) - (B*e*(a^2 - b^2*x^2)*(a*c - b
*c*x)^(1/2)*(a + b*x)^(1/2))/(3*b^2) - (B*a^4*c^(1/2)*f*atan(((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(2*b^3) - (C*
a^4*c^(1/2)*e*atan(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^
(1/2) - a^(1/2)))))/(2*b^3) - (A*a^2*b^(1/2)*c^2*e*log((-b*c)^(1/2)*(c*(a -
 b*x))^(1/2)*(a + b*x)^(1/2) - b^(3/2)*c*x))/(2*(-b*c)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\left(-a+bx\right)}\sqrt{a+bx}\left(e+fx\right)\left(A+Bx+Cx^2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(e + f*x)*(A + B*x + C*x**2), x)
```

3.23 $\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(A+Bx+Cx^2\right)\,dx$

Optimal. Leaf size=221

$$\frac{1}{8}x\sqrt{a+bx}\left(\frac{a^2C}{b^2}+4A\right)\sqrt{ac-bcx}+\frac{a^2\sqrt{c}\,\sqrt{a+bx}\left(a^2C+4Ab^2\right)\sqrt{ac-bcx}\,\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^3\sqrt{a^2c-b^2cx^2}}-\frac{B\sqrt{a+bx}\,(a$$

Rubi [A] time = 0.15, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {901, 1815, 641, 195, 217, 203}

$$\frac{a^2\sqrt{c}\,\sqrt{a+bx}\left(a^2C+4Ab^2\right)\sqrt{ac-bcx}\,\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{8b^3\sqrt{a^2c-b^2cx^2}}+\frac{1}{8}x\sqrt{a+bx}\left(\frac{a^2C}{b^2}+4A\right)\sqrt{ac-bcx}-\frac{B\sqrt{a+bx}\left(a^2-b^2x^2\right)\sqrt{ac-bcx}}{3b^2}-\frac{Cx\sqrt{a+bx}\left(a^2-b^2x^2\right)\sqrt{ac-bcx}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((4*A + (a^2*C)/b^2)*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 - (B*Sqrt[a + b*x
]*Sqrt[a*c - b*c*x]*(a^2 - b^2*x^2))/(3*b^2) - (C*x*Sqrt[a + b*x]*Sqrt[a*c
- b*c*x]*(a^2 - b^2*x^2))/(4*b^2) + (a^2*Sqrt[c]*(4*A*b^2 + a^2*C)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b
^3*Sqrt[a^2*c - b^2*c*x^2])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 901

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^Fr
acPart[m])/(d*f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(A+Bx+Cx^2\right)dx = \frac{\left(\sqrt{a+bx}\,\sqrt{ac-bcx}\right)\int \sqrt{a^2c-b^2cx^2}\,\left(A+Bx+Cx^2\right)dx}{\sqrt{a^2c-b^2cx^2}}$$

$$= -\frac{Cx\sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(a^2-b^2x^2\right)}{4b^2} - \frac{\left(\sqrt{a+bx}\,\sqrt{ac-bcx}\right)\int\left(-c\left(\right.\right.}{4b^2}$$

$$= -\frac{B\sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(a^2-b^2x^2\right)}{3b^2} - \frac{Cx\sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(a^2-b^2\right.}{}$$

$$= \frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\,\sqrt{ac-bcx} - \frac{B\sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(a^2-b^2\right.}{3b^2}$$

$$= \frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\,\sqrt{ac-bcx} - \frac{B\sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(a^2-b^2\right.}{3b^2}$$

$$= \frac{1}{8}\left(4A+\frac{a^2C}{b^2}\right)x\sqrt{a+bx}\,\sqrt{ac-bcx} - \frac{B\sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(a^2-b^2\right.}{3b^2}$$

Mathematica [A] time = 0.41, size = 142, normalized size = 0.64

$$-\frac{c\left(b\left(b^2x^2-a^2\right)\left(2b^2x\left(6A+4Bx+3Cx^2\right)-a^2(8B+3Cx)\right)+6a^{5/2}\sqrt{a-bx}\,\sqrt{\frac{bx}{a}+1}\,\left(a^2C+4Ab^2\right)\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\,\sqrt{a}}\right)\right)}{24b^3\sqrt{a+bx}\,\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2),x]
```

```
[Out] -1/24*(c*(b*(-a^2 + b^2*x^2)*(-(a^2*(8*B + 3*C*x)) + 2*b^2*x*(6*A + 4*B*x +
 3*C*x^2)) + 6*a^(5/2)*(4*A*b^2 + a^2*C)*Sqrt[a - b*x]*Sqrt[1 + (b*x)/a]*Ar
cSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]))/(b^3*Sqrt[c*(a - b*x)]*Sqrt[a + b*x
])
```

IntegrateAlgebraic [A] time = 0.41, size = 326, normalized size = 1.48

$$\frac{a^2c\sqrt{ac-bcx}\left(-\frac{21a^2c^2C(ac-bcx)}{a+bx}+\frac{21a^2cC(ac-bcx)^2}{(a+bx)^2}-\frac{3a^2C(ac-bcx)^3}{(a+bx)^3}+3a^2c^3C+\frac{12Ab^2c^2(ac-bcx)}{a+bx}-\frac{12Ab^2c(ac-bcx)^2}{(a+bx)^2}-\frac{12Ab^2(ac-bcx)^3}{(a+bx)^3}-\frac{32abBc^2(ac-bcx)}{a+bx}-\frac{32abBc(ac-bcx)^2}{(a+bx)^2}+12Ab^2c^3\right)}{12b^3\sqrt{a+bx}\left(\frac{ac-bcx}{a+bx}+c\right)^4}-\frac{\sqrt{c}\left(a^4C+4a^2Ab^2\right)\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\sqrt{a+bx}}\right)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(A + B*x + C*x^2),x]
```

```
[Out] (a^2*c*Sqrt[a*c - b*c*x]*(12*A*b^2*c^3 + 3*a^2*c^3*C + (12*A*b^2*c^2*(a*c -
 b*c*x))/(a + b*x) - (32*a*b*B*c^2*(a*c - b*c*x))/(a + b*x) - (21*a^2*c^2*C
*(a*c - b*c*x))/(a + b*x) - (12*A*b^2*c*(a*c - b*c*x)^2)/(a + b*x)^2 - (32*
a*b*B*c*(a*c - b*c*x)^2)/(a + b*x)^2 + (21*a^2*c*C*(a*c - b*c*x)^2)/(a + b*
x)^2 - (12*A*b^2*(a*c - b*c*x)^3)/(a + b*x)^3 - (3*a^2*C*(a*c - b*c*x)^3)/(
a + b*x)^3))/(12*b^3*Sqrt[a + b*x]*(c + (a*c - b*c*x)/(a + b*x))^4) - (Sqrt
[c]*(4*a^2*A*b^2 + a^4*C)*ArcTan[Sqrt[a*c - b*c*x]/(Sqrt[c]*Sqrt[a + b*x])]
)/(4*b^3)
```

fricas [A] time = 0.85, size = 265, normalized size = 1.20

$$\left[\frac{3\left(Ca^4+4\,Aa^2b^2\right)\sqrt{-c}\,\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\left(6\,Cb^3x^3+8\,Bb^3x^2-8\,Ba^2b-3\left(Ca^2b-4\,Ab^3\right)x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{48\,b^3},\ -\frac{3\left(Ca^4+4\,Aa^2b^2\right)\sqrt{c}\,\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)-\left(6\,Cb^3x^3+8\,Bb^3x^2-8\,Ba^2b-3\left(Ca^2b-4\,Ab^3\right)x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{24\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [1/48*(3*(C*a^4 + 4*A*a^2*b^2)*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a
*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(6*C*b^3*x^3 + 8*B*b^3*x^2 - 8*
B*a^2*b - 3*(C*a^2*b - 4*A*b^3)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^3, -
1/24*(3*(C*a^4 + 4*A*a^2*b^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x +
a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (6*C*b^3*x^3 + 8*B*b^3*x^2 - 8*B*a^2*
b - 3*(C*a^2*b - 4*A*b^3)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b^3]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 287, normalized size = 1.30

$$\frac{\sqrt{bx+a}\,\sqrt{-(bx-a)c}\,\left(12A\,a^2b^2c\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-\left(b^2x^2-a^2\right)c}}\right)+3C\,a^4c\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-\left(b^2x^2-a^2\right)c}}\right)+6\sqrt{-\left(b^2x^2-a^2\right)c}\,\sqrt{b^2c}\,C\,b^2x^3+8\sqrt{-\left(b^2x^2-a^2\right)c}\,\sqrt{b^2c}\,B\,b^2x^2+12\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,A\,b^2x-3\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,C\,a^2x-8\sqrt{-\left(b^2x^2-a^2\right)c}\,\sqrt{b^2c}\,B\,a^2\right)}{24\sqrt{-\left(b^2x^2-a^2\right)c}\,\sqrt{b^2c}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/24*(b*x+a)^(1/2)*(-(b*x-a)*c)^(1/2)*(6*C*x^3*b^2*(-(b^2*x^2-a^2)*c)^(1/2)
*(b^2*c)^(1/2)+12*A*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^2*b^
2*c+8*B*x^2*b^2*(-(b^2*x^2-a^2)*c)^(1/2)*(b^2*c)^(1/2)+3*C*arctan((b^2*c)^(
1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^4*c+12*A*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c
)^(1/2)*x*b^2-3*C*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*a^2-8*B*a^2*(-(b
^2*x^2-a^2)*c)^(1/2)*(b^2*c)^(1/2))/(-(b^2*x^2-a^2)*c)^(1/2)/b^2/(b^2*c)^(1
/2)
```

maxima [A] time = 2.03, size = 140, normalized size = 0.63

$$\frac{Ca^4\sqrt{c}\,\arcsin\left(\frac{bx}{a}\right)}{8\,b^3}+\frac{Aa^2\sqrt{c}\,\arcsin\left(\frac{bx}{a}\right)}{2\,b}+\frac{1}{2}\,\sqrt{-b^2cx^2+a^2c}\,Ax+\frac{\sqrt{-b^2cx^2+a^2c}\,Ca^2x}{8\,b^2}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}Cx}{4\,b^2c}-\frac{\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}B}{3\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 1/8*C*a^4*sqrt(c)*arcsin(b*x/a)/b^3 + 1/2*A*a^2*sqrt(c)*arcsin(b*x/a)/b + 1
/2*sqrt(-b^2*c*x^2 + a^2*c)*A*x + 1/8*sqrt(-b^2*c*x^2 + a^2*c)*C*a^2*x/b^2
- 1/4*(-b^2*c*x^2 + a^2*c)^(3/2)*C*x/(b^2*c) - 1/3*(-b^2*c*x^2 + a^2*c)^(3/
2)*B/(b^2*c)
```

mupad [B] time = 16.52, size = 876, normalized size = 3.96

$$\frac{\frac{C\,a^4\,c^8\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{2\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}-\frac{C\,a^4\,c\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^{15}}{2\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^{15}}-\frac{35\,C\,a^4\,c^7\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^3}{2\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^3}+\frac{273\,C\,a^4\,c^6\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^5}{2\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^5}-\frac{715\,C\,a^4\,c^5\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^7}{2\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^7}+\frac{715\,C\,a^4\,c^4\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^9}{2\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^9}-\frac{273\,C\,a^4\,c^3\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^{11}}{2\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^{11}}+\frac{35\,C\,a^4\,c^2\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^{13}}{2\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^{13}}}{b^3\,c^8+\frac{b^3\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^{16}}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^{16}}+\frac{8\,b^3\,c^7\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^2}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}+\frac{28\,b^3\,c^6\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^4}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^4}+\frac{56\,b^3\,c^5\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^6}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^6}+\frac{70\,b^3\,c^4\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^8}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^8}+\frac{56\,b^3\,c^3\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^{10}}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^{10}}+\frac{28\,b^3\,c^2\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^{12}}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^{12}}+\frac{8\,b^3\,c\,{\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}^{14}}{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^{14}}}+\frac{A\,x\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{2}-\frac{B\,\left(a^2-b^2\,x^2\right)\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{3\,b^2}-\frac{C\,a^4\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}}{\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{2\,b^3}-\frac{A\,a^2\,\sqrt{b}\,c^2\,\ln\left(\sqrt{-b\,c}\,\sqrt{c\,\left(a-b\,x\right)}\,\sqrt{a+b\,x}-b^{3/2}\,c\,x\right)}{2\,{\left(-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] ((C*a^4*c^8*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(2*((a + b*x)^(1/2) - a^(1
/2))) - (C*a^4*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^15)/(2*((a + b*x)^(1/2
) - a^(1/2))^15) - (35*C*a^4*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(2*
((a + b*x)^(1/2) - a^(1/2))^3) + (273*C*a^4*c^6*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))^5)/(2*((a + b*x)^(1/2) - a^(1/2))^5) - (715*C*a^4*c^5*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^7)/(2*((a + b*x)^(1/2) - a^(1/2))^7) + (715*C*a^4*c
^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(2*((a + b*x)^(1/2) - a^(1/2))^9)
 - (273*C*a^4*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^11)/(2*((a + b*x)^(1/
2) - a^(1/2))^11) + (35*C*a^4*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^13)/(
2*((a + b*x)^(1/2) - a^(1/2))^13))/(b^3*c^8 + (b^3*((a*c - b*c*x)^(1/2) - (
a*c)^(1/2))^16)/((a + b*x)^(1/2) - a^(1/2))^16 + (8*b^3*c^7*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (28*b^3*c^6*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (56*b^3*c^5
*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6 + (70
*b^3*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))
^8 + (56*b^3*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) -
 a^(1/2))^10 + (28*b^3*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12)/((a + b*
x)^(1/2) - a^(1/2))^12 + (8*b^3*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^14)/(
(a + b*x)^(1/2) - a^(1/2))^14) + (A*x*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/
2 - (B*(a^2 - b^2*x^2)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/(3*b^2) - (C*a^
4*c^(1/2)*atan(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2
) - a^(1/2)))))/(2*b^3) - (A*a^2*b^(1/2)*c^2*log((-b*c)^(1/2)*(c*(a - b*x))
^(1/2)*(a + b*x)^(1/2) - b^(3/2)*c*x))/(2*(-b*c)^(3/2))
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\left(-a+bx\right)}\,\sqrt{a+bx}\,\left(A+Bx+Cx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x)*(A + B*x + C*x**2), x)
```

3.24 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)}\,dx$

Optimal. Leaf size=278

$$\frac{\sqrt{a^2c-b^2cx^2}\left(Af^2-Bef+Ce^2\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\,\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}\,\sqrt{b^2e^2-a^2f^2}}-\frac{\sqrt{a^2c-b^2cx^2}\,(Ce-Bf)\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}}-$$

Rubi [A] time = 0.49, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1610, 1654, 844, 217, 203, 725, 204}

$$\frac{\sqrt{a^2c-b^2cx^2}\left(Af^2-Bef+Ce^2\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\,\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}\,\sqrt{b^2e^2-a^2f^2}}-\frac{\sqrt{a^2c-b^2cx^2}\,(Ce-Bf)\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] -((C*(a^2 - b^2*x^2))/(b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - ((C*e - B*
f)*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(
b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e^2 - B*e*f + A*f^2)*S
qrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a
^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])]])/(Sqrt[c]*f^2*Sqrt[b^2*e^2 - a^2*f^2]*Sqr
t[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{-Ab^2cf^2+b^2cf(Ce-Bf)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{b^2cf^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}\,dx}{f^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{((C\ldots}{}$$

$$= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int \frac{1}{1+b^2cx^2}\,dx, x, \ldots}{f^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{(Ce-Bf)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(C\ldots}{}$$

Mathematica [A] time = 0.77, size = 225, normalized size = 0.81

$$\frac{\sqrt{a-bx}\left(\frac{2\left(f(Af-Be)+Ce^2\right)\tanh^{-1}\left(\frac{\sqrt{a-bx}\sqrt{be-af}}{\sqrt{a+bx}\sqrt{-af-be}}\right)}{\sqrt{-af-be}\sqrt{be-af}}+\frac{2\tan^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{a+bx}}\right)(aCf-bBf+bCe)}{b^2}+\frac{Cf\sqrt{a+bx}\left(-\sqrt{a-bx}-\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{b^2}\right)}{f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] (Sqrt[a - b*x]*((C*f*Sqrt[a + b*x]*(-Sqrt[a - b*x] - (2*Sqrt[a]*ArcSin[Sqrt
[a - b*x]/(Sqrt[2]*Sqrt[a])])/Sqrt[1 + (b*x)/a]))/b^2 + (2*(b*C*e - b*B*f +
 a*C*f)*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x]])/b^2 + (2*(C*e^2 + f*(-(B*e) +
A*f))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a +
b*x])])/(Sqrt[-(b*e) - a*f]*Sqrt[b*e - a*f])))/(f^2*Sqrt[c*(a - b*x)])
```

IntegrateAlgebraic [A] time = 0.37, size = 205, normalized size = 0.74

$$-\frac{2\left(Af^2-Bef+Ce^2\right)\tanh^{-1}\left(\frac{\sqrt{ac-bcx}\,\sqrt{af-be}}{\sqrt{c}\,\sqrt{a+bx}\,\sqrt{af+be}}\right)}{\sqrt{c}\,f^2\sqrt{af-be}\,\sqrt{af+be}}-\frac{2aC\sqrt{ac-bcx}}{b^2f\sqrt{a+bx}\left(\frac{ac-bcx}{a+bx}+c\right)}-\frac{2(Bf-Ce)\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\,\sqrt{a+bx}}\right)}{b\sqrt{c}\,f^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e +
f*x)),x]
```

```
[Out] (-2*a*C*Sqrt[a*c - b*c*x])/(b^2*f*Sqrt[a + b*x]*(c + (a*c - b*c*x)/(a + b*x
))) - (2*(-(C*e) + B*f)*ArcTan[Sqrt[a*c - b*c*x]/(Sqrt[c]*Sqrt[a + b*x])])/
(b*Sqrt[c]*f^2) - (2*(C*e^2 - B*e*f + A*f^2)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sq
rt[a*c - b*c*x])/(Sqrt[c]*Sqrt[b*e + a*f]*Sqrt[a + b*x])])/(Sqrt[c]*f^2*Sqr
t[-(b*e) + a*f]*Sqrt[b*e + a*f])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.07, size = 503, normalized size = 1.81

$$\frac{\left(-\sqrt{b^2c}\,A\,b^2c\,f^2\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{\left(a^2f^2-b^2e^2\right)c}{f^2}}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,f}{fx+e}\right)+\sqrt{b^2c}\,B\,b^2cef\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{\left(a^2f^2-b^2e^2\right)c}{f^2}}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,f}{fx+e}\right)+\sqrt{\frac{\left(a^2f^2-b^2e^2\right)c}{f^2}}\,B\,b^2c\,f^2\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-\left(b^2x^2-a^2\right)c}}\right)-\sqrt{b^2c}\,C\,b^2c\,e^2\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{\left(a^2f^2-b^2e^2\right)c}{f^2}}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,f}{fx+e}\right)-\sqrt{\frac{\left(a^2f^2-b^2e^2\right)c}{f^2}}\,C\,b^2cef\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-\left(b^2x^2-a^2\right)c}}\right)-\sqrt{b^2c}\,\sqrt{\frac{\left(a^2f^2-b^2e^2\right)c}{f^2}}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,C\,f^2\right)\sqrt{bx+a}\,\sqrt{-(bx-a)\,c}}{\sqrt{\frac{\left(a^2f^2-b^2e^2\right)c}{f^2}}\,\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,b^2c\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] (-A*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-(b^2*x^2-a^2)
*c)^(1/2)*f)/(f*x+e))*b^2*c*f^2*(b^2*c)^(1/2)+B*ln(2*(b^2*c*e*x+a^2*c*f+(c*
(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))*b^2*c*e*f
*(b^2*c)^(1/2)+B*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*b^2*c*f^2
*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+(c*(a^2*f^2-b^2*
e^2)/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))*b^2*c*e^2*(b^2*c)^(1/2
)-C*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*b^2*c*e*f*(c*(a^2*f^2-
b^2*e^2)/f^2)^(1/2)-C*f^2*(b^2*c)^(1/2)*(c*(a^2*f^2-b^2*e^2)/f^2)^(1/2)*(-(
b^2*x^2-a^2)*c)^(1/2))*(b*x+a)^(1/2)*(-(b*x-a)*c)^(1/2)/(c*(a^2*f^2-b^2*e^2
)/f^2)^(1/2)/f^3/(b^2*c)^(1/2)/b^2/c/(-(b^2*x^2-a^2)*c)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for mor
e details)Is (4*b^2*c     *(a^2*c-(b^2*c*e^2)
   /f^2))    /f^2    +(4*b^4*c^2*e^2)/f^4    zero or nonzero?
```

mupad [B] time = 44.56, size = 9298, normalized size = 33.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] (B*a*e*atan(((B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*
a^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c
^2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) - (
B*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) +
 (B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4
*c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B
*a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((
a + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*
b^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^
4*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*
x)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a
^(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(
a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(
1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*
e*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^
6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) +
(16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f
*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^
(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*
b^4*c^5*e^4 - 144*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^
8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)
) + (458752*B^3*a^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a
 + b*x)^(1/2) - a^(1/2))))*1i)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (B*a
*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a^(15/2)*b^2*c^4*
e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^2*e*f^2*(a*c)^(5
/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) + (B*a*e*((4096*(16*
B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) - (B*a*e*((4096*(2
4*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*c^5*e^3*f^2*(a*c
)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*a^10*b^2*c^6*e^2
*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) -
a^(1/2))) - (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b^4*c^7*e^2*f^4))
/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4*c^6*e^2*f^4)*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2
))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^(15/2)*b^4*c^5*e
^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a +
 b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (4096*((a
*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e*f^4*(a*c)^(5/2)
 - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6*((a + b*x)^(1/2
) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (16384*(8*B^2*a^(
17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*(a*c)^(3/2))*((a
*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))
```

```
 + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*b^4*c^5*e^4 - 144
*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^8*e^6*((a + b*x)^
(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (458752*B^3*a
^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a + b*x)^(1/2) - a
^(1/2))))*1i)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)))/((131072*B^4*a^4*c^5)/
(b^8*e^4) - (B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a
^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^
2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) - (B
*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) +
(B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*
c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*
a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a
 + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b
^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4
*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x
)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^
(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a
^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1
/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e
*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6
*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (
16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*
(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(
1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*b
^4*c^5*e^4 - 144*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^8
*e^6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2))
 + (458752*B^3*a^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a
+ b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (B*a*e*(
(4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a^(15/2)*b^2*c^4*e^3*
(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^2*e*f^2*(a*c)^(5/2)
- 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) + (B*a*e*((4096*(16*B^2*
a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) - (B*a*e*((4096*(24*B*
a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*c^5*e^3*f^2*(a*c)^(3
/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*a^10*b^2*c^6*e^2*f^3
)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1
/2))) - (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b^4*c^7*e^2*f^4))/(a^
6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4*c^6*e^2*f^4)*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2
) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^(15/2)*b^4*c^5*e^3*f
^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x
)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (4096*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e*f^4*(a*c)^(5/2) - 9
0*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6*((a + b*x)^(1/2) -
a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (16384*(8*B^2*a^(17/2
)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*(a*c)^(3/2))*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2))) + (
4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*b^4*c^5*e^4 - 144*B^2
*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^8*e^6*((a + b*x)^(1/2
) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (458752*B^3*a^4*c
^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a + b*x)^(1/2) - a^(1/
2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (917504*B^4*a^4*c^4*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*e^4*((a + b*x)^(1/2) - a^(1/2))^2)))*2i
)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) - (C*e^2*atan(((C*e^2*((4096*(32*C^
3*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 24*C^3*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2
)))/(b^8*e^4*f^4) + (C*e^2*((4096*(16*C^2*a^6*c^6*f^6 + 9*C^2*a^2*b^4*c^6*e
^4*f^2))/(b^8*e^4*f^4) - (C*e^2*((4096*(24*C*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2
) - 30*C*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^8*e^4*f^4) + (C*e^2*((409
```

```
6*(7*a^4*b^4*c^7*f^8 - 9*a^2*b^6*c^7*e^2*f^6))/(b^8*e^4*f^4) + (16384*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))*(5*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 6*a^(3/
2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2)))
 + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(11*a^4*b^4*c^6*f^8 - 9*a^2*b
^6*c^6*e^2*f^6))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*
f^2 - b^2*c*e^2)^(1/2)) + (16384*(20*C*a^6*c^6*f^6 - 22*C*a^4*b^2*c^6*e^2*f
^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^
(1/2))) + (4096*(96*C*a^(5/2)*b^2*c^3*f^7*(a*c)^(5/2) - 90*C*a^(3/2)*b^4*c^
4*e^2*f^5*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*e^4*f^4*
((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (409
6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*C^2*a^2*b^4*c^5*e^4*f^2 - 144*C^
2*a^6*c^5*f^6 + 128*C^2*a^4*b^2*c^5*e^2*f^4))/(b^8*e^4*f^4*((a + b*x)^(1/2)
 - a^(1/2))^2) + (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(8*C^2*a^(5/2)*
c^3*e^2*f^3*(a*c)^(5/2) + 3*C^2*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^7*e^
5*f^2*((a + b*x)^(1/2) - a^(1/2)))))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) -
(4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(32*C^3*a^(5/2)*c^2*e^2*f^3*(a*
c)^(5/2) - 96*C^3*a^(3/2)*b^2*c^3*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4*((a + b*
x)^(1/2) - a^(1/2))^2) + (458752*C^3*a^4*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2)))/(b^7*e*f^2*((a + b*x)^(1/2) - a^(1/2))))*1i)/(f^2*(a^2*c*f^2 - b^2*c
*e^2)^(1/2)) + (C*e^2*((4096*(32*C^3*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 24*C
^3*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4) - (C*e^2*((4096*(16*C^
2*a^6*c^6*f^6 + 9*C^2*a^2*b^4*c^6*e^4*f^2))/(b^8*e^4*f^4) + (C*e^2*((4096*(
24*C*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 30*C*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(
3/2)))/(b^8*e^4*f^4) - (C*e^2*((4096*(7*a^4*b^4*c^7*f^8 - 9*a^2*b^6*c^7*e^2
*f^6))/(b^8*e^4*f^4) + (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(5*a^(5/2
)*b^2*c^4*f^7*(a*c)^(5/2) - 6*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^7*e^
5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^2*(11*a^4*b^4*c^6*f^8 - 9*a^2*b^6*c^6*e^2*f^6))/(b^8*e^4*f^4*((a + b*x)
^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (16384*(20*C*a
^6*c^6*f^6 - 22*C*a^4*b^2*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))
/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*(96*C*a^(5/2)*b^2*c^3*f^
7*(a*c)^(5/2) - 90*C*a^(3/2)*b^4*c^4*e^2*f^5*(a*c)^(3/2))*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^2)/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a
^2*c*f^2 - b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*
(9*C^2*a^2*b^4*c^5*e^4*f^2 - 144*C^2*a^6*c^5*f^6 + 128*C^2*a^4*b^2*c^5*e^2*
f^4))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (16384*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))*(8*C^2*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 3*C^2*a^(3/2)*
b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2)))))/(f^
2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)
)^2*(32*C^3*a^(5/2)*c^2*e^2*f^3*(a*c)^(5/2) - 96*C^3*a^(3/2)*b^2*c^3*e^4*f*
(a*c)^(3/2)))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (458752*C^3*a^4
*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e*f^2*((a + b*x)^(1/2) - a^(
1/2))))*1i)/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)))/((131072*C^4*a^4*c^5)/(b^8
*f^4) + (C*e^2*((4096*(32*C^3*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 24*C^3*a^(3
/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4) + (C*e^2*((4096*(16*C^2*a^6*c
^6*f^6 + 9*C^2*a^2*b^4*c^6*e^4*f^2))/(b^8*e^4*f^4) - (C*e^2*((4096*(24*C*a^
(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 30*C*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/
(b^8*e^4*f^4) + (C*e^2*((4096*(7*a^4*b^4*c^7*f^8 - 9*a^2*b^6*c^7*e^2*f^6))/
(b^8*e^4*f^4) + (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(5*a^(5/2)*b^2*c
^4*f^7*(a*c)^(5/2) - 6*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^7*e^5*f^2*(
(a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(
11*a^4*b^4*c^6*f^8 - 9*a^2*b^6*c^6*e^2*f^6))/(b^8*e^4*f^4*((a + b*x)^(1/2)
- a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (16384*(20*C*a^6*c^6*
f^6 - 22*C*a^4*b^2*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e
^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*(96*C*a^(5/2)*b^2*c^3*f^7*(a*c)
^(5/2) - 90*C*a^(3/2)*b^4*c^4*e^2*f^5*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (
a*c)^(1/2))^2)/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^
2 - b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*C^2*
a^2*b^4*c^5*e^4*f^2 - 144*C^2*a^6*c^5*f^6 + 128*C^2*a^4*b^2*c^5*e^2*f^4))/(
```

```
b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (16384*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))*(8*C^2*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 3*C^2*a^(3/2)*b^2*c^4
*e^4*f*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2)))))/(f^2*(a^2*
c*f^2 - b^2*c*e^2)^(1/2)) - (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(32
*C^3*a^(5/2)*c^2*e^2*f^3*(a*c)^(5/2) - 96*C^3*a^(3/2)*b^2*c^3*e^4*f*(a*c)^(
3/2)))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (458752*C^3*a^4*c^5*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e*f^2*((a + b*x)^(1/2) - a^(1/2))))
)/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (C*e^2*((4096*(32*C^3*a^(5/2)*c^3*e
^2*f^3*(a*c)^(5/2) + 24*C^3*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^
4) - (C*e^2*((4096*(16*C^2*a^6*c^6*f^6 + 9*C^2*a^2*b^4*c^6*e^4*f^2))/(b^8*e
^4*f^4) + (C*e^2*((4096*(24*C*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 30*C*a^(3/2
)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^8*e^4*f^4) - (C*e^2*((4096*(7*a^4*b^4*c^
7*f^8 - 9*a^2*b^6*c^7*e^2*f^6))/(b^8*e^4*f^4) + (16384*((a*c - b*c*x)^(1/2)
 - (a*c)^(1/2))*(5*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 6*a^(3/2)*b^4*c^5*e^2*
f^5*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^2*(11*a^4*b^4*c^6*f^8 - 9*a^2*b^6*c^6*e^2*f^6)
)/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2
)^(1/2)) + (16384*(20*C*a^6*c^6*f^6 - 22*C*a^4*b^2*c^6*e^2*f^4)*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096
*(96*C*a^(5/2)*b^2*c^3*f^7*(a*c)^(5/2) - 90*C*a^(3/2)*b^4*c^4*e^2*f^5*(a*c)
^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*e^4*f^4*((a + b*x)^(1/2
) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^2*(9*C^2*a^2*b^4*c^5*e^4*f^2 - 144*C^2*a^6*c^5*f^6 +
 128*C^2*a^4*b^2*c^5*e^2*f^4))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)
+ (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(8*C^2*a^(5/2)*c^3*e^2*f^3*(a*
c)^(5/2) + 3*C^2*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x
)^(1/2) - a^(1/2)))))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (4096*((a*c - b
*c*x)^(1/2) - (a*c)^(1/2))^2*(32*C^3*a^(5/2)*c^2*e^2*f^3*(a*c)^(5/2) - 96*C
^3*a^(3/2)*b^2*c^3*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1
/2))^2) + (458752*C^3*a^4*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e*f
^2*((a + b*x)^(1/2) - a^(1/2)))))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (91
7504*C^4*a^4*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*f^4*((a + b*x)
^(1/2) - a^(1/2))^2)))*2i)/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (4*B*atan(
(67108864*B^5*a^16*c^7*f^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)
^(1/2) - a^(1/2))*(67108864*B^5*a^16*c^(15/2)*f^4 + 37748736*B^5*a^12*b^4*c
^(15/2)*e^4 - 100663296*B^5*a^14*b^2*c^(15/2)*e^2*f^2)) + (37748736*B^5*a^1
2*b^4*c^7*e^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1
/2))*(67108864*B^5*a^16*c^(15/2)*f^4 + 37748736*B^5*a^12*b^4*c^(15/2)*e^4 -
 100663296*B^5*a^14*b^2*c^(15/2)*e^2*f^2)) - (100663296*B^5*a^14*b^2*c^7*e^
2*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))*(67
108864*B^5*a^16*c^(15/2)*f^4 + 37748736*B^5*a^12*b^4*c^(15/2)*e^4 - 1006632
96*B^5*a^14*b^2*c^(15/2)*e^2*f^2))))/(b*c^(1/2)*f) - (A*a*atan((a*c*(a*c -
b*c*x)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)*2i - (a*c)^(3/2)*(a^4*c*f^2
- a^2*b^2*c*e^2)^(1/2)*1i + a*c*(a*c)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/
2)*1i + b*c*x*(a*c)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)*2i - a^(1/2)*c*
(a*c)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)*(a + b*x)^(1/2)*2i)/(2*a^(5/2
)*b*c^2*e - 2*a^3*c^2*f*(a + b*x)^(1/2) - 2*a^2*b*c^2*e*(a + b*x)^(1/2) + 2
*a^(5/2)*b*c^2*f*x + 2*a^(5/2)*c*f*(a*c - b*c*x)^(1/2)*(a*c)^(1/2) - 2*a^(3
/2)*b*c*e*(a*c - b*c*x)^(1/2)*(a*c)^(1/2) + 2*a*b*c*e*(a*c - b*c*x)^(1/2)*(
a*c)^(1/2)*(a + b*x)^(1/2)))*2i)/(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2) + (4*C*e
*atan((67108864*C^5*a^8*c^7*f^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a +
 b*x)^(1/2) - a^(1/2))*(67108864*C^5*a^8*c^(15/2)*f^4 + 37748736*C^5*a^4*b^
4*c^(15/2)*e^4 - 100663296*C^5*a^6*b^2*c^(15/2)*e^2*f^2)) + (37748736*C^5*a
^4*b^4*c^7*e^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(
1/2))*(67108864*C^5*a^8*c^(15/2)*f^4 + 37748736*C^5*a^4*b^4*c^(15/2)*e^4 -
100663296*C^5*a^6*b^2*c^(15/2)*e^2*f^2)) - (100663296*C^5*a^6*b^2*c^7*e^2*f
^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))*(67108
864*C^5*a^8*c^(15/2)*f^4 + 37748736*C^5*a^4*b^4*c^(15/2)*e^4 - 100663296*C^
5*a^6*b^2*c^(15/2)*e^2*f^2))))/(b*c^(1/2)*f^2) - (8*C*a^(1/2)*(a*c)^(1/2)*(
```

```
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*f*((a + b*x)^(1/2) - a^(1/2))^2*
(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4/((a + b*x)^(1/2) - a^(1/2))^4 + c^2
+ (2*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

3.25 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx$

Optimal. Leaf size=322

$$\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{\sqrt{a+bx}\,(e+fx)\sqrt{ac-bcx}\,\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2c-b^2cx^2}\,\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\,(a}{\sqrt{a^2c-b^2c}}\right.}{\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(b^2e^2-a^2f^2\right)^{3/2}}$$

Rubi [A] time = 0.58, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1610, 1651, 844, 217, 203, 725, 204}

$$\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{\sqrt{a+bx}\,(e+fx)\sqrt{ac-bcx}\,\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2c-b^2cx^2}\,\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\,\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(b^2e^2-a^2f^2\right)^{3/2}}+\frac{C\sqrt{a^2c-b^2cx^2}\,\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/((b^2*e^2 - a^2*f^2)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + (C*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*S
qrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c -
 b*c*x]) + ((a^2*f^2*(2*C*e - B*f) - b^2*(C*e^3 - A*e*f^2))*Sqrt[a^2*c - b^
2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a
^2*c - b^2*c*x^2])])/(Sqrt[c]*f^2*(b^2*e^2 - a^2*f^2)^(3/2)*Sqrt[a + b*x]*S
qrt[a*c - b*c*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\,\int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)} + \frac{\sqrt{a^2c-b^2cx^2}\,\int \frac{c\left(Ab^2e+a^2(Ce-Bf\right.}{(e+fx)\sqrt{a}}}{c\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)} + \frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a^2c-b^2cx^2}\right)}{f\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)} + \frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a^2c-b^2cx^2}\right)}{f\left(b^2e^2-a^2f^2\right)}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)} + \frac{C\sqrt{a^2c-b^2cx^2}\,\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2c}}\right.}{b\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Mathematica [A]   time = 0.85, size = 309, normalized size = 0.96

$$\frac{-\frac{2b^2e\sqrt{a-bx}\left(f(Af-Be)+Ce^2\right)\tanh^{-1}\left(\frac{\sqrt{a-bx}\,\sqrt{be-af}}{\sqrt{a+bx}\,\sqrt{-af-be}}\right)}{(-af-be)^{3/2}(be-af)^{3/2}}+\frac{f(bx-a)\sqrt{a+bx}\left(f(Af-Be)+Ce^2\right)}{(e+fx)(af-be)(af+be)}-\frac{2\sqrt{a-bx}\,(2Ce-Bf)\tanh^{-1}\left(\frac{\sqrt{a-bx}\,\sqrt{be-af}}{\sqrt{a+bx}\,\sqrt{-af-be}}\right)}{\sqrt{-af-be}\,\sqrt{be-af}}-\frac{2C\sqrt{a-bx}\,\tan^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{a+bx}}\right)}{b}}{f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x
]
```

```
[Out] ((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/((-(b*e) + a*f)*(b
*e + a*f)*(e + f*x)) - (2*C*Sqrt[a - b*x]*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x
]])/b - (2*(2*C*e - B*f)*Sqrt[a - b*x]*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[a - b*
x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])])/(Sqrt[-(b*e) - a*f]*Sqrt[b*e - a*f
]) - (2*b^2*e*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a - b*x]*ArcTanh[(Sqrt[b*e -
a*f]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])])/((-(b*e) - a*f)^(3
/2)*(b*e - a*f)^(3/2)))/(f^2*Sqrt[c*(a - b*x)])
```

IntegrateAlgebraic [A] time = 1.12, size = 282, normalized size = 0.88

$$-\frac{2\left(a^2Bf^3-2a^2Cef^2-Ab^2ef^2+b^2Ce^3\right)\tanh^{-1}\left(\frac{\sqrt{ac-bcx}\sqrt{af-be}}{\sqrt{c}\sqrt{a+bx}\sqrt{af+be}}\right)}{\sqrt{c}f^2(af-be)^{3/2}(af+be)^{3/2}}+\frac{2ab\sqrt{ac-bcx}\left(Af^2-Bef+Ce^2\right)}{f\sqrt{a+bx}(af-be)(af+be)\left(-\frac{be(ac-bcx)}{a+bx}+\frac{af(ac-bcx)}{a+bx}-acf-bce\right)}-\frac{2C\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\sqrt{a+bx}}\right)}{b\sqrt{c}f^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e +
f*x)^2),x]
```

```
[Out] (2*a*b*(C*e^2 - B*e*f + A*f^2)*Sqrt[a*c - b*c*x])/(f*(-(b*e) + a*f)*(b*e +
a*f)*Sqrt[a + b*x]*(-(b*c*e) - a*c*f - (b*e*(a*c - b*c*x))/(a + b*x) + (a*f
*(a*c - b*c*x))/(a + b*x))) - (2*C*ArcTan[Sqrt[a*c - b*c*x]/(Sqrt[c]*Sqrt[a
 + b*x])])/(b*Sqrt[c]*f^2) - (2*(b^2*C*e^3 - A*b^2*e*f^2 - 2*a^2*C*e*f^2 +
a^2*B*f^3)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[a*c - b*c*x])/(Sqrt[c]*Sqrt[b*e
 + a*f]*Sqrt[a + b*x])])/(Sqrt[c]*f^2*(-(b*e) + a*f)^(3/2)*(b*e + a*f)^(3/2
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.04, size = 1200, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] (A*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*
c)^(1/2)*f)/(f*x+e))*x*b^2*c*e*f^3*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e*x+a^2*c*f+
((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))*x*a^2*
c*f^4*(b^2*c)^(1/2)+2*C*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(
1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))*x*a^2*c*e*f^3*(b^2*c)^(1/2)-C*ln(
2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/
2)*f)/(f*x+e))*x*b^2*c*e^3*f*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)/(-(b^2*x^
2-a^2)*c)^(1/2)*x)*x*a^2*c*f^4*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)-C*arctan((b^
2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*x*b^2*c*e^2*f^2*((a^2*f^2-b^2*e^2)*c
/f^2)^(1/2)+A*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^
2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))*b^2*c*e^2*f^2*(b^2*c)^(1/2)-B*ln(2*(b^2*c*e
*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x
+e))*a^2*c*e*f^3*(b^2*c)^(1/2)+2*C*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^
2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))*a^2*c*e^2*f^2*(b^2*c)^
(1/2)-C*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-
```

```
a^2)*c)^(1/2)*f)/(f*x+e))*b^2*c*e^4*(b^2*c)^(1/2)+C*arctan((b^2*c)^(1/2)/(-
(b^2*x^2-a^2)*c)^(1/2)*x)*a^2*c*e*f^3*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)-C*arc
tan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*b^2*c*e^3*f*((a^2*f^2-b^2*e^2
)*c/f^2)^(1/2)-A*f^4*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*((a^2*f^2-b^2*e
^2)*c/f^2)^(1/2)+B*e*f^3*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*((a^2*f^2-b
^2*e^2)*c/f^2)^(1/2)-C*e^2*f^2*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*((a^2
*f^2-b^2*e^2)*c/f^2)^(1/2))/c*(-(b*x-a)*c)^(1/2)*(b*x+a)^(1/2)/(-(b^2*x^2-a
^2)*c)^(1/2)/(a*f-b*e)/(b^2*c)^(1/2)/(a*f+b*e)/(f*x+e)/((a^2*f^2-b^2*e^2)*c
/f^2)^(1/2)/f^3
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for mor
e details)Is (4*b^2*c     *(a^2*c-(b^2*c*e^2)
   /f^2))    /f^2    +(4*b^4*c^2*e^2)/f^4    zero or nonzero?
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^2*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

3.26 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$

Optimal. Leaf size=363

$$\frac{\left(a^2-b^2x^2\right)\left(2a^2f^2(2Ce-Bf)-b^2e\left(f(Be-3Af)+Ce^2\right)\right)}{2f\sqrt{a+bx}\,(e+fx)\sqrt{ac-bcx}\,\left(b^2e^2-a^2f^2\right)^2}+\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2\sqrt{a+bx}\,(e+fx)^2\sqrt{ac-bcx}\,\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2}\cdots}{\cdots}$$

Rubi [A] time = 0.68, antiderivative size = 361, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1610, 1651, 807, 725, 204}

$$\frac{\left(a^2-b^2x^2\right)\left(2a^2f^2(2Ce-Bf)-b^2\left(ef(Be-3Af)+Ce^3\right)\right)}{2f\sqrt{a+bx}\,(e+fx)\sqrt{ac-bcx}\,\left(b^2e^2-a^2f^2\right)^2}+\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2\sqrt{a+bx}\,(e+fx)^2\sqrt{ac-bcx}\,\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2c-b^2cx^2}\,\left(A\left(a^2b^2f^2+2b^4e^2\right)+a^2b^2e(Ce-3Bf)+2a^4Cf^2\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\,\sqrt{b^2e^2-a^2f^2}}\right)}{2\sqrt{c}\,\sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(b^2e^2-a^2f^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/(2*(b^2*e^2 - a^2*f^2)*Sqrt[a
 + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2) + ((2*a^2*f^2*(2*C*e - B*f) - b^2*(C
*e^3 + e*f*(B*e - 3*A*f)))*(a^2 - b^2*x^2))/(2*f*(b^2*e^2 - a^2*f^2)^2*Sqrt
[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3
*B*f) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c
]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(2
*Sqrt[c]*(b^2*e^2 - a^2*f^2)^(5/2)*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
```

```
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}\,(e+fx)^3}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}\,(e+fx)^2} + \frac{\sqrt{a^2c-b^2cx^2}\int \frac{2c\left(Ab^2e+a^2(Ce \cdots\right.}{\cdots}}{2c\left(b^2e^2-a^2f^2\right)\cdots}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}\,(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2\left(Ce^3 \cdots\right.\right.}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a \cdots}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}\,(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2\left(Ce^3 \cdots\right.\right.}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a \cdots}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}\,(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2\left(Ce^3 \cdots\right.\right.}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a \cdots}}$$

Mathematica [A] time = 1.79, size = 492, normalized size = 1.36

$$\frac{\frac{b^2\sqrt{a-bx}\left(f(Af-Be)+Ce^2\right)\left(2(e+fx)\left(a^2f^2+2b^2e^2\right)\tanh^{-1}\left(\frac{\sqrt{a-bx}\sqrt{be-af}}{\sqrt{a+bx}\sqrt{-af-be}}\right)+3ef\sqrt{a-bx}\sqrt{a+bx}\sqrt{-af-be}\sqrt{be-af}\right)}{(e+fx)(-af-be)^{5/2}(be-af)^{5/2}}+\frac{2f(bx-a)\sqrt{a+bx}(Bf-2Ce)}{(e+fx)\left(a^2f^2-b^2e^2\right)}+\frac{f(bx-a)\sqrt{a+bx}\left(f(Af-Be)+Ce^2\right)}{(e+fx)^2(af-be)(af+be)}+\frac{4b^2e\sqrt{a-bx}(2Ce-Bf)\tanh^{-1}\left(\frac{\sqrt{a-bx}\sqrt{be-af}}{\sqrt{a+bx}\sqrt{-af-be}}\right)}{(-af-be)^{3/2}(be-af)^{3/2}}+\frac{4C\sqrt{a-bx}\tanh^{-1}\left(\frac{\sqrt{a-bx}\sqrt{be-af}}{\sqrt{a+bx}\sqrt{-af-be}}\right)}{\sqrt{-af-be}\sqrt{be-af}}}{2f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x
]
```

```
[Out] ((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/((-(b*e) + a*f)*(b
*e + a*f)*(e + f*x)^2) + (2*f*(-2*C*e + B*f)*(-a + b*x)*Sqrt[a + b*x])/((-(
b^2*e^2) + a^2*f^2)*(e + f*x)) + (4*C*Sqrt[a - b*x]*ArcTanh[(Sqrt[b*e - a*f
]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])]])/(Sqrt[-(b*e) - a*f]*S
qrt[b*e - a*f]) + (4*b^2*e*(2*C*e - B*f)*Sqrt[a - b*x]*ArcTanh[(Sqrt[b*e -
a*f]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])]])/((-(b*e) - a*f)^(3
/2)*(b*e - a*f)^(3/2)) + (b^2*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a - b*x]*(3*e
*f*Sqrt[-(b*e) - a*f]*Sqrt[b*e - a*f]*Sqrt[a - b*x]*Sqrt[a + b*x] + 2*(2*b^
2*e^2 + a^2*f^2)*(e + f*x)*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[a - b*x])/(Sqrt[-(
b*e) - a*f]*Sqrt[a + b*x])]]))/((-(b*e) - a*f)^(5/2)*(b*e - a*f)^(5/2)*(e +
f*x)))/(2*f^2*Sqrt[c*(a - b*x)])
```

IntegrateAlgebraic [A] time = 1.47, size = 610, normalized size = 1.68

$$\frac{\left(-2a^4Cf^2-a^2Ab^2f^2+3a^2b^2Bef-a^2b^2Ce^2-2Ab^4e^2\right)\tanh^{-1}\left(\frac{\sqrt{ac-bcx}\sqrt{af-be}}{\sqrt{c}\sqrt{a+bx}\sqrt{af+be}}\right)}{\sqrt{c}(be-af)^2\sqrt{af-be}(af+be)^{5/2}}-\frac{ab\sqrt{ac-bcx}\left(-\frac{2a^3Bf^3(ac-bcx)}{a+bx}+\frac{4a^3Cef^2(ac-bcx)}{a+bx}+2a^3Bcf^3-4a^3cCef^2+\frac{a^2Abf^3(ac-bcx)}{a+bx}+a^2Abcf^3+\frac{a^2bBef^2(ac-bcx)}{a+bx}+a^2bBcef^2-\frac{3a^2bCe^2f(ac-bcx)}{a+bx}-3a^2bcCe^2f-\frac{4Ab^3e^2f(ac-bcx)}{a+bx}+\frac{3aAb^2ef^2(ac-bcx)}{a+bx}-3aAb^2cef^2+\frac{2b^3Be^3(ac-bcx)}{a+bx}-\frac{ab^2Be^2f(ac-bcx)}{a+bx}+ab^2Bce^2f-\frac{ab^2Ce^3(ac-bcx)}{a+bx}+ab^2cCe^3-4Ab^3ce^2f+2b^3Bce^3\right)}{\sqrt{a+bx}(be-af)^2(af+be)^2\left(\frac{be(ac-bcx)}{a+bx}-\frac{af(ac-bcx)}{a+bx}+acf+bce\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e +
f*x)^3),x]
```

```
[Out] -((a*b*Sqrt[a*c - b*c*x]*(2*b^3*B*c*e^3 + a*b^2*c*C*e^3 - 4*A*b^3*c*e^2*f +
 a*b^2*B*c*e^2*f - 3*a^2*b*c*C*e^2*f - 3*a*A*b^2*c*e*f^2 + a^2*b*B*c*e*f^2
- 4*a^3*c*C*e*f^2 + a^2*A*b*c*f^3 + 2*a^3*B*c*f^3 + (2*b^3*B*e^3*(a*c - b*c
*x))/(a + b*x) - (a*b^2*C*e^3*(a*c - b*c*x))/(a + b*x) - (4*A*b^3*e^2*f*(a*
c - b*c*x))/(a + b*x) - (a*b^2*B*e^2*f*(a*c - b*c*x))/(a + b*x) - (3*a^2*b*
C*e^2*f*(a*c - b*c*x))/(a + b*x) + (3*a*A*b^2*e*f^2*(a*c - b*c*x))/(a + b*x
) + (a^2*b*B*e*f^2*(a*c - b*c*x))/(a + b*x) + (4*a^3*C*e*f^2*(a*c - b*c*x))
/(a + b*x) + (a^2*A*b*f^3*(a*c - b*c*x))/(a + b*x) - (2*a^3*B*f^3*(a*c - b*
c*x))/(a + b*x)))/((b*e - a*f)^2*(b*e + a*f)^2*Sqrt[a + b*x]*(b*c*e + a*c*f
 + (b*e*(a*c - b*c*x))/(a + b*x) - (a*f*(a*c - b*c*x))/(a + b*x))^2)) + ((-
2*A*b^4*e^2 - a^2*b^2*C*e^2 + 3*a^2*b^2*B*e*f - a^2*A*b^2*f^2 - 2*a^4*C*f^2
)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[a*c - b*c*x])/(Sqrt[c]*Sqrt[b*e + a*f]*S
qrt[a + b*x])])/(Sqrt[c]*(b*e - a*f)^2*Sqrt[-(b*e) + a*f]*(b*e + a*f)^(5/2)
)
```

fricas [A] time = 163.67, size = 1355, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*((3*B*a^2*b^2*e^3*f - (C*a^2*b^2 + 2*A*b^4)*e^4 - (2*C*a^4 + A*a^2*b^2
)*e^2*f^2 + (3*B*a^2*b^2*e*f^3 - (C*a^2*b^2 + 2*A*b^4)*e^2*f^2 - (2*C*a^4 +
 A*a^2*b^2)*f^4)*x^2 + 2*(3*B*a^2*b^2*e^2*f^2 - (C*a^2*b^2 + 2*A*b^4)*e^3*f
 - (2*C*a^4 + A*a^2*b^2)*e*f^3)*x)*sqrt(-b^2*c*e^2 + a^2*c*f^2)*log((2*a^2*
b^2*c*e*f*x - a^2*b^2*c*e^2 + 2*a^4*c*f^2 + (2*b^4*c*e^2 - a^2*b^2*c*f^2)*x
^2 - 2*sqrt(-b^2*c*e^2 + a^2*c*f^2)*(b^2*e*x + a^2*f)*sqrt(-b*c*x + a*c)*sq
rt(b*x + a))/(f^2*x^2 + 2*e*f*x + e^2)) - 2*(2*B*b^4*e^5 - B*a^2*b^2*e^3*f^
2 - B*a^4*e*f^4 - A*a^4*f^5 - (3*C*a^2*b^2 + 4*A*b^4)*e^4*f + (3*C*a^4 + 5*
A*a^2*b^2)*e^2*f^3 + (C*b^4*e^5 + B*b^4*e^4*f + B*a^2*b^2*e^2*f^3 - 2*B*a^4
*f^5 - (5*C*a^2*b^2 + 3*A*b^4)*e^3*f^2 + (4*C*a^4 + 3*A*a^2*b^2)*e*f^4)*x)*
sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^6*c*e^8 - 3*a^2*b^4*c*e^6*f^2 + 3*a^4*
b^2*c*e^4*f^4 - a^6*c*e^2*f^6 + (b^6*c*e^6*f^2 - 3*a^2*b^4*c*e^4*f^4 + 3*a^
4*b^2*c*e^2*f^6 - a^6*c*f^8)*x^2 + 2*(b^6*c*e^7*f - 3*a^2*b^4*c*e^5*f^3 + 3
*a^4*b^2*c*e^3*f^5 - a^6*c*e*f^7)*x), -1/2*((3*B*a^2*b^2*e^3*f - (C*a^2*b^2
 + 2*A*b^4)*e^4 - (2*C*a^4 + A*a^2*b^2)*e^2*f^2 + (3*B*a^2*b^2*e*f^3 - (C*a
^2*b^2 + 2*A*b^4)*e^2*f^2 - (2*C*a^4 + A*a^2*b^2)*f^4)*x^2 + 2*(3*B*a^2*b^2
*e^2*f^2 - (C*a^2*b^2 + 2*A*b^4)*e^3*f - (2*C*a^4 + A*a^2*b^2)*e*f^3)*x)*sq
rt(b^2*c*e^2 - a^2*c*f^2)*arctan(sqrt(b^2*c*e^2 - a^2*c*f^2)*(b^2*e*x + a^2
*f)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^2*b^2*c*e^2 - a^4*c*f^2 - (b^4*c*e^
2 - a^2*b^2*c*f^2)*x^2)) + (2*B*b^4*e^5 - B*a^2*b^2*e^3*f^2 - B*a^4*e*f^4 -
 A*a^4*f^5 - (3*C*a^2*b^2 + 4*A*b^4)*e^4*f + (3*C*a^4 + 5*A*a^2*b^2)*e^2*f^
3 + (C*b^4*e^5 + B*b^4*e^4*f + B*a^2*b^2*e^2*f^3 - 2*B*a^4*f^5 - (5*C*a^2*b
^2 + 3*A*b^4)*e^3*f^2 + (4*C*a^4 + 3*A*a^2*b^2)*e*f^4)*x)*sqrt(-b*c*x + a*c
)*sqrt(b*x + a))/(b^6*c*e^8 - 3*a^2*b^4*c*e^6*f^2 + 3*a^4*b^2*c*e^4*f^4 - a
^6*c*e^2*f^6 + (b^6*c*e^6*f^2 - 3*a^2*b^4*c*e^4*f^4 + 3*a^4*b^2*c*e^2*f^6 -
 a^6*c*f^8)*x^2 + 2*(b^6*c*e^7*f - 3*a^2*b^4*c*e^5*f^3 + 3*a^4*b^2*c*e^3*f^
5 - a^6*c*e*f^7)*x)]
```

giac [B] time = 7.02, size = 1658, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -(2*C*a^4*sqrt(-c)*c^2*f^2 + A*a^2*b^2*sqrt(-c)*c^2*f^2 - 3*B*a^2*b^2*sqrt(
-c)*c^2*f*e + C*a^2*b^2*sqrt(-c)*c^2*e^2 + 2*A*b^4*sqrt(-c)*c^2*e^2)*arctan
```

```
(1/2*(2*b*c^2*e + (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*
c)*c))^2*f)/(sqrt(a^2*f^2 - b^2*e^2)*c^2))/((a^4*f^4*abs(c) - 2*a^2*b^2*f^2
*abs(c)*e^2 + b^4*abs(c)*e^4)*sqrt(a^2*f^2 - b^2*e^2)*c^2) + 2*(16*B*a^6*b*
sqrt(-c)*c^8*f^5 - 32*C*a^6*b*sqrt(-c)*c^8*f^4*e - 24*A*a^4*b^3*sqrt(-c)*c^
8*f^4*e + 4*A*a^4*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x
- a*c)*c))^2*sqrt(-c)*c^6*f^5 + 8*B*a^4*b^3*sqrt(-c)*c^8*f^3*e^2 + 20*B*a^4
*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt
(-c)*c^6*f^4*e + 4*B*a^4*b*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b
*c*x - a*c)*c))^4*sqrt(-c)*c^4*f^5 + 8*C*a^4*b^3*sqrt(-c)*c^8*f^2*e^3 - 44*
C*a^4*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2
*sqrt(-c)*c^6*f^3*e^2 - 40*A*a^2*b^4*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*
a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)*c^6*f^3*e^2 - 8*C*a^4*b*(sqrt(-b*c*x +
 a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sqrt(-c)*c^4*f^4*e - 6*
A*a^2*b^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4
*sqrt(-c)*c^4*f^4*e - A*a^2*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2
 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f^5 + 16*B*a^2*b^4*(sqrt(-b*c*x + a*c)*
sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)*c^6*f^2*e^3 + 10*B*a
^2*b^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sq
rt(-c)*c^4*f^3*e^2 + 3*B*a^2*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^
2 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f^4*e + 8*C*a^2*b^4*(sqrt(-b*c*x + a*c
)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)*c^6*f*e^4 - 14*C*a
^2*b^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sq
rt(-c)*c^4*f^2*e^3 - 12*A*b^5*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 +
 (b*c*x - a*c)*c))^4*sqrt(-c)*c^4*f^2*e^3 - 5*C*a^2*b^2*(sqrt(-b*c*x + a*c)
*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f^3*e^2 - 2*A*b
^4*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*sqrt(-
c)*c^2*f^3*e^2 + 4*B*b^5*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c
*x - a*c)*c))^4*sqrt(-c)*c^4*f*e^4 + 4*C*b^5*(sqrt(-b*c*x + a*c)*sqrt(-c) -
 sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sqrt(-c)*c^4*e^5 + 2*C*b^4*(sqrt(-b*c*x
 + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f*e^4)/(
(a^4*f^6*abs(c) - 2*a^2*b^2*f^4*abs(c)*e^2 + b^4*f^2*abs(c)*e^4)*(4*a^2*c^4
*f + 4*b*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*
c^2*e + (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*f
)^2)
```

maple [B] time = 0.06, size = 1848, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] -1/2*(C*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-
a^2)*c)^(1/2)*f)/(f*x+e))*x^2*a^2*b^2*c*e^2*f^2+2*A*ln(2*(b^2*c*e*x+a^2*c*f
+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))*x*a^2
*b^2*c*e*f^3+2*A*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-
(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))*b^4*c*e^4+2*C*ln(2*(b^2*c*e*x+a^2*c*f+((
a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))*x^2*a^4*
c*f^4+2*C*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^
2-a^2)*c)^(1/2)*f)/(f*x+e))*a^4*c*e^2*f^2+C*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f
^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^4
+2*B*x*a^2*f^4*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)-4*A
*b^2*e^2*f^2*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+B*a^2
*e*f^3*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+2*B*b^2*e^3
*f*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)-3*C*a^2*e^2*f^2
*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)-3*B*ln(2*(b^2*c*e
*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x
+e))*x^2*a^2*b^2*c*e*f^3+A*a^2*f^4*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x
^2-a^2)*c)^(1/2)-6*B*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2
)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))*x*a^2*b^2*c*e^2*f^2+2*C*ln(2*(b^2*c*
```

```
e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*
x+e))*x*a^2*b^2*c*e^3*f-4*C*x*a^2*e*f^3*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(
b^2*x^2-a^2)*c)^(1/2)+A*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(
1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))*x^2*a^2*b^2*c*f^4+2*A*ln(2*(b^2*c
*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f
*x+e))*x^2*b^4*c*e^2*f^2+4*A*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f
^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))*x*b^4*c*e^3*f+4*C*ln(2*(b^2*
c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(
f*x+e))*x*a^4*c*e*f^3+A*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(
1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))*a^2*b^2*c*e^2*f^2-3*B*ln(2*(b^2*c
*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f
*x+e))*a^2*b^2*c*e^3*f+C*x*b^2*e^3*f*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2
*x^2-a^2)*c)^(1/2)-3*A*x*b^2*e*f^3*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x
^2-a^2)*c)^(1/2)+B*x*b^2*e^2*f^2*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2
-a^2)*c)^(1/2))/c*(-(b*x-a)*c)^(1/2)*(b*x+a)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)
/(a*f-b*e)/(a*f+b*e)/(a^2*f^2-b^2*e^2)/(f*x+e)^2/((a^2*f^2-b^2*e^2)*c/f^2)^
(1/2)/f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for more
 details)Is a*f-b*e positive, negative or zero?
```

mupad [B] time = 86.67, size = 9344, normalized size = 25.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^3*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] ((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(4*C*a^4*c^3*f^2 + 2*C*a^2*b^2*c^3*e^
2))/(((a + b*x)^(1/2) - a^(1/2))*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a^4*b*e*f^4
)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(68*C*a^4*c^2*f^2 - 14*C*a^2*b^
2*c^2*e^2))/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a
^4*b*e*f^4)) - ((68*C*a^4*c*f^2 - 14*C*a^2*b^2*c*e^2)*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^5 - 2*a^2*b^3*e^3*f
^2 + a^4*b*e*f^4)) - ((4*C*a^4*f^2 + 2*C*a^2*b^2*e^2)*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(1/2))^7*(b^5*e^5 - 2*a^2*b^3*e^3*f
^2 + a^4*b*e*f^4)) - (a^(1/2)*(a*c)^(1/2)*(48*C*a^4*c*f^3 - 24*C*a^2*b^2*c*
e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(((a + b*x)^(1/2) - a^(1/2))^
4*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)) + (a^(1/2)*(a*c)^(1/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(24*C*a^4*f^3 + 12*C*a^2*b^2*e^2*f))/(
((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^
4)) + (a^(1/2)*(a*c)^(1/2)*(24*C*a^4*c^2*f^3 + 12*C*a^2*b^2*c^2*e^2*f)*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^6
- 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^8/((a + b*x)^(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^6*(16*a^2*c*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6
) + ((16*a^2*c^3*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2
)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/
2))^4) - (8*a^(1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b
*e*((a + b*x)^(1/2) - a^(1/2))^7) + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*
```

```
c*x)^(1/2) - (a*c)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c
*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2)
 - a^(1/2))^5) + (8*a^(1/2)*c^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^3)/(b*e*((a + b*x)^(1/2) - a^(1/2))^3)) + (((4*A*a^4*f^4 - 10*A*a^2*
b^2*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(
1/2))^7*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*f^2)) - ((4*A*a^4*c^3*f^4
 - 10*A*a^2*b^2*c^3*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x
)^(1/2) - a^(1/2))*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*f^2)) - ((4*A*a
^4*c^2*f^4 - 58*A*a^2*b^2*c^2*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*
f^2)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5*(4*A*a^4*c*f^4 - 58*A*a^2*b^
2*c*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a
^2*b^3*e^5*f^2)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^6*(16*A*b^4*e^4*f - 8*A*a^4*f^5 + 28*A*a^2*b^2*e^2*f^3))/(((a + b*x)^(1/2)
 - a^(1/2))^6*(b^6*e^8 - 2*a^2*b^4*e^6*f^2 + a^4*b^2*e^4*f^4)) + (a^(1/2)*(
a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4*(16*A*a^4*c*f^5 + 32*A*b^4
*c*e^4*f - 72*A*a^2*b^2*c*e^2*f^3))/(((a + b*x)^(1/2) - a^(1/2))^4*(b^6*e^8
 - 2*a^2*b^4*e^6*f^2 + a^4*b^2*e^4*f^4)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2))^2*(16*A*b^4*c^2*e^4*f - 8*A*a^4*c^2*f^5 + 28*A*a^2
*b^2*c^2*e^2*f^3))/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^8 - 2*a^2*b^4*e^6*
f^2 + a^4*b^2*e^4*f^4)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8/((a + b*x)^
(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(16*a^2*c
*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6) + ((16*a^2*c^3
*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*e^2*((a +
 b*x)^(1/2) - a^(1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^2)*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^4) - (8*a^(1
/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b*e*((a + b*x)^(1
/2) - a^(1/2))^7) + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c*f*(a*c)^(1/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2) - a^(1/2))^5) +
 (8*a^(1/2)*c^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*(
(a + b*x)^(1/2) - a^(1/2))^3)) - (((32*B*a^4*c^2*f^3 + 22*B*a^2*b^2*c^2*e^2
*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(((a + b*x)^(1/2) - a^(1/2))^3*(
b^5*e^6 + a^4*b*e^2*f^4 - 2*a^2*b^3*e^4*f^2)) - ((32*B*a^4*c*f^3 + 22*B*a^2
*b^2*c*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^
(1/2))^5*(b^5*e^6 + a^4*b*e^2*f^4 - 2*a^2*b^3*e^4*f^2)) + (a^(1/2)*(a*c)^(1
/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(8*B*a^4*c^2*f^4 + 8*B*b^4*c^2*e^
4 + 20*B*a^2*b^2*c^2*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^7 - 2*
a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^4)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^6*(8*B*a^4*f^4 + 8*B*b^4*e^4 + 20*B*a^2*b^2*e^2*f^2))/(
((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^7 - 2*a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^
4)) - (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4*(16*B*a^4*
c*f^4 - 16*B*b^4*c*e^4 + 24*B*a^2*b^2*c*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/
2))^4*(b^6*e^7 - 2*a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^4)) - (6*B*a^2*b*f*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(1/2))^7*(a^4*f^4 +
 b^4*e^4 - 2*a^2*b^2*e^2*f^2)) + (6*B*a^2*b*c^3*f*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))*(a^4*f^4 + b^4*e^4 - 2*a^2*b^2*e^2
*f^2)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8/((a + b*x)^(1/2) - a^(1/2))^
8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(16*a^2*c*f^2 + 4*b^2*c*e^
2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6) + ((16*a^2*c^3*f^2 + 4*b^2*c^3*
e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*e^2*((a + b*x)^(1/2) - a^(
1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^4) - (8*a^(1/2)*f*(a*c)^(1/2)
*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b*e*((a + b*x)^(1/2) - a^(1/2))^7)
 + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b*e*(
(a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/
2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2) - a^(1/2))^5) + (8*a^(1/2)*c^2*f
*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*((a + b*x)^(1/2) -
 a^(1/2))^3)) + (C*a^2*(2*a^2*f^2 + b^2*e^2)*(2*atan(((((a*c - b*c*x)^(1/2)
```

```
 - (a*c)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (a^2
*c*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + 2
*a^(1/2)*b*c*e*f*(a*c)^(1/2))/(2*b*c*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2))) + 2*
atan((((((4*(4*C^2*a^8*f^4 + C^2*a^4*b^4*e^4 + 4*C^2*a^6*b^2*e^2*f^2))/(b^
10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b
^2*e^2*f^8) - (C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2*(12*a^10*c*f^10 - 4*b^10*c*e
^10 + 28*a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f^4 + 88*a^6*b^4*c*e^4*f^6 -
52*a^8*b^2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)
*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a
^8*b^2*e^2*f^8)))/(4*b*c^2*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (C*a^(3/2)*(2
*a^2*f^2 + b^2*e^2)*(8*C*a^(17/2)*f^7*(a*c)^(1/2) - 12*C*a^(13/2)*b^2*e^2*f
^5*(a*c)^(1/2) + 4*C*a^(5/2)*b^6*e^6*f*(a*c)^(1/2)))/(2*b*c^2*e*f*(a*c)^(1/
2)*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 - 4
*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)
))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 + (
((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(((4*(4*C^2*a^8*c*f^4 + C^2*a^4*b^4*c*e
^4 + 4*C^2*a^6*b^2*c*e^2*f^2))/(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e
^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8) + (C^2*a^4*(2*a^2*f^2 + b^2*e
^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c^2*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^4*b
^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^4*f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f + b
*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2
+ 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)))/(4*b*c^2*e*(b^
2*c*e^2 - a^2*c*f^2)^(1/2)) + (8*C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2)/(b*e*(a*f
 + b*e)^4*(a*f - b*e)^4*(b^2*c*e^2 - a^2*c*f^2)^(3/2)) - (C*a^(3/2)*(2*a^2*
f^2 + b^2*e^2)*(8*C*a^(17/2)*c*f^7*(a*c)^(1/2) + 4*C*a^(5/2)*b^6*c*e^6*f*(a
*c)^(1/2) - 12*C*a^(13/2)*b^2*c*e^2*f^5*(a*c)^(1/2)))/(2*b*c^2*e*f*(a*c)^(1
/2)*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 -
4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8
))))/((a + b*x)^(1/2) - a^(1/2)) - ((((4*(4*C^2*a^8*f^4 + C^2*a^4*b^4*e^4 +
 4*C^2*a^6*b^2*e^2*f^2))/(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4
 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8) - (C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2*
(12*a^10*c*f^10 - 4*b^10*c*e^10 + 28*a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f
^4 + 88*a^6*b^4*c*e^4*f^6 - 52*a^8*b^2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*
e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6
*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)))/(2*a^(1/2)*c*f*(a*c)^(1/2)*(b
^2*c*e^2 - a^2*c*f^2)^(1/2)) + (4*C^2*a^(9/2)*f*(a*c)^(1/2)*(2*a^2*f^2 + b^
2*e^2)^2)/(b^2*c*e^2*(a*f + b*e)^4*(a*f - b*e)^4*(b^2*c*e^2 - a^2*c*f^2)^(3
/2)))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2
- ((4*(4*C^2*a^8*c*f^4 + C^2*a^4*b^4*c*e^4 + 4*C^2*a^6*b^2*c*e^2*f^2))/(b^1
0*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^
2*e^2*f^8) + (C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c^2
*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^4*b^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^4*
f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^
2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4
*f^6 + a^8*b^2*e^2*f^8)))/(2*a^(1/2)*c*f*(a*c)^(1/2)*(b^2*c*e^2 - a^2*c*f^2
)^(1/2)))*(b^10*e^10*(a^2*c*f^2 - b^2*c*e^2) - 4*a^2*b^8*e^8*f^2*(a^2*c*f^2
 - b^2*c*e^2) + 6*a^4*b^6*e^6*f^4*(a^2*c*f^2 - b^2*c*e^2) - 4*a^6*b^4*e^4*f
^6*(a^2*c*f^2 - b^2*c*e^2) + a^8*b^2*e^2*f^8*(a^2*c*f^2 - b^2*c*e^2)))/(16*
C^2*a^8*f^4 + 4*C^2*a^4*b^4*e^4 + 16*C^2*a^6*b^2*e^2*f^2))))/(2*(a*f + b*e)
^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (A*b^2*(a^2*f^2 + 2*b^2*e
^2)*(2*atan(((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/
((a + b*x)^(1/2) - a^(1/2)) - (a^2*c*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)
))/((a + b*x)^(1/2) - a^(1/2)) + 2*a^(1/2)*b*c*e*f*(a*c)^(1/2))/(2*b*c*e*(b
^2*c*e^2 - a^2*c*f^2)^(1/2))) + 2*atan((((((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))*(((4*(4*A^2*b^8*c*e^4 + A^2*a^4*b^4*c*f^4 + 4*A^2*a^2*b^6*c*e^2*f^2))/(b
^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*
b^2*e^2*f^8) + (A^2*b^4*(a^2*f^2 + 2*b^2*e^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c
^2*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^4*b^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^
4*f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 -
```

```
b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e
^4*f^6 + a^8*b^2*e^2*f^8)))/(4*b*c^2*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (8*
A^2*b^3*(a^2*f^2 + 2*b^2*e^2)^2)/(e*(a*f + b*e)^4*(a*f - b*e)^4*(b^2*c*e^2
- a^2*c*f^2)^(3/2)) - (A*b*(a^2*f^2 + 2*b^2*e^2)*(4*A*a^(13/2)*b^2*c*f^7*(a
*c)^(1/2) + 8*A*a^(1/2)*b^8*c*e^6*f*(a*c)^(1/2) - 12*A*a^(5/2)*b^6*c*e^4*f^
3*(a*c)^(1/2)))/(2*a^(1/2)*c^2*e*f*(a*c)^(1/2)*(a*f + b*e)^2*(a*f - b*e)^2*
(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^
6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8))))/((a + b*x)^(1/2) - a^(1/2))
 + ((((4*(4*A^2*b^8*e^4 + A^2*a^4*b^4*f^4 + 4*A^2*a^2*b^6*e^2*f^2))/(b^10*e
^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e
^2*f^8) - (A^2*b^4*(a^2*f^2 + 2*b^2*e^2)^2*(12*a^10*c*f^10 - 4*b^10*c*e^10
+ 28*a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f^4 + 88*a^6*b^4*c*e^4*f^6 - 52*a
^8*b^2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^
10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b
^2*e^2*f^8)))/(4*b*c^2*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (A*b*(a^2*f^2 + 2
*b^2*e^2)*(4*A*a^(13/2)*b^2*f^7*(a*c)^(1/2) - 12*A*a^(5/2)*b^6*e^4*f^3*(a*c
)^(1/2) + 8*A*a^(1/2)*b^8*e^6*f*(a*c)^(1/2)))/(2*a^(1/2)*c^2*e*f*(a*c)^(1/2
)*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 - 4*
a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8))
)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 - ((
((4*(4*A^2*b^8*e^4 + A^2*a^4*b^4*f^4 + 4*A^2*a^2*b^6*e^2*f^2))/(b^10*e^10 -
 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^
8) - (A^2*b^4*(a^2*f^2 + 2*b^2*e^2)^2*(12*a^10*c*f^10 - 4*b^10*c*e^10 + 28*
a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f^4 + 88*a^6*b^4*c*e^4*f^6 - 52*a^8*b^
2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^
10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^
2*f^8)))/(2*a^(1/2)*c*f*(a*c)^(1/2)*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (4*A^2
*a^(1/2)*b^2*f*(a*c)^(1/2)*(a^2*f^2 + 2*b^2*e^2)^2)/(c*e^2*(a*f + b*e)^4*(a
*f - b*e)^4*(b^2*c*e^2 - a^2*c*f^2)^(3/2)))*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - ((4*(4*A^2*b^8*c*e^4 + A^2*a^4*b^4*
c*f^4 + 4*A^2*a^2*b^6*c*e^2*f^2))/(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^
6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8) + (A^2*b^4*(a^2*f^2 + 2*b^
2*e^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c^2*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^
4*b^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^4*f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f
+ b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f
^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)))/(2*a^(1/2)*
c*f*(a*c)^(1/2)*(b^2*c*e^2 - a^2*c*f^2)^(1/2)))*(b^8*e^10*(a^2*c*f^2 - b^2*
c*e^2) + a^8*e^2*f^8*(a^2*c*f^2 - b^2*c*e^2) - 4*a^2*b^6*e^8*f^2*(a^2*c*f^2
 - b^2*c*e^2) + 6*a^4*b^4*e^6*f^4*(a^2*c*f^2 - b^2*c*e^2) - 4*a^6*b^2*e^4*f
^6*(a^2*c*f^2 - b^2*c*e^2)))/(16*A^2*b^6*e^4 + 4*A^2*a^4*b^2*f^4 + 16*A^2*a
^2*b^4*e^2*f^2))))/(2*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(
1/2)) + (3*B*a^2*b^2*e*f*(2*atan((2*b^3*c^3*e^3 + 2*b*c^2*e*(a^2*c*f^2 - b^
2*c*e^2) + 2*a^2*b*c^3*e*f^2 + (3*a^(3/2)*f^3*(a*c)^(3/2)*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 + (2*b^3*c^2*e^3*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - (3*a^(1/2)
*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(a^2*c*f^2 - b^2*c*e^2
))/((a + b*x)^(1/2) - a^(1/2))^3 - (a^(3/2)*c*f^3*(a*c)^(3/2)*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + (2*b*c*e*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^2*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/
2))^2 + (a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(a^2*c
*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (10*a^2*b*c^2*e*f^2*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (7*a^(1/2)
*b^2*c^2*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^
(1/2) - a^(1/2)) - (a^(1/2)*b^2*c*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3)/(4*a^(1/2)*b*c^2*e*f*(a*c)^(
1/2)*(b^2*c*e^2 - a^2*c*f^2)^(1/2))) - 2*atan(((((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (a^2*c*f^2*
((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + 2*a^(1/2
)*b*c*e*f*(a*c)^(1/2))/(2*b*c*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)))))/(2*(a*f +
```

```
b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
[Out] Timed out
```

3.27 $\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx$

Optimal. Leaf size=501

$$-\frac{\left(a^2-b^2x^2\right)(e+fx)^2\left(16a^2Cf^2-b^2\left(3Ce^2-5f(4Af+3Be)\right)\right)}{60b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{\sqrt{a^2c-b^2cx^2}\,\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)\left(4A\left(3a^2b^2ef^2\right.\right.}{8b^5\sqrt{c}\,\sqrt{a}}$$

Rubi [A] time = 1.28, antiderivative size = 496, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1610, 1654, 833, 780, 217, 203}

$$\frac{\left(a^2-b^2x^2\right)(e+fx)^2\left(-\frac{16a^2Cf^2}{b^2}-5f(4Af+3Be)+3Ce^2\right)}{60b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{\left(a^2-b^2x^2\right)\left(b^2fx\left(a^2f^2(45Bf+71Ce)-b^2\left(6Ce^3-10ef(10Af+3Be)\right)\right)+4\left(4a^2b^2f^2\left(5f(Af+3Be)+13Ce^2\right)+16a^4Cf^4+b^4\left(-e^2\right)\left(3Ce^2-5f(16Af+3Be)\right)\right)\right)}{120b^6f\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{\sqrt{a^2c-b^2cx^2}\,\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)\left(4A\left(3a^2b^2ef^2+2b^4e^3\right)+4a^2b^2e^2(3Bf+Ce)+3a^4f^2(Bf+3Ce)\right)}{8b^5\sqrt{c}\,\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{\left(a^2-b^2x^2\right)(e+fx)^3(Ce-5Bf)}{20b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{C\left(a^2-b^2x^2\right)(e+fx)^4}{5b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] ((3*C*e^2 - (16*a^2*C*f^2)/b^2 - 5*f*(3*B*e + 4*A*f))*(e + f*x)^2*(a^2 - b^
2*x^2))/(60*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e - 5*B*f)*(e + f*
x)^3*(a^2 - b^2*x^2))/(20*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) - (C*(e +
f*x)^4*(a^2 - b^2*x^2))/(5*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) - ((4*(16
*a^4*C*f^4 + 4*a^2*b^2*f^2*(13*C*e^2 + 5*f*(3*B*e + A*f)) - b^4*e^2*(3*C*e^
2 - 5*f*(3*B*e + 16*A*f))) + b^2*f*(a^2*f^2*(71*C*e + 45*B*f) - b^2*(6*C*e^
3 - 10*e*f*(3*B*e + 10*A*f)))*x)*(a^2 - b^2*x^2))/(120*b^6*f*Sqrt[a + b*x]*
Sqrt[a*c - b*c*x]) + ((3*a^4*f^2*(3*C*e + B*f) + 4*a^2*b^2*e^2*(C*e + 3*B*f
) + 4*A*(2*b^4*e^3 + 3*a^2*b^2*e*f^2))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sq
rt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^5*Sqrt[c]*Sqrt[a + b*x]*Sqrt[a*c -
b*c*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^3\left(A+Bx+Cx^2\right)}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^4\left(a^2-b^2x^2\right)}{5b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^3\left(-c\left(5Ab^2+4a^2C\right)f^2+b^2cf(Ce-\cdots}{\sqrt{a^2c-b^2cx^2}}}{5b^2cf^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{(Ce-5Bf)(e+fx)^3\left(a^2-b^2x^2\right)}{20b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^4\left(a^2-b^2x^2\right)}{5b^2f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\sqrt{a^2c-b^2cx\cdots}}{\cdots}$$

$$= -\frac{\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce-5\cdots}{20b^2\cdots}$$

$$= -\frac{\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce-5\cdots}{20b^2\cdots}$$

$$= -\frac{\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce-5\cdots}{20b^2\cdots}$$

$$= -\frac{\left(16a^2Cf^2-b^2\left(3Ce^2-5f(3Be+4Af)\right)\right)(e+fx)^2\left(a^2-b^2x^2\right)}{60b^4f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{(Ce-5\cdots}{20b^2\cdots}$$

Mathematica [A] time = 4.90, size = 727, normalized size = 1.45

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
,x]
```

```
[Out] (-120*(b*e - a*f)^2*(5*a^2*C*f + b^2*(B*e + 3*A*f) - 2*a*b*(C*e + 2*B*f))*S
qrt[a - b*x]*Sqrt[a + b*x]*(Sqrt[a - b*x]*Sqrt[1 + (b*x)/a] + 2*Sqrt[a]*Arc
```

```
Sin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]) - 60*(b*e - a*f)*(10*a^2*C*f^2 - 2*a*
b*f*(4*C*e + 3*B*f) + b^2*(C*e^2 + 3*f*(B*e + A*f)))*Sqrt[a - b*x]*Sqrt[a +
 b*x]*(Sqrt[a - b*x]*(4*a + b*x)*Sqrt[1 + (b*x)/a] + 6*a^(3/2)*ArcSin[Sqrt[
a - b*x]/(Sqrt[2]*Sqrt[a])]) - 20*f*(10*a^2*C*f^2 - 4*a*b*f*(3*C*e + B*f) +
 b^2*(3*C*e^2 + f*(3*B*e + A*f)))*Sqrt[a - b*x]*Sqrt[a + b*x]*(Sqrt[a - b*x
]*Sqrt[1 + (b*x)/a]*(22*a^2 + 9*a*b*x + 2*b^2*x^2) + 30*a^(5/2)*ArcSin[Sqrt
[a - b*x]/(Sqrt[2]*Sqrt[a])]) - 5*f^2*(3*b*C*e + b*B*f - 5*a*C*f)*Sqrt[a -
b*x]*Sqrt[a + b*x]*(Sqrt[a - b*x]*Sqrt[1 + (b*x)/a]*(160*a^3 + 81*a^2*b*x +
 32*a*b^2*x^2 + 6*b^3*x^3) + 210*a^(7/2)*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt
[a])]) - 3*C*f^3*Sqrt[a + b*x]*((a - b*x)*Sqrt[1 + (b*x)/a]*(488*a^4 + 275*
a^3*b*x + 144*a^2*b^2*x^2 + 50*a*b^3*x^3 + 8*b^4*x^4) + 630*a^(9/2)*Sqrt[a
- b*x]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]) - 240*(A*b^2 + a*(-(b*B) +
a*C))*(b*e - a*f)^3*Sqrt[a - b*x]*Sqrt[1 + (b*x)/a]*ArcTan[Sqrt[a - b*x]/Sq
rt[a + b*x]])/(120*b^6*Sqrt[c*(a - b*x)]*Sqrt[1 + (b*x)/a])
```

IntegrateAlgebraic [B] time = 1.26, size = 1909, normalized size = 3.81

result too large to display

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e + f*x)^3*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c
 - b*c*x]),x]
```

```
[Out] ((-120*a*b^4*B*c^4*e^3*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] - (60*a^2*b^3*c^4*C
*e^3*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] - (360*a*A*b^4*c^4*e^2*f*Sqrt[a*c - b
*c*x])/Sqrt[a + b*x] - (180*a^2*b^3*B*c^4*e^2*f*Sqrt[a*c - b*c*x])/Sqrt[a +
 b*x] - (360*a^3*b^2*c^4*C*e^2*f*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] - (180*a^
2*A*b^3*c^4*e*f^2*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] - (360*a^3*b^2*B*c^4*e*f
^2*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] - (225*a^4*b*c^4*C*e*f^2*Sqrt[a*c - b*c
*x])/Sqrt[a + b*x] - (120*a^3*A*b^2*c^4*f^3*Sqrt[a*c - b*c*x])/Sqrt[a + b*x
] - (75*a^4*b*B*c^4*f^3*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] - (120*a^5*c^4*C*f
^3*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] - (480*a*b^4*B*c^3*e^3*(a*c - b*c*x)^(3
/2))/(a + b*x)^(3/2) - (120*a^2*b^3*c^3*C*e^3*(a*c - b*c*x)^(3/2))/(a + b*x
)^(3/2) - (1440*a*A*b^4*c^3*e^2*f*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (3
60*a^2*b^3*B*c^3*e^2*f*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (960*a^3*b^2*
c^3*C*e^2*f*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (360*a^2*A*b^3*c^3*e*f^2
*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (960*a^3*b^2*B*c^3*e*f^2*(a*c - b*c
*x)^(3/2))/(a + b*x)^(3/2) - (90*a^4*b*c^3*C*e*f^2*(a*c - b*c*x)^(3/2))/(a
+ b*x)^(3/2) - (320*a^3*A*b^2*c^3*f^3*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2)
- (30*a^4*b*B*c^3*f^3*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (160*a^5*c^3*C
*f^3*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (720*a*b^4*B*c^2*e^3*(a*c - b*c
*x)^(5/2))/(a + b*x)^(5/2) - (2160*a*A*b^4*c^2*e^2*f*(a*c - b*c*x)^(5/2))/(
a + b*x)^(5/2) - (1200*a^3*b^2*c^2*C*e^2*f*(a*c - b*c*x)^(5/2))/(a + b*x)^(
5/2) - (1200*a^3*b^2*B*c^2*e*f^2*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (40
0*a^3*A*b^2*c^2*f^3*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (464*a^5*c^2*C*f
^3*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (480*a*b^4*B*c*e^3*(a*c - b*c*x)^
(7/2))/(a + b*x)^(7/2) + (120*a^2*b^3*c*C*e^3*(a*c - b*c*x)^(7/2))/(a + b*x
)^(7/2) - (1440*a*A*b^4*c*e^2*f*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) + (360
*a^2*b^3*B*c*e^2*f*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) - (960*a^3*b^2*c*C*
e^2*f*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) + (360*a^2*A*b^3*c*e*f^2*(a*c -
b*c*x)^(7/2))/(a + b*x)^(7/2) - (960*a^3*b^2*B*c*e*f^2*(a*c - b*c*x)^(7/2))
/(a + b*x)^(7/2) + (90*a^4*b*c*C*e*f^2*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2)
 - (320*a^3*A*b^2*c*f^3*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) + (30*a^4*b*B*
c*f^3*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) - (160*a^5*c*C*f^3*(a*c - b*c*x)
^(7/2))/(a + b*x)^(7/2) - (120*a*b^4*B*e^3*(a*c - b*c*x)^(9/2))/(a + b*x)^(
9/2) + (60*a^2*b^3*C*e^3*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2) - (360*a*A*b^
4*e^2*f*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2) + (180*a^2*b^3*B*e^2*f*(a*c -
b*c*x)^(9/2))/(a + b*x)^(9/2) - (360*a^3*b^2*C*e^2*f*(a*c - b*c*x)^(9/2))/(
a + b*x)^(9/2) + (180*a^2*A*b^3*e*f^2*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2)
- (360*a^3*b^2*B*e*f^2*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2) + (225*a^4*b*C*
```

```
e*f^2*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2) - (120*a^3*A*b^2*f^3*(a*c - b*c*
x)^(9/2))/(a + b*x)^(9/2) + (75*a^4*b*B*f^3*(a*c - b*c*x)^(9/2))/(a + b*x)^
(9/2) - (120*a^5*C*f^3*(a*c - b*c*x)^(9/2))/(a + b*x)^(9/2))/(60*b^6*(c + (
a*c - b*c*x)/(a + b*x))^5) + ((-8*A*b^4*e^3 - 4*a^2*b^2*C*e^3 - 12*a^2*b^2*
B*e^2*f - 12*a^2*A*b^2*e*f^2 - 9*a^4*C*e*f^2 - 3*a^4*B*f^3)*ArcTan[Sqrt[a*c
 - b*c*x]/(Sqrt[c]*Sqrt[a + b*x])])/(4*b^5*Sqrt[c])
```

fricas [A] time = 0.78, size = 700, normalized size = 1.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/240*(15*(12*B*a^2*b^3*e^2*f + 3*B*a^4*b*f^3 + 4*(C*a^2*b^3 + 2*A*b^5)*e
^3 + 3*(3*C*a^4*b + 4*A*a^2*b^3)*e*f^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-
b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(24*C*b^4*f^3*x^4 + 12
0*B*b^4*e^3 + 240*B*a^2*b^2*e*f^2 + 120*(2*C*a^2*b^2 + 3*A*b^4)*e^2*f + 16*
(4*C*a^4 + 5*A*a^2*b^2)*f^3 + 30*(3*C*b^4*e*f^2 + B*b^4*f^3)*x^3 + 8*(15*C*
b^4*e^2*f + 15*B*b^4*e*f^2 + (4*C*a^2*b^2 + 5*A*b^4)*f^3)*x^2 + 15*(4*C*b^4
*e^3 + 12*B*b^4*e^2*f + 3*B*a^2*b^2*f^3 + 3*(3*C*a^2*b^2 + 4*A*b^4)*e*f^2)*
x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^6*c), -1/120*(15*(12*B*a^2*b^3*e^2*
f + 3*B*a^4*b*f^3 + 4*(C*a^2*b^3 + 2*A*b^5)*e^3 + 3*(3*C*a^4*b + 4*A*a^2*b^
3)*e*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*
c*x^2 - a^2*c)) + (24*C*b^4*f^3*x^4 + 120*B*b^4*e^3 + 240*B*a^2*b^2*e*f^2 +
 120*(2*C*a^2*b^2 + 3*A*b^4)*e^2*f + 16*(4*C*a^4 + 5*A*a^2*b^2)*f^3 + 30*(3
*C*b^4*e*f^2 + B*b^4*f^3)*x^3 + 8*(15*C*b^4*e^2*f + 15*B*b^4*e*f^2 + (4*C*a
^2*b^2 + 5*A*b^4)*f^3)*x^2 + 15*(4*C*b^4*e^3 + 12*B*b^4*e^2*f + 3*B*a^2*b^2
*f^3 + 3*(3*C*a^2*b^2 + 4*A*b^4)*e*f^2)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)
)/(b^6*c)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.03, size = 965, normalized size = 1.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/120*(b*x+a)^(1/2)*(-(b*x-a)*c)^(1/2)/c*(-24*C*x^4*b^4*f^3*(b^2*c)^(1/2)*(
-(b^2*x^2-a^2)*c)^(1/2)-30*B*x^3*b^4*f^3*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(
1/2)-90*C*x^3*b^4*e*f^2*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+180*A*arctan
((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^2*b^4*c*e*f^2+120*A*arctan((b^
2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*b^6*c*e^3-40*A*x^2*b^4*f^3*(b^2*c)^(
1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+45*B*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^
(1/2)*x)*a^4*b^2*c*f^3+180*B*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*
x)*a^2*b^4*c*e^2*f-120*B*x^2*b^4*e*f^2*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/
2)+135*C*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^4*b^2*c*e*f^2+6
0*C*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^2*b^4*c*e^3-32*C*x^2
```

```
*a^2*b^2*f^3*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)-120*C*x^2*b^4*e^2*f*(b^
2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)-180*A*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^
(1/2)*x*b^4*e*f^2-45*B*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*a^2*b^2*f^3
-180*B*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*b^4*e^2*f-135*C*(b^2*c)^(1/
2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*a^2*b^2*e*f^2-60*C*(b^2*c)^(1/2)*(-(b^2*x^2-a
^2)*c)^(1/2)*x*b^4*e^3-80*A*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*a^2*b^2*
f^3-360*A*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*b^4*e^2*f-240*B*(b^2*c)^(1
/2)*(-(b^2*x^2-a^2)*c)^(1/2)*a^2*b^2*e*f^2-120*B*(b^2*c)^(1/2)*(-(b^2*x^2-a
^2)*c)^(1/2)*b^4*e^3-64*C*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*a^4*f^3-24
0*C*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*a^2*b^2*e^2*f)/b^6/(-(b^2*x^2-a^
2)*c)^(1/2)/(b^2*c)^(1/2)
```

maxima [A] time = 1.97, size = 471, normalized size = 0.94

$$-\frac{\sqrt{-b^2cx^2+a^2c}\,Cf^3x^4}{5\,b^2c}-\frac{4\,\sqrt{-b^2cx^2+a^2c}\,Ca^2f^3x^2}{15\,b^4c}+\frac{Ae^3\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}}-\frac{\sqrt{-b^2cx^2+a^2c}\,Be^3}{b^2c}-\frac{3\,\sqrt{-b^2cx^2+a^2c}\,Ae^2f}{b^2c}-\frac{8\,\sqrt{-b^2cx^2+a^2c}\,Ca^4f^3}{15\,b^6c}-\frac{\sqrt{-b^2cx^2+a^2c}\,(3\,Cef^2+Bf^3)x^3}{4\,b^2c}-\frac{\sqrt{-b^2cx^2+a^2c}\,(3\,Ce^2f+3\,Bef^2+Af^3)x^2}{3\,b^2c}+\frac{3\,(3\,Cef^2+Bf^3)a^4\arcsin\left(\frac{bx}{a}\right)}{8\,b^5\sqrt{c}}+\frac{(Ce^3+3\,Be^2f+3\,Aef^2)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}}-\frac{3\,\sqrt{-b^2cx^2+a^2c}\,(3\,Cef^2+Bf^3)a^2x}{8\,b^4c}-\frac{\sqrt{-b^2cx^2+a^2c}\,(Ce^3+3\,Be^2f+3\,Aef^2)x}{2\,b^2c}-\frac{2\,\sqrt{-b^2cx^2+a^2c}\,(3\,Ce^2f+3\,Bef^2+Af^3)a^2}{3\,b^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/5*sqrt(-b^2*c*x^2 + a^2*c)*C*f^3*x^4/(b^2*c) - 4/15*sqrt(-b^2*c*x^2 + a^
2*c)*C*a^2*f^3*x^2/(b^4*c) + A*e^3*arcsin(b*x/a)/(b*sqrt(c)) - sqrt(-b^2*c*
x^2 + a^2*c)*B*e^3/(b^2*c) - 3*sqrt(-b^2*c*x^2 + a^2*c)*A*e^2*f/(b^2*c) - 8
/15*sqrt(-b^2*c*x^2 + a^2*c)*C*a^4*f^3/(b^6*c) - 1/4*sqrt(-b^2*c*x^2 + a^2*
c)*(3*C*e*f^2 + B*f^3)*x^3/(b^2*c) - 1/3*sqrt(-b^2*c*x^2 + a^2*c)*(3*C*e^2*
f + 3*B*e*f^2 + A*f^3)*x^2/(b^2*c) + 3/8*(3*C*e*f^2 + B*f^3)*a^4*arcsin(b*x
/a)/(b^5*sqrt(c)) + 1/2*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*a^2*arcsin(b*x/a)/(
b^3*sqrt(c)) - 3/8*sqrt(-b^2*c*x^2 + a^2*c)*(3*C*e*f^2 + B*f^3)*a^2*x/(b^4*
c) - 1/2*sqrt(-b^2*c*x^2 + a^2*c)*(C*e^3 + 3*B*e^2*f + 3*A*e*f^2)*x/(b^2*c)
 - 2/3*sqrt(-b^2*c*x^2 + a^2*c)*(3*C*e^2*f + 3*B*e*f^2 + A*f^3)*a^2/(b^4*c)
```

mupad [B] time = 161.43, size = 4167, normalized size = 8.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^3*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x
)
```

```
[Out] - ((((23*B*a^4*c*f^3)/2 - 18*B*a^2*b^2*c*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))^13)/(b^5*((a + b*x)^(1/2) - a^(1/2))^13) + (((a*c - b*c*x)^(1/2) -
 (a*c)^(1/2))^15*((3*B*a^4*f^3)/2 + 6*B*a^2*b^2*e^2*f))/(b^5*((a + b*x)^(1/
2) - a^(1/2))^15) - (((3*B*a^4*c^7*f^3)/2 + 6*B*a^2*b^2*c^7*e^2*f)*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2)))/(b^5*((a + b*x)^(1/2) - a^(1/2))) - (((23*B*a^
4*c^6*f^3)/2 - 18*B*a^2*b^2*c^6*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/(b^5*((a + b*x)^(1/2) - a^(1/2))^3) + (((333*B*a^4*c^5*f^3)/2 + 90*B*a^2
*b^2*c^5*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b^5*((a + b*x)^(1/2
) - a^(1/2))^5) - (((333*B*a^4*c^2*f^3)/2 + 90*B*a^2*b^2*c^2*e^2*f)*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^11)/(b^5*((a + b*x)^(1/2) - a^(1/2))^11) - (((
671*B*a^4*c^4*f^3)/2 - 66*B*a^2*b^2*c^4*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^7)/(b^5*((a + b*x)^(1/2) - a^(1/2))^7) + (((671*B*a^4*c^3*f^3)/2 -
66*B*a^2*b^2*c^3*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(b^5*((a + b
*x)^(1/2) - a^(1/2))^9) + (a^(1/2)*(a*c)^(1/2)*(48*B*b^2*c^5*e^3 + 192*B*a^
2*c^5*e*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^4*((a + b*x)^(1/2) -
 a^(1/2))^4) + (a^(1/2)*(a*c)^(1/2)*(160*B*b^2*c^3*e^3 + 128*B*a^2*c^3*e*f^
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/(b^4*((a + b*x)^(1/2) - a^(1/2))^
8) + (a^(1/2)*(a*c)^(1/2)*(120*B*b^2*c^4*e^3 + 256*B*a^2*c^4*e*f^2)*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^6)/(b^4*((a + b*x)^(1/2) - a^(1/2))^6) + (a^(1
/2)*(a*c)^(1/2)*(120*B*b^2*c^2*e^3 + 256*B*a^2*c^2*e*f^2)*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^10)/(b^4*((a + b*x)^(1/2) - a^(1/2))^10) + (a^(1/2)*(a*c
)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12*(48*B*b^2*c*e^3 + 192*B*a^2*
```

c*e*f^2))/(b^4*((a + b*x)^(1/2) - a^(1/2))^12) + (8*B*a^(1/2)*e^3*(a*c)^(1/
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^14)/(b^2*((a + b*x)^(1/2) - a^(1/2))
^14) + (8*B*a^(1/2)*c^6*e^3*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^2)/(b^2*((a + b*x)^(1/2) - a^(1/2))^2))/((((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^16/((a + b*x)^(1/2) - a^(1/2))^16 + c^8 + (8*c*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^14)/((a + b*x)^(1/2) - a^(1/2))^14 + (8*c^7*((a*c - b*c*x)^(1/2)
 - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (28*c^6*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (56*c^5*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6 + (70*c^4*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (56*c^3*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (28*c^
2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12)/((a + b*x)^(1/2) - a^(1/2))^12) -
 ((a^(1/2)*(a*c)^(1/2)*(64*A*a^2*c^3*f^3 + 96*A*b^2*c^3*e^2*f)*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^4)/(b^4*((a + b*x)^(1/2) - a^(1/2))^4) - (a^(1/2)*(
a*c)^(1/2)*((128*A*a^2*c^2*f^3)/3 - 144*A*b^2*c^2*e^2*f)*((a*c - b*c*x)^(1/
2) - (a*c)^(1/2))^6)/(b^4*((a + b*x)^(1/2) - a^(1/2))^6) + (a^(1/2)*(a*c)^(
1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8*(64*A*a^2*c*f^3 + 96*A*b^2*c*e^2
f))/(b^4((a + b*x)^(1/2) - a^(1/2))^8) + (6*A*a^2*e*f^2*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^11)/(b^3*((a + b*x)^(1/2) - a^(1/2))^11) - (6*A*a^2*c^5*
e*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^3*((a + b*x)^(1/2) - a^(1/2))
) - (30*A*a^2*c*e*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(b^3*((a + b*x
)^(1/2) - a^(1/2))^9) + (24*A*a^(1/2)*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))^10)/(b^2*((a + b*x)^(1/2) - a^(1/2))^10) + (30*A*a^2*c^4*e
f^2((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b^3*((a + b*x)^(1/2) - a^(1/2)
)^3) + (36*A*a^2*c^3*e*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b^3*((a
+ b*x)^(1/2) - a^(1/2))^5) - (36*A*a^2*c^2*e*f^2*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))^7)/(b^3*((a + b*x)^(1/2) - a^(1/2))^7) + (24*A*a^(1/2)*c^4*e^2*f*
(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*((a + b*x)^(1/2) -
a^(1/2))^2))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12/((a + b*x)^(1/2) - a^(
1/2))^12 + c^6 + (6*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1
/2) - a^(1/2))^10 + (6*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x
)^(1/2) - a^(1/2))^2 + (15*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a +
 b*x)^(1/2) - a^(1/2))^4 + (20*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/(
(a + b*x)^(1/2) - a^(1/2))^6 + (15*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
8)/((a + b*x)^(1/2) - a^(1/2))^8) - ((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^1
9*((9*C*a^4*e*f^2)/2 + 2*C*a^2*b^2*e^3))/(b^5*((a + b*x)^(1/2) - a^(1/2))^1
9) - ((2*C*a^2*b^2*c*e^3 - (87*C*a^4*c*e*f^2)/2)*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))^17)/(b^5*((a + b*x)^(1/2) - a^(1/2))^17) - (((9*C*a^4*c^9*e*f^2)/
2 + 2*C*a^2*b^2*c^9*e^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^5*((a + b*
x)^(1/2) - a^(1/2))) - (((87*C*a^4*c^8*e*f^2)/2 - 2*C*a^2*b^2*c^8*e^3)*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b^5*((a + b*x)^(1/2) - a^(1/2))^3) - ((
42*C*a^4*c^6*e*f^2 - 88*C*a^2*b^2*c^6*e^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^7)/(b^5*((a + b*x)^(1/2) - a^(1/2))^7) + ((42*C*a^4*c^3*e*f^2 - 88*C*a^
2*b^2*c^3*e^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^13)/(b^5*((a + b*x)^(1/2
) - a^(1/2))^13) + ((426*C*a^4*c^7*e*f^2 + 40*C*a^2*b^2*c^7*e^3)*((a*c - b*
c*x)^(1/2) - (a*c)^(1/2))^5)/(b^5*((a + b*x)^(1/2) - a^(1/2))^5) - ((426*C*
a^4*c^2*e*f^2 + 40*C*a^2*b^2*c^2*e^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^1
5)/(b^5*((a + b*x)^(1/2) - a^(1/2))^15) - ((507*C*a^4*c^5*e*f^2 - 52*C*a^2*
b^2*c^5*e^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^9)/(b^5*((a + b*x)^(1/2) -
 a^(1/2))^9) + ((507*C*a^4*c^4*e*f^2 - 52*C*a^2*b^2*c^4*e^3)*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^11)/(b^5*((a + b*x)^(1/2) - a^(1/2))^11) + (a^(1/2)*(
a*c)^(1/2)*((2048*C*a^4*c^6*f^3)/3 + 640*C*a^2*b^2*c^6*e^2*f)*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^6)/(b^6*((a + b*x)^(1/2) - a^(1/2))^6) + (a^(1/2)*(a
c)^(1/2)((2048*C*a^4*c^2*f^3)/3 + 640*C*a^2*b^2*c^2*e^2*f)*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^14)/(b^6*((a + b*x)^(1/2) - a^(1/2))^14) - (a^(1/2)*(
a*c)^(1/2)*((4096*C*a^4*c^5*f^3)/3 - 832*C*a^2*b^2*c^5*e^2*f)*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^8)/(b^6*((a + b*x)^(1/2) - a^(1/2))^8) - (a^(1/2)*(a
c)^(1/2)((4096*C*a^4*c^3*f^3)/3 - 832*C*a^2*b^2*c^3*e^2*f)*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^12)/(b^6*((a + b*x)^(1/2) - a^(1/2))^12) + (a^(1/2)*(

```
a*c)^(1/2)*((12288*C*a^4*c^4*f^3)/5 + 768*C*a^2*b^2*c^4*e^2*f)*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^10)/(b^6*((a + b*x)^(1/2) - a^(1/2))^10) + (192*C*a
^(5/2)*c*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^16)/(b^4*((a
 + b*x)^(1/2) - a^(1/2))^16) + (192*C*a^(5/2)*c^7*e^2*f*(a*c)^(1/2)*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^4*((a + b*x)^(1/2) - a^(1/2))^4))/(((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))^20/((a + b*x)^(1/2) - a^(1/2))^20 + c^10 + (
10*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^18)/((a + b*x)^(1/2) - a^(1/2))^18
 + (10*c^9*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2
))^2 + (45*c^8*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^
(1/2))^4 + (120*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2)
 - a^(1/2))^6 + (210*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^
(1/2) - a^(1/2))^8 + (252*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a +
 b*x)^(1/2) - a^(1/2))^10 + (210*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12
)/((a + b*x)^(1/2) - a^(1/2))^12 + (120*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^14)/((a + b*x)^(1/2) - a^(1/2))^14 + (45*c^2*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^16)/((a + b*x)^(1/2) - a^(1/2))^16) - (2*A*e*atan((A*e*(3*a^2*f^
2 + 2*b^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(c^(1/2)*(2*A*b^2*e^3 +
 3*A*a^2*e*f^2)*((a + b*x)^(1/2) - a^(1/2))))*(3*a^2*f^2 + 2*b^2*e^2))/(b^3
*c^(1/2)) - (3*B*a^2*f*atan((B*a^2*f*(a^2*f^2 + 4*b^2*e^2)*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2)))/(c^(1/2)*(B*a^4*f^3 + 4*B*a^2*b^2*e^2*f)*((a + b*x)^(1
/2) - a^(1/2))))*(a^2*f^2 + 4*b^2*e^2))/(2*b^5*c^(1/2)) - (C*a^2*e*atan((C*
a^2*e*(9*a^2*f^2 + 4*b^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(c^(1/2)
*(9*C*a^4*e*f^2 + 4*C*a^2*b^2*e^3)*((a + b*x)^(1/2) - a^(1/2))))*(9*a^2*f^2
 + 4*b^2*e^2))/(2*b^5*c^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
[Out] Timed out
```

3.28 $\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx$

Optimal. Leaf size=368

$$-\frac{\left(a^2-b^2x^2\right)\left(fx\left(9a^2Cf^2-b^2\left(2Ce^2-4f(3Af+2Be)\right)\right)+4\left(4a^2f^2(Bf+2Ce)-b^2e\left(Ce^2-4f(3Af+Be)\right)\right)\right)}{24b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Rubi [A] time = 0.88, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1610, 1654, 833, 780, 217, 203}

$$-\frac{\left(a^2-b^2x^2\right)\left(fx\left(9a^2Cf^2-b^2\left(2Ce^2-4f(3Af+2Be)\right)\right)+4\left(4a^2f^2(Bf+2Ce)-\frac{1}{4}b^2\left(4Ce^3-16ef(3Af+Be)\right)\right)\right)}{24b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{\sqrt{a^2c-b^2cx^2}\,\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)\left(4A\left(a^2b^2f^2+2b^4e^2\right)+4a^2b^2e(2Bf+Ce)+3a^4Cf^2\right)}{8b^5\sqrt{c}\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{\left(a^2-b^2x^2\right)(e+fx)^2(Ce-4Bf)}{12b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{C\left(a^2-b^2x^2\right)(e+fx)^3}{4b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] ((C*e - 4*B*f)*(e + f*x)^2*(a^2 - b^2*x^2))/(12*b^2*f*Sqrt[a + b*x]*Sqrt[a*
c - b*c*x]) - (C*(e + f*x)^3*(a^2 - b^2*x^2))/(4*b^2*f*Sqrt[a + b*x]*Sqrt[a
*c - b*c*x]) - ((4*(4*a^2*f^2*(2*C*e + B*f) - (b^2*(4*C*e^3 - 16*e*f*(B*e +
 3*A*f)))/4) + f*(9*a^2*C*f^2 - b^2*(2*C*e^2 - 4*f*(2*B*e + 3*A*f)))*x)*(a^
2 - b^2*x^2))/(24*b^4*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((3*a^4*C*f^2 +
4*a^2*b^2*e*(C*e + 2*B*f) + 4*A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a^2*c - b^2
*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(8*b^5*Sqrt[c]*Sqrt[
a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
```

```
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)^2\left(-c\left(4Ab^2+3a^2C\right)f^2+b^2cf(Ce-4Bf\cdots\right.}{\sqrt{a^2c-b^2cx^2}}}{4b^2cf^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\left(a^2-b^2x^2\right)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\sqrt{a^2c-b^2cx^2}\int\cdots}{\cdots}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\left(a^2-b^2x^2\right)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(4\left(4a^2f^2(2Ce\cdots\right.\right.}{\cdots}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\left(a^2-b^2x^2\right)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(4\left(4a^2f^2(2Ce\cdots\right.\right.}{\cdots}$$

$$= \frac{(Ce-4Bf)(e+fx)^2\left(a^2-b^2x^2\right)}{12b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{C(e+fx)^3\left(a^2-b^2x^2\right)}{4b^2f\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\left(4\left(4a^2f^2(2Ce\cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 2.68, size = 555, normalized size = 1.51

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
,x]
```

```
[Out] (-24*(b*e - a*f)*(4*a^2*C*f + b^2*(B*e + 2*A*f) - a*b*(2*C*e + 3*B*f))*Sqrt
[a - b*x]*Sqrt[a + b*x]*(Sqrt[a - b*x]*Sqrt[1 + (b*x)/a] + 2*Sqrt[a]*ArcSin
[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]) - 12*(6*a^2*C*f^2 - 3*a*b*f*(2*C*e + B*f
) + b^2*(C*e^2 + f*(2*B*e + A*f)))*Sqrt[a - b*x]*Sqrt[a + b*x]*(Sqrt[a - b*
x]*(4*a + b*x)*Sqrt[1 + (b*x)/a] + 6*a^(3/2)*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*
Sqrt[a])]) - 4*f*(2*b*C*e + b*B*f - 4*a*C*f)*Sqrt[a - b*x]*Sqrt[a + b*x]*(S
qrt[a - b*x]*Sqrt[1 + (b*x)/a]*(22*a^2 + 9*a*b*x + 2*b^2*x^2) + 30*a^(5/2)*
ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]) - C*f^2*Sqrt[a + b*x]*((a - b*x)*S
```

```
qrt[1 + (b*x)/a]*(160*a^3 + 81*a^2*b*x + 32*a*b^2*x^2 + 6*b^3*x^3) + 210*a^
(7/2)*Sqrt[a - b*x]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]) - 48*(A*b^2 +
a*(-(b*B) + a*C))*(b*e - a*f)^2*Sqrt[a - b*x]*Sqrt[1 + (b*x)/a]*ArcTan[Sqrt
[a - b*x]/Sqrt[a + b*x]])/(24*b^5*Sqrt[c*(a - b*x)]*Sqrt[1 + (b*x)/a])
```

IntegrateAlgebraic [B] time = 0.82, size = 1213, normalized size = 3.30

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e + f*x)^2*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c
- b*c*x]),x]
```

```
[Out] ((-24*a*b^3*B*c^3*e^2*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] - (12*a^2*b^2*c^3*C*
e^2*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] - (48*a*A*b^3*c^3*e*f*Sqrt[a*c - b*c*x
])/Sqrt[a + b*x] - (24*a^2*b^2*B*c^3*e*f*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] -
 (48*a^3*b*c^3*C*e*f*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] - (12*a^2*A*b^2*c^3*f
^2*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] - (24*a^3*b*B*c^3*f^2*Sqrt[a*c - b*c*x]
)/Sqrt[a + b*x] - (15*a^4*c^3*C*f^2*Sqrt[a*c - b*c*x])/Sqrt[a + b*x] - (72*
a*b^3*B*c^2*e^2*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (12*a^2*b^2*c^2*C*e^
2*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (144*a*A*b^3*c^2*e*f*(a*c - b*c*x)
^(3/2))/(a + b*x)^(3/2) - (24*a^2*b^2*B*c^2*e*f*(a*c - b*c*x)^(3/2))/(a + b
*x)^(3/2) - (80*a^3*b*c^2*C*e*f*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (12*
a^2*A*b^2*c^2*f^2*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) - (40*a^3*b*B*c^2*f^
2*(a*c - b*c*x)^(3/2))/(a + b*x)^(3/2) + (9*a^4*c^2*C*f^2*(a*c - b*c*x)^(3/
2))/(a + b*x)^(3/2) - (72*a*b^3*B*c*e^2*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2
) + (12*a^2*b^2*c*C*e^2*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (144*a*A*b^3
*c*e*f*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) + (24*a^2*b^2*B*c*e*f*(a*c - b*
c*x)^(5/2))/(a + b*x)^(5/2) - (80*a^3*b*c*C*e*f*(a*c - b*c*x)^(5/2))/(a + b
*x)^(5/2) + (12*a^2*A*b^2*c*f^2*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (40*
a^3*b*B*c*f^2*(a*c - b*c*x)^(5/2))/(a + b*x)^(5/2) - (9*a^4*c*C*f^2*(a*c -
b*c*x)^(5/2))/(a + b*x)^(5/2) - (24*a*b^3*B*e^2*(a*c - b*c*x)^(7/2))/(a + b
*x)^(7/2) + (12*a^2*b^2*C*e^2*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) - (48*a*
A*b^3*e*f*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) + (24*a^2*b^2*B*e*f*(a*c - b
*c*x)^(7/2))/(a + b*x)^(7/2) - (48*a^3*b*C*e*f*(a*c - b*c*x)^(7/2))/(a + b*
x)^(7/2) + (12*a^2*A*b^2*f^2*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) - (24*a^3
*b*B*f^2*(a*c - b*c*x)^(7/2))/(a + b*x)^(7/2) + (15*a^4*C*f^2*(a*c - b*c*x)
^(7/2))/(a + b*x)^(7/2))/(12*b^5*(c + (a*c - b*c*x)/(a + b*x))^4) + ((-8*A*
b^4*e^2 - 4*a^2*b^2*C*e^2 - 8*a^2*b^2*B*e*f - 4*a^2*A*b^2*f^2 - 3*a^4*C*f^2
)*ArcTan[Sqrt[a*c - b*c*x]/(Sqrt[c]*Sqrt[a + b*x])])/(4*b^5*Sqrt[c])
```

fricas [A] time = 1.22, size = 482, normalized size = 1.31

$$\left[-\frac{3\left(8Ba^2b^2ef + 4\left(Ca^2b^2 + 2Ab^4\right)e^2 + \left(3Ca^4 + 4Aa^2b^2\right)f^2\right)\sqrt{-c}\log\left(2b^2cx^2 - 2\sqrt{-bcx + ac}\sqrt{bx + a}\,b\sqrt{-c}x - a^2c\right) + 2\left(6Cb^3f^2x^3 + 24Bb^3e^2 + 16Ba^2bf^2 + 16\left(2Ca^2b + 3Ab^3\right)ef + 8\left(2Cb^3ef + Bb^3f^2\right)x^2 + 3\left(4Cb^3e^2 + 8Bb^3ef + \left(3Ca^2b + 4Ab^3\right)f^2\right)x\right)\sqrt{-bcx + ac}\sqrt{bx + a}}{48b^5c},\ -\frac{3\left(8Ba^2b^2ef + 4\left(Ca^2b^2 + 2Ab^4\right)e^2 + \left(3Ca^4 + 4Aa^2b^2\right)f^2\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx + ac}\sqrt{bx + a}\,b\sqrt{c}x}{b^2cx^2 - a^2c}\right) + \left(6Cb^3f^2x^3 + 24Bb^3e^2 + 16Ba^2bf^2 + 16\left(2Ca^2b + 3Ab^3\right)ef + 8\left(2Cb^3ef + Bb^3f^2\right)x^2 + 3\left(4Cb^3e^2 + 8Bb^3ef + \left(3Ca^2b + 4Ab^3\right)f^2\right)x\right)\sqrt{-bcx + ac}\sqrt{bx + a}}{24b^5c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/48*(3*(8*B*a^2*b^2*e*f + 4*(C*a^2*b^2 + 2*A*b^4)*e^2 + (3*C*a^4 + 4*A*a
^2*b^2)*f^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*
b*sqrt(-c)*x - a^2*c) + 2*(6*C*b^3*f^2*x^3 + 24*B*b^3*e^2 + 16*B*a^2*b*f^2
+ 16*(2*C*a^2*b + 3*A*b^3)*e*f + 8*(2*C*b^3*e*f + B*b^3*f^2)*x^2 + 3*(4*C*b
^3*e^2 + 8*B*b^3*e*f + (3*C*a^2*b + 4*A*b^3)*f^2)*x)*sqrt(-b*c*x + a*c)*sqr
t(b*x + a))/(b^5*c), -1/24*(3*(8*B*a^2*b^2*e*f + 4*(C*a^2*b^2 + 2*A*b^4)*e^
2 + (3*C*a^4 + 4*A*a^2*b^2)*f^2)*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x
 + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (6*C*b^3*f^2*x^3 + 24*B*b^3*e^2 +
16*B*a^2*b*f^2 + 16*(2*C*a^2*b + 3*A*b^3)*e*f + 8*(2*C*b^3*e*f + B*b^3*f^2)
*x^2 + 3*(4*C*b^3*e^2 + 8*B*b^3*e*f + (3*C*a^2*b + 4*A*b^3)*f^2)*x)*sqrt(-b
*c*x + a*c)*sqrt(b*x + a))/(b^5*c)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 635, normalized size = 1.73

$$\frac{\sqrt{bx+a}\,\sqrt{-\left(bx-a\right)c}\,\left(12A\,a^{2}b^{2}c\,f^{2}\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}}\right)+24A\,b^{4}c\,e^{2}\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}}\right)+24B\,a^{2}b^{2}cef\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}}\right)+9C\,a^{4}c\,f^{2}\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}}\right)+12C\,a^{2}b^{2}c\,e^{2}\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}}\right)-6\sqrt{b^{2}c}\,\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,C\,b^{2}f^{2}x^{3}-8\sqrt{b^{2}c}\,\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,B\,b^{2}f^{2}x^{2}-16\sqrt{b^{2}c}\,\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,C\,b^{2}ef\,x^{2}-12\sqrt{b^{2}c}\,\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,A\,b^{2}f^{2}x-24\sqrt{b^{2}c}\,\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,B\,b^{2}efx-9\sqrt{b^{2}c}\,\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,C\,a^{2}f^{2}x-12\sqrt{b^{2}c}\,\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,C\,b^{2}e^{2}x-48\sqrt{b^{2}c}\,\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,A\,b^{2}ef-16\sqrt{b^{2}c}\,\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,B\,a^{2}f^{2}-24\sqrt{b^{2}c}\,\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,B\,b^{2}e^{2}-32\sqrt{b^{2}c}\,\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,C\,a^{2}ef\right)}{24\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,\sqrt{b^{2}c}\,b^{4}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/24*(b*x+a)^(1/2)*(-(b*x-a)*c)^(1/2)/c*(-6*C*x^3*b^2*f^2*(b^2*c)^(1/2)*(-(
b^2*x^2-a^2)*c)^(1/2)+12*A*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)
*a^2*b^2*c*f^2+24*A*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*b^4*c*
e^2+24*B*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^2*b^2*c*e*f-8*B
*x^2*b^2*f^2*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)+9*C*arctan((b^2*c)^(1/2
)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^4*c*f^2+12*C*arctan((b^2*c)^(1/2)/(-(b^2*x^
2-a^2)*c)^(1/2)*x)*a^2*b^2*c*e^2-16*C*x^2*b^2*e*f*(b^2*c)^(1/2)*(-(b^2*x^2-
a^2)*c)^(1/2)-12*A*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*b^2*f^2-24*B*(b
^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*b^2*e*f-9*C*(b^2*c)^(1/2)*(-(b^2*x^2
-a^2)*c)^(1/2)*x*a^2*f^2-12*C*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*x*b^2*
e^2-48*A*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*b^2*e*f-16*B*(b^2*c)^(1/2)*
(-(b^2*x^2-a^2)*c)^(1/2)*a^2*f^2-24*B*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2
)*b^2*e^2-32*C*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*a^2*e*f)/b^4/(-(b^2*x
^2-a^2)*c)^(1/2)/(b^2*c)^(1/2)
```

maxima [A] time = 2.02, size = 317, normalized size = 0.86

$$-\frac{\sqrt{-b^{2}cx^{2}+a^{2}c}\,Cf^{2}x^{3}}{4\,b^{2}c}+\frac{Ae^{2}\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}}+\frac{3\,Ca^{4}f^{2}\arcsin\left(\frac{bx}{a}\right)}{8\,b^{5}\sqrt{c}}-\frac{3\sqrt{-b^{2}cx^{2}+a^{2}c}\,Ca^{2}f^{2}x}{8\,b^{4}c}-\frac{\sqrt{-b^{2}cx^{2}+a^{2}c}\,Be^{2}}{b^{2}c}-\frac{2\sqrt{-b^{2}cx^{2}+a^{2}c}\,Aef}{b^{2}c}-\frac{\sqrt{-b^{2}cx^{2}+a^{2}c}\left(2\,Cef+Bf^{2}\right)x^{2}}{3\,b^{2}c}+\frac{\left(Ce^{2}+2\,Bef+Af^{2}\right)a^{2}\arcsin\left(\frac{bx}{a}\right)}{2\,b^{3}\sqrt{c}}-\frac{\sqrt{-b^{2}cx^{2}+a^{2}c}\left(Ce^{2}+2\,Bef+Af^{2}\right)x}{2\,b^{2}c}-\frac{2\sqrt{-b^{2}cx^{2}+a^{2}c}\left(2\,Cef+Bf^{2}\right)a^{2}}{3\,b^{4}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*sqrt(-b^2*c*x^2 + a^2*c)*C*f^2*x^3/(b^2*c) + A*e^2*arcsin(b*x/a)/(b*sq
rt(c)) + 3/8*C*a^4*f^2*arcsin(b*x/a)/(b^5*sqrt(c)) - 3/8*sqrt(-b^2*c*x^2 +
a^2*c)*C*a^2*f^2*x/(b^4*c) - sqrt(-b^2*c*x^2 + a^2*c)*B*e^2/(b^2*c) - 2*sqr
t(-b^2*c*x^2 + a^2*c)*A*e*f/(b^2*c) - 1/3*sqrt(-b^2*c*x^2 + a^2*c)*(2*C*e*f
 + B*f^2)*x^2/(b^2*c) + 1/2*(C*e^2 + 2*B*e*f + A*f^2)*a^2*arcsin(b*x/a)/(b^
3*sqrt(c)) - 1/2*sqrt(-b^2*c*x^2 + a^2*c)*(C*e^2 + 2*B*e*f + A*f^2)*x/(b^2*
c) - 2/3*sqrt(-b^2*c*x^2 + a^2*c)*(2*C*e*f + B*f^2)*a^2/(b^4*c)
```

mupad [B] time = 81.65, size = 2799, normalized size = 7.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^2*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x
)
```

```
[Out] - ((a^(1/2)*(a*c)^(1/2)*(64*B*a^2*c*f^2 + 32*B*b^2*c*e^2)*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^8)/(b^4*((a + b*x)^(1/2) - a^(1/2))^8) + (a^(1/2)*(a*c)^
(1/2)*(64*B*a^2*c^3*f^2 + 32*B*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1
```

```
/2))^4)/(b^4*((a + b*x)^(1/2) - a^(1/2))^4) - (a^(1/2)*(a*c)^(1/2)*((128*B*
a^2*c^2*f^2)/3 - 48*B*b^2*c^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/(
b^4*((a + b*x)^(1/2) - a^(1/2))^6) + (4*B*a^2*e*f*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^11)/(b^3*((a + b*x)^(1/2) - a^(1/2))^11) + (8*B*a^(1/2)*e^2*(a*c
)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/(b^2*((a + b*x)^(1/2) - a^(
1/2))^10) + (20*B*a^2*c^4*e*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b^3*(
(a + b*x)^(1/2) - a^(1/2))^3) + (24*B*a^2*c^3*e*f*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2))^5)/(b^3*((a + b*x)^(1/2) - a^(1/2))^5) - (24*B*a^2*c^2*e*f*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b^3*((a + b*x)^(1/2) - a^(1/2))^7) + (8*
B*a^(1/2)*c^4*e^2*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*(
(a + b*x)^(1/2) - a^(1/2))^2) - (4*B*a^2*c^5*e*f*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2)))/(b^3*((a + b*x)^(1/2) - a^(1/2))) - (20*B*a^2*c*e*f*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^9)/(b^3*((a + b*x)^(1/2) - a^(1/2))^9))/(((a*c - b*
c*x)^(1/2) - (a*c)^(1/2))^12/((a + b*x)^(1/2) - a^(1/2))^12 + c^6 + (6*c*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (6*c
^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (
15*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4
 + (20*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2
))^6 + (15*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^
(1/2))^8) - ((2*A*a^2*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b^3*((a +
 b*x)^(1/2) - a^(1/2))^7) + (14*A*a^2*c^2*f^2*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^3)/(b^3*((a + b*x)^(1/2) - a^(1/2))^3) - (2*A*a^2*c^3*f^2*((a*c - b*
c*x)^(1/2) - (a*c)^(1/2)))/(b^3*((a + b*x)^(1/2) - a^(1/2))) - (14*A*a^2*c*
f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b^3*((a + b*x)^(1/2) - a^(1/2))
^5) + (16*A*a^(1/2)*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/
(b^2*((a + b*x)^(1/2) - a^(1/2))^6) + (32*A*a^(1/2)*c*e*f*(a*c)^(1/2)*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^2*((a + b*x)^(1/2) - a^(1/2))^4) + (16
*A*a^(1/2)*c^2*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*
((a + b*x)^(1/2) - a^(1/2))^2))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8/((a
+ b*x)^(1/2) - a^(1/2))^8 + c^4 + (4*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
6)/((a + b*x)^(1/2) - a^(1/2))^6 + (4*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4) - ((((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^5*((333*C*a^4*c^5*f^2)/2 + 30*C*a^2*b^2*c^5*e^2))/(b^5*((a + b*x)^(1/2
) - a^(1/2))^5) - (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*((23*C*a^4*c^6*f^2
)/2 - 6*C*a^2*b^2*c^6*e^2))/(b^5*((a + b*x)^(1/2) - a^(1/2))^3) - (((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))*((3*C*a^4*c^7*f^2)/2 + 2*C*a^2*b^2*c^7*e^2))/(b
^5*((a + b*x)^(1/2) - a^(1/2))) - (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^11*(
(333*C*a^4*c^2*f^2)/2 + 30*C*a^2*b^2*c^2*e^2))/(b^5*((a + b*x)^(1/2) - a^(1
/2))^11) - (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7*((671*C*a^4*c^4*f^2)/2 -
22*C*a^2*b^2*c^4*e^2))/(b^5*((a + b*x)^(1/2) - a^(1/2))^7) + (((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^9*((671*C*a^4*c^3*f^2)/2 - 22*C*a^2*b^2*c^3*e^2))/(b
^5*((a + b*x)^(1/2) - a^(1/2))^9) + (((23*C*a^4*c*f^2)/2 - 6*C*a^2*b^2*c*e^
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^13)/(b^5*((a + b*x)^(1/2) - a^(1/2))
^13) + (((3*C*a^4*f^2)/2 + 2*C*a^2*b^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^15)/(b^5*((a + b*x)^(1/2) - a^(1/2))^15) + (128*C*a^(5/2)*c*e*f*(a*c)^
(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^12)/(b^4*((a + b*x)^(1/2) - a^(1/
2))^12) + (128*C*a^(5/2)*c^5*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^4)/(b^4*((a + b*x)^(1/2) - a^(1/2))^4) + (512*C*a^(5/2)*c^4*e*f*(a*c)
^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/(3*b^4*((a + b*x)^(1/2) - a^(
1/2))^6) + (256*C*a^(5/2)*c^3*e*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^8)/(3*b^4*((a + b*x)^(1/2) - a^(1/2))^8) + (512*C*a^(5/2)*c^2*e*f*(a
*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^10)/(3*b^4*((a + b*x)^(1/2) -
 a^(1/2))^10))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^16/((a + b*x)^(1/2) - a
^(1/2))^16 + c^8 + (8*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^14)/((a + b*x)^
(1/2) - a^(1/2))^14 + (8*c^7*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b
*x)^(1/2) - a^(1/2))^2 + (28*c^6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a
 + b*x)^(1/2) - a^(1/2))^4 + (56*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)
/((a + b*x)^(1/2) - a^(1/2))^6 + (70*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)
```

```
)^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (56*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^10)/((a + b*x)^(1/2) - a^(1/2))^10 + (28*c^2*((a*c - b*c*x)^(1/2) - (
a*c)^(1/2))^12)/((a + b*x)^(1/2) - a^(1/2))^12) - (2*A*atan((A*(a^2*f^2 + 2
*b^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(c^(1/2)*(A*a^2*f^2 + 2*A*b^
2*e^2)*((a + b*x)^(1/2) - a^(1/2))))*(a^2*f^2 + 2*b^2*e^2))/(b^3*c^(1/2)) -
 (C*a^2*atan((C*a^2*(3*a^2*f^2 + 4*b^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2)))/(c^(1/2)*(3*C*a^4*f^2 + 4*C*a^2*b^2*e^2)*((a + b*x)^(1/2) - a^(1/2)))
)*(3*a^2*f^2 + 4*b^2*e^2))/(2*b^5*c^(1/2)) - (4*B*a^2*e*f*atan(((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b^3*c^(1/2
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

3.29 $\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx$

Optimal. Leaf size=246

$$-\frac{\left(a^2-b^2x^2\right)\left(2\left(2a^2Cf^2-b^2\left(Ce^2-3f(Af+Be)\right)\right)-b^2fx(Ce-3Bf)\right)}{6b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{\sqrt{a^2c-b^2cx^2}\,\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)\left(a^2\right.}{2b^3\sqrt{c}\,\sqrt{a+bx}\,\sqrt{ac-\right.}$$

Rubi [A] time = 0.40, antiderivative size = 249, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1610, 1654, 780, 217, 203}

$$-\frac{\left(a^2-b^2x^2\right)\left(2\left(2a^2Cf^2-\frac{1}{2}b^2\left(2Ce^2-6f(Af+Be)\right)\right)-b^2fx(Ce-3Bf)\right)}{6b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{\sqrt{a^2c-b^2cx^2}\,\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)\left(a^2(Bf+Ce)+2Ab^2e\right)}{2b^3\sqrt{c}\,\sqrt{a+bx}\,\sqrt{ac-bcx}}-\frac{C\left(a^2-b^2x^2\right)(e+fx)^2}{3b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] -(C*(e + f*x)^2*(a^2 - b^2*x^2))/(3*b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
- ((2*(2*a^2*C*f^2 - (b^2*(2*C*e^2 - 6*f*(B*e + A*f)))/2) - b^2*f*(C*e - 3*
B*f)*x)*(a^2 - b^2*x^2))/(6*b^4*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((2*A*
b^2*e + a^2*(C*e + B*f))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[
a^2*c - b^2*c*x^2]])/(2*b^3*Sqrt[c]*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
```

```
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx}\,\sqrt{ac-bcx}}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)\left(A+Bx+Cx^2\right)}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^2\left(a^2-b^2x^2\right)}{3b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{(e+fx)\left(-c\left(3Ab^2+2a^2C\right)f^2+b^2cf(Ce-3Bf)x\right)}{\sqrt{a^2c-b^2cx^2}} \cdots}{3b^2cf^2\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^2\left(a^2-b^2x^2\right)}{3b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\left(2\left(2a^2Cf^2-\frac{1}{2}b^2\left(2Ce^2-6f(Be+Af)\right)\right)-b^2f(C \cdots}{6b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^2\left(a^2-b^2x^2\right)}{3b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\left(2\left(2a^2Cf^2-\frac{1}{2}b^2\left(2Ce^2-6f(Be+Af)\right)\right)-b^2f(C \cdots}{6b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= -\frac{C(e+fx)^2\left(a^2-b^2x^2\right)}{3b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\left(2\left(2a^2Cf^2-\frac{1}{2}b^2\left(2Ce^2-6f(Be+Af)\right)\right)-b^2f(C \cdots}{6b^4f\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Mathematica [A] time = 1.43, size = 390, normalized size = 1.59

$$-\frac{3\sqrt{a-bx}\,\sqrt{a+bx}\left(6a^{3/2}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\,\sqrt{a}}\right)+\sqrt{a-bx}\,(4a+bx)\sqrt{\frac{bx}{a}+1}\right)(-3aCf+bBf+bCe)+6\sqrt{a-bx}\,\sqrt{a+bx}\left(\sqrt{a-bx}\,\sqrt{\frac{bx}{a}+1}+2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\,\sqrt{a}}\right)\right)\left(3a^2Cf-2ab(Bf+Ce)+b^2(Af+Be)\right)+Cf\sqrt{a+bx}\left(30a^{5/2}\sqrt{a-bx}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\,\sqrt{a}}\right)+(a-bx)\sqrt{\frac{bx}{a}+1}\left(22a^2+9abx+2b^2x^2\right)\right)+12\sqrt{a-bx}\,\sqrt{\frac{bx}{a}+1}\,(be-af)\tan^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{a+bx}}\right)\left(a(aC-bB)+Ab^2\right)}{6b^4\sqrt{\frac{bx}{a}+1}\,\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x
]
```

```
[Out] -1/6*(6*(3*a^2*C*f + b^2*(B*e + A*f) - 2*a*b*(C*e + B*f))*Sqrt[a - b*x]*Sqr
t[a + b*x]*(Sqrt[a - b*x]*Sqrt[1 + (b*x)/a] + 2*Sqrt[a]*ArcSin[Sqrt[a - b*x
]/(Sqrt[2]*Sqrt[a])]) + 3*(b*C*e + b*B*f - 3*a*C*f)*Sqrt[a - b*x]*Sqrt[a +
b*x]*(Sqrt[a - b*x]*(4*a + b*x)*Sqrt[1 + (b*x)/a] + 6*a^(3/2)*ArcSin[Sqrt[a
 - b*x]/(Sqrt[2]*Sqrt[a])]) + C*f*Sqrt[a + b*x]*((a - b*x)*Sqrt[1 + (b*x)/a
]*(22*a^2 + 9*a*b*x + 2*b^2*x^2) + 30*a^(5/2)*Sqrt[a - b*x]*ArcSin[Sqrt[a -
 b*x]/(Sqrt[2]*Sqrt[a])]) + 12*(A*b^2 + a*(-(b*B) + a*C))*(b*e - a*f)*Sqrt[
a - b*x]*Sqrt[1 + (b*x)/a]*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x]])/(b^4*Sqrt[c
*(a - b*x)]*Sqrt[1 + (b*x)/a])
```

IntegrateAlgebraic [A] time = 0.41, size = 356, normalized size = 1.45

$$\frac{\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\,\sqrt{a+bx}}\right)\left(-a^2Bf+a^2(-C)e-2Ab^2e\right)}{b^3\sqrt{c}}-\frac{a\sqrt{ac-bcx}\left(\frac{6a^2Cf(ac-bcx)^2}{(a+bx)^2}+\frac{4a^2cCf(ac-bcx)}{a+bx}+6a^2c^2Cf+\frac{6Ab^2f(ac-bcx)^2}{(a+bx)^2}+\frac{12Ab^2cf(ac-bcx)}{a+bx}+\frac{6b^2Be(ac-bcx)^2}{(a+bx)^2}+\frac{12b^2Bce(ac-bcx)}{a+bx}+3abBc^2f-\frac{3abBf(ac-bcx)^2}{(a+bx)^2}+3abc^2Ce-\frac{3abCe(ac-bcx)^2}{(a+bx)^2}+6Ab^2c^2f+6b^2Bc^2e\right)}{3b^4\sqrt{a+bx}\left(\frac{ac-bcx}{a+bx}+c\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((e + f*x)*(A + B*x + C*x^2))/(Sqrt[a + b*x]*Sqrt[a*c -
b*c*x]),x]
```

```
[Out] -1/3*(a*Sqrt[a*c - b*c*x]*(6*b^2*B*c^2*e + 3*a*b*c^2*C*e + 6*A*b^2*c^2*f +
3*a*b*B*c^2*f + 6*a^2*c^2*C*f + (12*b^2*B*c*e*(a*c - b*c*x))/(a + b*x) + (1
2*A*b^2*c*f*(a*c - b*c*x))/(a + b*x) + (4*a^2*c*C*f*(a*c - b*c*x))/(a + b*x
) + (6*b^2*B*e*(a*c - b*c*x)^2)/(a + b*x)^2 - (3*a*b*C*e*(a*c - b*c*x)^2)/(
a + b*x)^2 + (6*A*b^2*f*(a*c - b*c*x)^2)/(a + b*x)^2 - (3*a*b*B*f*(a*c - b*
```

```
c*x)^2)/(a + b*x)^2 + (6*a^2*C*f*(a*c - b*c*x)^2)/(a + b*x)^2))/(b^4*Sqrt[a
 + b*x]*(c + (a*c - b*c*x)/(a + b*x))^3) + ((-2*A*b^2*e - a^2*C*e - a^2*B*f
)*ArcTan[Sqrt[a*c - b*c*x]/(Sqrt[c]*Sqrt[a + b*x])])/(b^3*Sqrt[c])
```

fricas [A] time = 0.71, size = 302, normalized size = 1.23

$$\left[-\frac{3\left(Ba^2bf+\left(Ca^2b+2\,Ab^3\right)e\right)\sqrt{-c}\,\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\left(2\,Cb^2fx^2+6\,Bb^2e+2\left(2\,Ca^2+3\,Ab^2\right)f+3\left(Cb^2e+Bb^2f\right)x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{12\,b^4c},\ -\frac{3\left(Ba^2bf+\left(Ca^2b+2\,Ab^3\right)e\right)\sqrt{c}\,\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)+\left(2\,Cb^2fx^2+6\,Bb^2e+2\left(2\,Ca^2+3\,Ab^2\right)f+3\left(Cb^2e+Bb^2f\right)x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{6\,b^4c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/12*(3*(B*a^2*b*f + (C*a^2*b + 2*A*b^3)*e)*sqrt(-c)*log(2*b^2*c*x^2 - 2*
sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(2*C*b^2*f*x^2 +
 6*B*b^2*e + 2*(2*C*a^2 + 3*A*b^2)*f + 3*(C*b^2*e + B*b^2*f)*x)*sqrt(-b*c*x
 + a*c)*sqrt(b*x + a))/(b^4*c), -1/6*(3*(B*a^2*b*f + (C*a^2*b + 2*A*b^3)*e)
*sqrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a
^2*c)) + (2*C*b^2*f*x^2 + 6*B*b^2*e + 2*(2*C*a^2 + 3*A*b^2)*f + 3*(C*b^2*e
+ B*b^2*f)*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/(b^4*c)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 365, normalized size = 1.48

$$\frac{\sqrt{bx+a}\,\sqrt{-(bx-a)\,c}\,\left(6A\,b^4ce\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-\left(b^2x^2-a^2\right)c}}\right)+3B\,a^2b^2cf\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-\left(b^2x^2-a^2\right)c}}\right)+3C\,a^2b^2ce\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-\left(b^2x^2-a^2\right)c}}\right)-2\sqrt{-\left(b^2x^2-a^2\right)c}\,\sqrt{b^2c}\,C\,b^2fx^2-3\sqrt{-\left(b^2x^2-a^2\right)c}\,\sqrt{b^2c}\,B\,b^2fx-3\sqrt{-\left(b^2x^2-a^2\right)c}\,\sqrt{b^2c}\,C\,b^2ex-6\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,A\,b^2f-6\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,B\,b^2e-4\sqrt{b^2c}\,\sqrt{-\left(b^2x^2-a^2\right)c}\,C\,a^2f\right)}{6\sqrt{-\left(b^2x^2-a^2\right)c}\,\sqrt{b^2c}\,b^4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/6*(b*x+a)^(1/2)*(-(b*x-a)*c)^(1/2)/c*(6*A*arctan((b^2*c)^(1/2)/(-(b^2*x^2
-a^2)*c)^(1/2)*x)*b^4*c*e+3*B*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)
*x)*a^2*b^2*c*f+3*C*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)*a^2*b^
2*c*e-2*C*x^2*b^2*f*(-(b^2*x^2-a^2)*c)^(1/2)*(b^2*c)^(1/2)-3*B*(-(b^2*x^2-a
^2)*c)^(1/2)*(b^2*c)^(1/2)*x*b^2*f-3*C*(-(b^2*x^2-a^2)*c)^(1/2)*(b^2*c)^(1/
2)*x*b^2*e-6*A*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*b^2*f-6*B*(b^2*c)^(1/
2)*(-(b^2*x^2-a^2)*c)^(1/2)*b^2*e-4*C*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2
)*a^2*f)/(-(b^2*x^2-a^2)*c)^(1/2)/b^4/(b^2*c)^(1/2)
```

maxima [A] time = 2.05, size = 189, normalized size = 0.77

$$-\frac{\sqrt{-b^2cx^2+a^2c}\,Cfx^2}{3\,b^2c}+\frac{Ae\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}}+\frac{\left(Ce+Bf\right)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3\sqrt{c}}-\frac{\sqrt{-b^2cx^2+a^2c}\,Be}{b^2c}-\frac{2\,\sqrt{-b^2cx^2+a^2c}\,Ca^2f}{3\,b^4c}-\frac{\sqrt{-b^2cx^2+a^2c}\,Af}{b^2c}-\frac{\sqrt{-b^2cx^2+a^2c}\,\left(Ce+Bf\right)x}{2\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -1/3*sqrt(-b^2*c*x^2 + a^2*c)*C*f*x^2/(b^2*c) + A*e*arcsin(b*x/a)/(b*sqrt(c
)) + 1/2*(C*e + B*f)*a^2*arcsin(b*x/a)/(b^3*sqrt(c)) - sqrt(-b^2*c*x^2 + a^
2*c)*B*e/(b^2*c) - 2/3*sqrt(-b^2*c*x^2 + a^2*c)*C*a^2*f/(b^4*c) - sqrt(-b^2
```

```
*c*x^2 + a^2*c)*A*f/(b^2*c) - 1/2*sqrt(-b^2*c*x^2 + a^2*c)*(C*e + B*f)*x/(b
^2*c)
```

mupad [B] time = 30.74, size = 1011, normalized size = 4.11

$$-\frac{\frac{2Ba^2f\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^7}{\left(\sqrt{a+bx}-\sqrt{a}\right)^7}-\frac{2Ba^2c^3f\left(\sqrt{ac-bcx}-\sqrt{ac}\right)}{\sqrt{a+bx}-\sqrt{a}}-\frac{14Ba^2cf\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^5}{\left(\sqrt{a+bx}-\sqrt{a}\right)^5}+\frac{14Ba^2c^2f\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^3}{\left(\sqrt{a+bx}-\sqrt{a}\right)^3}}{b^3c^4+\frac{b^3\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^8}{\left(\sqrt{a+bx}-\sqrt{a}\right)^8}+\frac{4b^3c^3\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^2}{\left(\sqrt{a+bx}-\sqrt{a}\right)^2}+\frac{6b^3c^2\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^4}{\left(\sqrt{a+bx}-\sqrt{a}\right)^4}+\frac{4b^3c\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^6}{\left(\sqrt{a+bx}-\sqrt{a}\right)^6}}-\frac{\frac{2Ca^2e\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^7}{\left(\sqrt{a+bx}-\sqrt{a}\right)^7}-\frac{2Ca^2c^3e\left(\sqrt{ac-bcx}-\sqrt{ac}\right)}{\sqrt{a+bx}-\sqrt{a}}-\frac{14Ca^2ce\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^5}{\left(\sqrt{a+bx}-\sqrt{a}\right)^5}+\frac{14Ca^2c^2e\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^3}{\left(\sqrt{a+bx}-\sqrt{a}\right)^3}}{b^3c^4+\frac{b^3\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^8}{\left(\sqrt{a+bx}-\sqrt{a}\right)^8}+\frac{4b^3c^3\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^2}{\left(\sqrt{a+bx}-\sqrt{a}\right)^2}+\frac{6b^3c^2\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^4}{\left(\sqrt{a+bx}-\sqrt{a}\right)^4}+\frac{4b^3c\left(\sqrt{ac-bcx}-\sqrt{ac}\right)^6}{\left(\sqrt{a+bx}-\sqrt{a}\right)^6}}-\frac{\sqrt{ac-bcx}\,\left(\frac{2Ca^3f}{3b^4c}+\frac{Cfx^3}{3bc}+\frac{Cafx^2}{3b^2c}+\frac{2Ca^2fx}{3b^3c}\right)}{\sqrt{a+bx}}-\frac{4Ae\,\mathrm{atan}\left(\frac{b\left(\sqrt{ac-bcx}-\sqrt{ac}\right)}{\sqrt{b^2c}\,\left(\sqrt{a+bx}-\sqrt{a}\right)}\right)}{\sqrt{b^2c}}-\frac{Af\sqrt{ac-bcx}\,\sqrt{a+bx}}{b^2c}-\frac{Be\sqrt{ac-bcx}\,\sqrt{a+bx}}{b^2c}-\frac{2Ba^2f\,\mathrm{atan}\left(\frac{\sqrt{ac-bcx}-\sqrt{ac}}{\sqrt{c}\,\left(\sqrt{a+bx}-\sqrt{a}\right)}\right)}{b^3\sqrt{c}}-\frac{2Ca^2e\,\mathrm{atan}\left(\frac{\sqrt{ac-bcx}-\sqrt{ac}}{\sqrt{c}\,\left(\sqrt{a+bx}-\sqrt{a}\right)}\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(A + B*x + C*x^2))/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] - ((2*B*a^2*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) - a^(
1/2))^7 - (2*B*a^2*c^3*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1
/2) - a^(1/2)) - (14*B*a^2*c*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/((a +
 b*x)^(1/2) - a^(1/2))^5 + (14*B*a^2*c^2*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^3)/((a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^2*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (4*b^3*c*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6) - ((2*C
*a^2*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) - a^(1/2))^7
 - (2*C*a^2*c^3*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a
^(1/2)) - (14*C*a^2*c*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/((a + b*x)^(
1/2) - a^(1/2))^5 + (14*C*a^2*c^2*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/
((a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^2*((a*c - b*c*x)
^(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (4*b^3*c*((a*c - b
*c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6) - ((a*c - b*c*
x)^(1/2)*((2*C*a^3*f)/(3*b^4*c) + (C*f*x^3)/(3*b*c) + (C*a*f*x^2)/(3*b^2*c)
 + (2*C*a^2*f*x)/(3*b^3*c)))/(a + b*x)^(1/2) - (4*A*e*atan((b*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2)))/((b^2*c)^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b^2*
c)^(1/2) - (A*f*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/(b^2*c) - (B*e*(a*c -
b*c*x)^(1/2)*(a + b*x)^(1/2))/(b^2*c) - (2*B*a^2*f*atan(((a*c - b*c*x)^(1/2
) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b^3*c^(1/2)) - (2
*C*a^2*e*atan(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2)
 - a^(1/2)))))/(b^3*c^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

3.30 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}} dx$

Optimal. Leaf size=177

$$\frac{(a^2C+2Ab^2)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{B(a^2-b^2x^2)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

Rubi [A] time = 0.12, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {901, 1815, 641, 217, 203}

$$\frac{(a^2C+2Ab^2)\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{B(a^2-b^2x^2)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{Cx(a^2-b^2x^2)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] -((B*(a^2 - b^2*x^2))/(b^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - (C*x*(a^2 -
b^2*x^2))/(2*b^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((2*A*b^2 + a^2*C)*Sqrt
[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(2*b^3*S
qrt[c]*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 901

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^Fr
acPart[m])/(d*f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{Cx\left(a^2-b^2x^2\right)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{-c\left(2Ab^2+a^2C\right)-2b^2Bcx}{\sqrt{a^2c-b^2cx^2}}\,dx}{2b^2c\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= -\frac{B\left(a^2-b^2x^2\right)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{Cx\left(a^2-b^2x^2\right)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\left(\left(2Ab^2+a^2C\right)\sqrt{a^2c-b^2cx^2}\right)\int}{2b^2\sqrt{a+bx}\sqrt{ac-bc}}$$

$$= -\frac{B\left(a^2-b^2x^2\right)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{Cx\left(a^2-b^2x^2\right)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\left(\left(2Ab^2+a^2C\right)\sqrt{a^2c-b^2cx^2}\right)\mathrm{S}}{2b^2\sqrt{a+bx}}$$

$$= -\frac{B\left(a^2-b^2x^2\right)}{b^2\sqrt{a+bx}\sqrt{ac-bcx}} - \frac{Cx\left(a^2-b^2x^2\right)}{2b^2\sqrt{a+bx}\sqrt{ac-bcx}} + \frac{\left(2Ab^2+a^2C\right)\sqrt{a^2c-b^2cx^2}\,\tan}{2b^3\sqrt{c}\sqrt{a+bx}\sqrt{ac-}}$$

Mathematica [A] time = 0.44, size = 169, normalized size = 0.95

$$-\frac{\sqrt{a-bx}\left(\sqrt{\frac{bx}{a}+1}\left(4\tan^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{a+bx}}\right)\left(a(aC-bB)+Ab^2\right)+b\sqrt{a-bx}\sqrt{a+bx}(2B+Cx)\right)-2\sqrt{a}\sqrt{a+bx}(aC-2bB)\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)\right)}{2b^3\sqrt{\frac{bx}{a}+1}\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] -1/2*(Sqrt[a - b*x]*(-2*Sqrt[a]*(-2*b*B + a*C)*Sqrt[a + b*x]*ArcSin[Sqrt[a
- b*x]/(Sqrt[2]*Sqrt[a])] + Sqrt[1 + (b*x)/a]*(b*Sqrt[a - b*x]*Sqrt[a + b*x
]*(2*B + C*x) + 4*(A*b^2 + a*(-(b*B) + a*C))*ArcTan[Sqrt[a - b*x]/Sqrt[a +
b*x]])))/(b^3*Sqrt[c*(a - b*x)]*Sqrt[1 + (b*x)/a])
```

IntegrateAlgebraic [A] time = 0.23, size = 150, normalized size = 0.85

$$\frac{\left(a^2(-C)-2Ab^2\right)\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\sqrt{a+bx}}\right)}{b^3\sqrt{c}} + \frac{a\sqrt{ac-bcx}\left(-\frac{2bB(ac-bcx)}{a+bx}+\frac{aC(ac-bcx)}{a+bx}-acC-2bBc\right)}{b^3\sqrt{a+bx}\left(\frac{ac-bcx}{a+bx}+c\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] (a*Sqrt[a*c - b*c*x]*(-2*b*B*c - a*c*C - (2*b*B*(a*c - b*c*x))/(a + b*x) +
(a*C*(a*c - b*c*x))/(a + b*x)))/(b^3*Sqrt[a + b*x]*(c + (a*c - b*c*x)/(a +
b*x))^2) + ((-2*A*b^2 - a^2*C)*ArcTan[Sqrt[a*c - b*c*x]/(Sqrt[c]*Sqrt[a + b
*x])])/(b^3*Sqrt[c])
```

fricas [A] time = 0.77, size = 196, normalized size = 1.11

$$\left[-\frac{\left(Ca^2+2\,Ab^2\right)\sqrt{-c}\log\left(2\,b^2cx^2-2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\,(Cbx+2\,Bb)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{4\,b^3c}, -\frac{\left(Ca^2+2\,Ab^2\right)\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)+(Cbx+2\,Bb)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{2\,b^3c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="fric
as")
```

```
[Out] [-1/4*((C*a^2 + 2*A*b^2)*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sq
rt(b*x + a)*b*sqrt(-c)*x - a^2*c) + 2*(C*b*x + 2*B*b)*sqrt(-b*c*x + a*c)*sq
```

```
rt(b*x + a))/(b^3*c), -1/2*((C*a^2 + 2*A*b^2)*sqrt(c)*arctan(sqrt(-b*c*x +
a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (C*b*x + 2*B*b)*sqrt(
-b*c*x + a*c)*sqrt(b*x + a))/(b^3*c)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 180, normalized size = 1.02

$$\frac{\sqrt{bx+a}\,\sqrt{-\left(bx-a\right)c}\,\left(2A\,b^{2}c\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}}\right)+C\,a^{2}c\arctan\left(\frac{\sqrt{b^{2}c}\,x}{\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}}\right)-\sqrt{b^{2}c}\,\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,Cx-2\sqrt{b^{2}c}\,\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,B\right)}{2\sqrt{-\left(b^{2}x^{2}-a^{2}\right)c}\,\sqrt{b^{2}c}\,b^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] 1/2*(b*x+a)^(1/2)*(-(b*x-a)*c)^(1/2)/b^2*(2*A*arctan((b^2*c)^(1/2)/(-(b^2*x
^2-a^2)*c)^(1/2)*x)*b^2*c+C*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x
)*a^2*c-C*(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*x-2*B*(b^2*c)^(1/2)*(-(b^2
*x^2-a^2)*c)^(1/2))/(-(b^2*x^2-a^2)*c)^(1/2)/c/(b^2*c)^(1/2)
```

maxima [A] time = 2.50, size = 88, normalized size = 0.50

$$\frac{Ca^{2}\arcsin\left(\frac{bx}{a}\right)}{2\,b^{3}\sqrt{c}}+\frac{A\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}}-\frac{\sqrt{-b^{2}cx^{2}+a^{2}c}\,Cx}{2\,b^{2}c}-\frac{\sqrt{-b^{2}cx^{2}+a^{2}c}\,B}{b^{2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 1/2*C*a^2*arcsin(b*x/a)/(b^3*sqrt(c)) + A*arcsin(b*x/a)/(b*sqrt(c)) - 1/2*s
qrt(-b^2*c*x^2 + a^2*c)*C*x/(b^2*c) - sqrt(-b^2*c*x^2 + a^2*c)*B/(b^2*c)
```

mupad [B] time = 14.95, size = 489, normalized size = 2.76

$$-\frac{\frac{2\,C\,a^{2}\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^{7}}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{7}}-\frac{2\,C\,a^{2}\,c^{3}\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{a+b\,x}-\sqrt{a}}-\frac{14\,C\,a^{2}\,c\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^{5}}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{5}}+\frac{14\,C\,a^{2}\,c^{2}\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^{3}}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{3}}}{b^{3}\,c^{4}+\frac{b^{3}\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^{8}}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{8}}+\frac{4\,b^{3}\,c^{3}\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^{2}}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{2}}+\frac{6\,b^{3}\,c^{2}\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^{4}}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{4}}+\frac{4\,b^{3}\,c\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)^{6}}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{6}}}-\frac{4\,A\,\mathrm{atan}\left(\frac{b\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{b^{2}\,c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{b^{2}\,c}}-\frac{2\,C\,a^{2}\,\mathrm{atan}\left(\frac{\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}}{\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{b^{3}\,\sqrt{c}}-\frac{B\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}{b^{2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] - ((2*C*a^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/((a + b*x)^(1/2) - a^(1/
2))^7 - (2*C*a^2*c^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2)
- a^(1/2)) - (14*C*a^2*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/((a + b*x)^
(1/2) - a^(1/2))^5 + (14*C*a^2*c^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(
(a + b*x)^(1/2) - a^(1/2))^3)/(b^3*c^4 + (b^3*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^8)/((a + b*x)^(1/2) - a^(1/2))^8 + (4*b^3*c^3*((a*c - b*c*x)^(1/2) -
 (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (6*b^3*c^2*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^4)/((a + b*x)^(1/2) - a^(1/2))^4 + (4*b^3*c*((a*c - b*
c*x)^(1/2) - (a*c)^(1/2))^6)/((a + b*x)^(1/2) - a^(1/2))^6) - (4*A*atan((b*
```

```
((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((b^2*c)^(1/2)*((a + b*x)^(1/2) - a^(1
/2)))))/(b^2*c)^(1/2) - (2*C*a^2*atan(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))/(
c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b^3*c^(1/2)) - (B*(a*c - b*c*x)^(1/
2)*(a + b*x)^(1/2))/(b^2*c)
```

sympy **[C]** time = 56.83, size = 338, normalized size = 1.91

$$-\frac{iAG^{6,2}_{6,6}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle| \frac{a^2}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}+\frac{AG^{2,6}_{6,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle| \frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}-\frac{iBaG^{6,2}_{6,6}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle| \frac{a^2}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b^2\sqrt{c}}-\frac{BaG^{2,6}_{6,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle| \frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b^2\sqrt{c}}-\frac{iCa^2G^{6,2}_{6,6}\left(\begin{matrix} -\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1 \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle| \frac{a^2}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b^3\sqrt{c}}+\frac{Ca^2G^{2,6}_{6,6}\left(\begin{matrix} -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0 \end{matrix}\middle| \frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] -I*A*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, a**2/(b**2*x**2))/(4*pi**(3/2)*b*sqrt(c)) + A*meijerg(((-1/2, -1/4, 0, 1/
4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(-2*I*pi)/(b
**2*x**2))/(4*pi**(3/2)*b*sqrt(c)) - I*B*a*meijerg(((-1/4, 1/4), (0, 0, 1/2
, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), a**2/(b**2*x**2))/(4*pi**(3/2)*b
**2*sqrt(c)) - B*a*meijerg(((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4
), (-1, -1/2, -1/2, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*
b**2*sqrt(c)) - I*C*a**2*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1,
-3/4, -1/2, -1/4, 0, 0), ()), a**2/(b**2*x**2))/(4*pi**(3/2)*b**3*sqrt(c))
+ C*a**2*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/
2, -1, -1, 0)), a**2*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b**3*sqrt
(c))
```

3.31 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)}\,dx$

Optimal. Leaf size=278

$$\frac{\sqrt{a^2c-b^2cx^2}\left(Af^2-Bef+Ce^2\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}\sqrt{b^2e^2-a^2f^2}}-\frac{\sqrt{a^2c-b^2cx^2}(Ce-Bf)\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{}{b^2}$$

Rubi [A] time = 0.46, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1610, 1654, 844, 217, 203, 725, 204}

$$\frac{\sqrt{a^2c-b^2cx^2}\left(Af^2-Bef+Ce^2\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}\sqrt{b^2e^2-a^2f^2}}-\frac{\sqrt{a^2c-b^2cx^2}(Ce-Bf)\tan^{-1}\left(\frac{b\sqrt{c}x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}f^2\sqrt{a+bx}\sqrt{ac-bcx}}-\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] -((C*(a^2 - b^2*x^2))/(b^2*f*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])) - ((C*e - B*
f)*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*Sqrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(
b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c - b*c*x]) + ((C*e^2 - B*e*f + A*f^2)*S
qrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a
^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])]])/(Sqrt[c]*f^2*Sqrt[b^2*e^2 - a^2*f^2]*Sqr
t[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1654

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(d + e*x
)^(m + q - 1)*(a + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q + 2*p + 1)), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\sqrt{a^2c-b^2cx^2}\int \frac{-Ab^2cf^2+b^2cf(Ce-Bf)x}{(e+fx)\sqrt{a^2c-b^2cx^2}}\,dx}{b^2cf^2\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\int \frac{1}{\sqrt{a^2c-b^2cx^2}}\,dx}{f^2\sqrt{a+bx}\,\sqrt{ac-bcx}} + \frac{\left((C\ldots\right.}{\ldots}$$

$$= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{\left((Ce-Bf)\sqrt{a^2c-b^2cx^2}\right)\text{Subst}\left(\int \frac{1}{1+b^2cx^2}\,dx, x, \ldots\right.}{f^2\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= -\frac{C\left(a^2-b^2x^2\right)}{b^2f\sqrt{a+bx}\,\sqrt{ac-bcx}} - \frac{(Ce-Bf)\sqrt{a^2c-b^2cx^2}\,\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}} + \frac{(C\ldots}{\ldots}$$

Mathematica [A] time = 0.71, size = 225, normalized size = 0.81

$$\frac{\sqrt{a-bx}\left(\frac{2\left(f(Af-Be)+Ce^2\right)\tanh^{-1}\left(\frac{\sqrt{a-bx}\,\sqrt{be-af}}{\sqrt{a+bx}\,\sqrt{-af-be}}\right)}{\sqrt{-af-be}\,\sqrt{be-af}} + \frac{2\tan^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{a+bx}}\right)(aCf-bBf+bCe)}{b^2} + \frac{Cf\sqrt{a+bx}\left(-\sqrt{a-bx}-\frac{2\sqrt{a}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{b^2}\right)}{f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)),x]
```

```
[Out] (Sqrt[a - b*x]*((C*f*Sqrt[a + b*x]*(-Sqrt[a - b*x] - (2*Sqrt[a]*ArcSin[Sqrt
[a - b*x]/(Sqrt[2]*Sqrt[a])])/Sqrt[1 + (b*x)/a]))/b^2 + (2*(b*C*e - b*B*f +
 a*C*f)*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x]])/b^2 + (2*(C*e^2 + f*(-(B*e) +
A*f))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a +
b*x])])/(Sqrt[-(b*e) - a*f]*Sqrt[b*e - a*f])))/(f^2*Sqrt[c*(a - b*x)])
```

IntegrateAlgebraic [A] time = 0.00, size = 205, normalized size = 0.74

$$-\frac{2\left(Af^2-Bef+Ce^2\right)\tanh^{-1}\left(\frac{\sqrt{ac-bcx}\,\sqrt{af-be}}{\sqrt{c}\,\sqrt{a+bx}\,\sqrt{af+be}}\right)}{\sqrt{c}\,f^2\sqrt{af-be}\,\sqrt{af+be}}-\frac{2aC\sqrt{ac-bcx}}{b^2f\sqrt{a+bx}\left(\frac{ac-bcx}{a+bx}+c\right)}-\frac{2(Bf-Ce)\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\,\sqrt{a+bx}}\right)}{b\sqrt{c}\,f^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e +
f*x)),x]
```

```
[Out] (-2*a*C*Sqrt[a*c - b*c*x])/(b^2*f*Sqrt[a + b*x]*(c + (a*c - b*c*x)/(a + b*x
))) - (2*(-(C*e) + B*f)*ArcTan[Sqrt[a*c - b*c*x]/(Sqrt[c]*Sqrt[a + b*x])])/
(b*Sqrt[c]*f^2) - (2*(C*e^2 - B*e*f + A*f^2)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sq
rt[a*c - b*c*x])/(Sqrt[c]*Sqrt[b*e + a*f]*Sqrt[a + b*x])])/(Sqrt[c]*f^2*Sqr
t[-(b*e) + a*f]*Sqrt[b*e + a*f])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.00, size = 503, normalized size = 1.81

$$\frac{\left(-\sqrt{b^2c}\,A\,b^2c\,f^2\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{(a^2f^2-b^2e^2)c}{f^2}}\,\sqrt{-(b^2x^2-a^2)c}\,f}{fx+e}\right)+\sqrt{b^2c}\,B\,b^2cef\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{(a^2f^2-b^2e^2)c}{f^2}}\,\sqrt{-(b^2x^2-a^2)c}\,f}{fx+e}\right)+\sqrt{\frac{(a^2f^2-b^2e^2)c}{f^2}}\,B\,b^2c\,f^2\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-(b^2x^2-a^2)c}}\right)-\sqrt{b^2c}\,C\,b^2c\,e^2\ln\left(\frac{2b^2cex+2a^2cf+2\sqrt{\frac{(a^2f^2-b^2e^2)c}{f^2}}\,\sqrt{-(b^2x^2-a^2)c}\,f}{fx+e}\right)-\sqrt{\frac{(a^2f^2-b^2e^2)c}{f^2}}\,C\,b^2cef\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-(b^2x^2-a^2)c}}\right)-\sqrt{b^2c}\,\sqrt{\frac{(a^2f^2-b^2e^2)c}{f^2}}\,\sqrt{-(b^2x^2-a^2)c}\,C\,f^2\right)\sqrt{bx+a}\,\sqrt{-(bx-a)c}}{\sqrt{\frac{(a^2f^2-b^2e^2)c}{f^2}}\,\sqrt{b^2c}\,\sqrt{-(b^2x^2-a^2)c}\,b^2c\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] (-(b^2*c)^(1/2)*A*b^2*c*f^2*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^
2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))+(b^2*c)^(1/2)*B*b^2*c*e*f*ln(
2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/
2)*f)/(f*x+e))+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*B*b^2*c*f^2*arctan((b^2*c)^(
1/2)/(-(b^2*x^2-a^2)*c)^(1/2)*x)-(b^2*c)^(1/2)*C*b^2*c*e^2*ln(2*(b^2*c*e*x+
a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e)
)-((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*C*b^2*c*e*f*arctan((b^2*c)^(1/2)/(-(b^2*x
^2-a^2)*c)^(1/2)*x)-(b^2*c)^(1/2)*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^
2-a^2)*c)^(1/2)*C*f^2)*(b*x+a)^(1/2)*(-(b*x-a)*c)^(1/2)/((a^2*f^2-b^2*e^2)*
c/f^2)^(1/2)/(b^2*c)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)/b^2/c/f^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for mor
e details)Is (4*b^2*c     *(a^2*c-(b^2*c*e^2)
   /f^2))    /f^2    +(4*b^4*c^2*e^2)/f^4    zero or nonzero?
```

mupad [B] time = 0.01, size = 9298, normalized size = 33.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] (B*a*e*atan(((B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*
a^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c
^2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) - (
B*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) +
 (B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4
*c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B
*a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((
a + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*
b^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^
4*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*
x)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a
^(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(
a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(
1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*
e*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^
6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) +
(16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f
*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^
(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*
b^4*c^5*e^4 - 144*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^
8*e^6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)
) + (458752*B^3*a^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a
 + b*x)^(1/2) - a^(1/2))))*1i)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (B*a
*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a^(15/2)*b^2*c^4*
e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^2*e*f^2*(a*c)^(5
/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) + (B*a*e*((4096*(16*
B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) - (B*a*e*((4096*(2
4*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*c^5*e^3*f^2*(a*c
)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*a^10*b^2*c^6*e^2
*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) -
a^(1/2))) - (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b^4*c^7*e^2*f^4))
/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4*c^6*e^2*f^4)*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2
))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^(15/2)*b^4*c^5*e
^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a +
 b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (4096*((a
*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e*f^4*(a*c)^(5/2)
 - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6*((a + b*x)^(1/2
) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (16384*(8*B^2*a^(
17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*(a*c)^(3/2))*((a
*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))
```

```
 + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*b^4*c^5*e^4 - 144
*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^8*e^6*((a + b*x)^
(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (458752*B^3*a
^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a + b*x)^(1/2) - a
^(1/2))))*1i)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)))/((131072*B^4*a^4*c^5)/
(b^8*e^4) - (B*a*e*((4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a
^(15/2)*b^2*c^4*e^3*(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^
2*e*f^2*(a*c)^(5/2) - 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) - (B
*a*e*((4096*(16*B^2*a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) +
(B*a*e*((4096*(24*B*a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*
c^5*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*
a^10*b^2*c^6*e^2*f^3)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a
 + b*x)^(1/2) - a^(1/2))) + (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b
^4*c^7*e^2*f^4))/(a^6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4
*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x
)^(1/2) - a^(1/2))^2) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^
(15/2)*b^4*c^5*e^3*f^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a
^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1
/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e
*f^4*(a*c)^(5/2) - 90*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6
*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (
16384*(8*B^2*a^(17/2)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*
(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(
1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*b
^4*c^5*e^4 - 144*B^2*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^8
*e^6*((a + b*x)^(1/2) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2))
 + (458752*B^3*a^4*c^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a
+ b*x)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (B*a*e*(
(4096*(32*B^3*a^(17/2)*c^3*e*f^2*(a*c)^(5/2) + 24*B^3*a^(15/2)*b^2*c^4*e^3*
(a*c)^(3/2)))/(a^6*b^8*e^6) - (4096*(32*B^3*a^(17/2)*c^2*e*f^2*(a*c)^(5/2)
- 96*B^3*a^(15/2)*b^2*c^3*e^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2) + (B*a*e*((4096*(16*B^2*
a^12*c^6*f^4 + 9*B^2*a^8*b^4*c^6*e^4))/(a^6*b^8*e^6) - (B*a*e*((4096*(24*B*
a^(17/2)*b^2*c^4*e*f^4*(a*c)^(5/2) - 30*B*a^(15/2)*b^4*c^5*e^3*f^2*(a*c)^(3
/2)))/(a^6*b^8*e^6) + (16384*(20*B*a^12*c^6*f^5 - 22*B*a^10*b^2*c^6*e^2*f^3
)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1
/2))) - (B*a*e*((4096*(9*a^8*b^6*c^7*e^4*f^2 - 7*a^10*b^4*c^7*e^2*f^4))/(a^
6*b^8*e^6) + (4096*(9*a^8*b^6*c^6*e^4*f^2 - 11*a^10*b^4*c^6*e^2*f^4)*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^2)/(a^6*b^8*e^6*((a + b*x)^(1/2) - a^(1/2))^2
) - (16384*(5*a^(17/2)*b^2*c^4*e*f^5*(a*c)^(5/2) - 6*a^(15/2)*b^4*c^5*e^3*f
^3*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x
)^(1/2) - a^(1/2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (4096*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^2*(96*B*a^(17/2)*b^2*c^3*e*f^4*(a*c)^(5/2) - 9
0*B*a^(15/2)*b^4*c^4*e^3*f^2*(a*c)^(3/2)))/(a^6*b^8*e^6*((a + b*x)^(1/2) -
a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (16384*(8*B^2*a^(17/2
)*c^3*e*f^3*(a*c)^(5/2) + 3*B^2*a^(15/2)*b^2*c^4*e^3*f*(a*c)^(3/2))*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2)))/(a^6*b^7*e^6*((a + b*x)^(1/2) - a^(1/2))) + (
4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*B^2*a^8*b^4*c^5*e^4 - 144*B^2
*a^12*c^5*f^4 + 128*B^2*a^10*b^2*c^5*e^2*f^2))/(a^6*b^8*e^6*((a + b*x)^(1/2
) - a^(1/2))^2)))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (458752*B^3*a^4*c
^5*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^4*((a + b*x)^(1/2) - a^(1/
2)))))/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) + (917504*B^4*a^4*c^4*((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*e^4*((a + b*x)^(1/2) - a^(1/2))^2)))*2i
)/(f*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)) - (C*e^2*atan(((C*e^2*((4096*(32*C^
3*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 24*C^3*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2
)))/(b^8*e^4*f^4) + (C*e^2*((4096*(16*C^2*a^6*c^6*f^6 + 9*C^2*a^2*b^4*c^6*e
^4*f^2))/(b^8*e^4*f^4) - (C*e^2*((4096*(24*C*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2
) - 30*C*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^8*e^4*f^4) + (C*e^2*((409
```

```
6*(7*a^4*b^4*c^7*f^8 - 9*a^2*b^6*c^7*e^2*f^6))/(b^8*e^4*f^4) + (16384*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))*(5*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 6*a^(3/
2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2)))
 + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(11*a^4*b^4*c^6*f^8 - 9*a^2*b
^6*c^6*e^2*f^6))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*
f^2 - b^2*c*e^2)^(1/2)) + (16384*(20*C*a^6*c^6*f^6 - 22*C*a^4*b^2*c^6*e^2*f
^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^
(1/2))) + (4096*(96*C*a^(5/2)*b^2*c^3*f^7*(a*c)^(5/2) - 90*C*a^(3/2)*b^4*c^
4*e^2*f^5*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*e^4*f^4*
((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (409
6*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*C^2*a^2*b^4*c^5*e^4*f^2 - 144*C^
2*a^6*c^5*f^6 + 128*C^2*a^4*b^2*c^5*e^2*f^4))/(b^8*e^4*f^4*((a + b*x)^(1/2)
 - a^(1/2))^2) + (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(8*C^2*a^(5/2)*
c^3*e^2*f^3*(a*c)^(5/2) + 3*C^2*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^7*e^
5*f^2*((a + b*x)^(1/2) - a^(1/2)))))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) -
(4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(32*C^3*a^(5/2)*c^2*e^2*f^3*(a*
c)^(5/2) - 96*C^3*a^(3/2)*b^2*c^3*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4*((a + b*
x)^(1/2) - a^(1/2))^2) + (458752*C^3*a^4*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2)))/(b^7*e*f^2*((a + b*x)^(1/2) - a^(1/2))))*1i)/(f^2*(a^2*c*f^2 - b^2*c
*e^2)^(1/2)) + (C*e^2*((4096*(32*C^3*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 24*C
^3*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4) - (C*e^2*((4096*(16*C^
2*a^6*c^6*f^6 + 9*C^2*a^2*b^4*c^6*e^4*f^2))/(b^8*e^4*f^4) + (C*e^2*((4096*(
24*C*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 30*C*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(
3/2)))/(b^8*e^4*f^4) - (C*e^2*((4096*(7*a^4*b^4*c^7*f^8 - 9*a^2*b^6*c^7*e^2
*f^6))/(b^8*e^4*f^4) + (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(5*a^(5/2
)*b^2*c^4*f^7*(a*c)^(5/2) - 6*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^7*e^
5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^2*(11*a^4*b^4*c^6*f^8 - 9*a^2*b^6*c^6*e^2*f^6))/(b^8*e^4*f^4*((a + b*x)
^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (16384*(20*C*a
^6*c^6*f^6 - 22*C*a^4*b^2*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))
/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*(96*C*a^(5/2)*b^2*c^3*f^
7*(a*c)^(5/2) - 90*C*a^(3/2)*b^4*c^4*e^2*f^5*(a*c)^(3/2))*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^2)/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a
^2*c*f^2 - b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*
(9*C^2*a^2*b^4*c^5*e^4*f^2 - 144*C^2*a^6*c^5*f^6 + 128*C^2*a^4*b^2*c^5*e^2*
f^4))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (16384*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))*(8*C^2*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 3*C^2*a^(3/2)*
b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2)))))/(f^
2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)
)^2*(32*C^3*a^(5/2)*c^2*e^2*f^3*(a*c)^(5/2) - 96*C^3*a^(3/2)*b^2*c^3*e^4*f*
(a*c)^(3/2)))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (458752*C^3*a^4
*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e*f^2*((a + b*x)^(1/2) - a^(
1/2))))*1i)/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)))/((131072*C^4*a^4*c^5)/(b^8
*f^4) + (C*e^2*((4096*(32*C^3*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 24*C^3*a^(3
/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4) + (C*e^2*((4096*(16*C^2*a^6*c
^6*f^6 + 9*C^2*a^2*b^4*c^6*e^4*f^2))/(b^8*e^4*f^4) - (C*e^2*((4096*(24*C*a^
(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 30*C*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/
(b^8*e^4*f^4) + (C*e^2*((4096*(7*a^4*b^4*c^7*f^8 - 9*a^2*b^6*c^7*e^2*f^6))/
(b^8*e^4*f^4) + (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(5*a^(5/2)*b^2*c
^4*f^7*(a*c)^(5/2) - 6*a^(3/2)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^7*e^5*f^2*(
(a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(
11*a^4*b^4*c^6*f^8 - 9*a^2*b^6*c^6*e^2*f^6))/(b^8*e^4*f^4*((a + b*x)^(1/2)
- a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (16384*(20*C*a^6*c^6*
f^6 - 22*C*a^4*b^2*c^6*e^2*f^4)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e
^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*(96*C*a^(5/2)*b^2*c^3*f^7*(a*c)
^(5/2) - 90*C*a^(3/2)*b^4*c^4*e^2*f^5*(a*c)^(3/2))*((a*c - b*c*x)^(1/2) - (
a*c)^(1/2))^2)/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^
2 - b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(9*C^2*
a^2*b^4*c^5*e^4*f^2 - 144*C^2*a^6*c^5*f^6 + 128*C^2*a^4*b^2*c^5*e^2*f^4))/(
```

```
b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (16384*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))*(8*C^2*a^(5/2)*c^3*e^2*f^3*(a*c)^(5/2) + 3*C^2*a^(3/2)*b^2*c^4
*e^4*f*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2)))))/(f^2*(a^2*
c*f^2 - b^2*c*e^2)^(1/2)) - (4096*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(32
*C^3*a^(5/2)*c^2*e^2*f^3*(a*c)^(5/2) - 96*C^3*a^(3/2)*b^2*c^3*e^4*f*(a*c)^(
3/2)))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2) + (458752*C^3*a^4*c^5*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e*f^2*((a + b*x)^(1/2) - a^(1/2))))
)/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (C*e^2*((4096*(32*C^3*a^(5/2)*c^3*e
^2*f^3*(a*c)^(5/2) + 24*C^3*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^
4) - (C*e^2*((4096*(16*C^2*a^6*c^6*f^6 + 9*C^2*a^2*b^4*c^6*e^4*f^2))/(b^8*e
^4*f^4) + (C*e^2*((4096*(24*C*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 30*C*a^(3/2
)*b^4*c^5*e^2*f^5*(a*c)^(3/2)))/(b^8*e^4*f^4) - (C*e^2*((4096*(7*a^4*b^4*c^
7*f^8 - 9*a^2*b^6*c^7*e^2*f^6))/(b^8*e^4*f^4) + (16384*((a*c - b*c*x)^(1/2)
 - (a*c)^(1/2))*(5*a^(5/2)*b^2*c^4*f^7*(a*c)^(5/2) - 6*a^(3/2)*b^4*c^5*e^2*
f^5*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^2*(11*a^4*b^4*c^6*f^8 - 9*a^2*b^6*c^6*e^2*f^6)
)/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2
)^(1/2)) + (16384*(20*C*a^6*c^6*f^6 - 22*C*a^4*b^2*c^6*e^2*f^4)*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2)))/(b^7*e^5*f^2*((a + b*x)^(1/2) - a^(1/2))) + (4096
*(96*C*a^(5/2)*b^2*c^3*f^7*(a*c)^(5/2) - 90*C*a^(3/2)*b^4*c^4*e^2*f^5*(a*c)
^(3/2))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*e^4*f^4*((a + b*x)^(1/2
) - a^(1/2))^2)))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (4096*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))^2*(9*C^2*a^2*b^4*c^5*e^4*f^2 - 144*C^2*a^6*c^5*f^6 +
 128*C^2*a^4*b^2*c^5*e^2*f^4))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1/2))^2)
+ (16384*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(8*C^2*a^(5/2)*c^3*e^2*f^3*(a*
c)^(5/2) + 3*C^2*a^(3/2)*b^2*c^4*e^4*f*(a*c)^(3/2)))/(b^7*e^5*f^2*((a + b*x
)^(1/2) - a^(1/2)))))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (4096*((a*c - b
*c*x)^(1/2) - (a*c)^(1/2))^2*(32*C^3*a^(5/2)*c^2*e^2*f^3*(a*c)^(5/2) - 96*C
^3*a^(3/2)*b^2*c^3*e^4*f*(a*c)^(3/2)))/(b^8*e^4*f^4*((a + b*x)^(1/2) - a^(1
/2))^2) + (458752*C^3*a^4*c^5*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b^7*e*f
^2*((a + b*x)^(1/2) - a^(1/2)))))/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) + (91
7504*C^4*a^4*c^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^8*f^4*((a + b*x)
^(1/2) - a^(1/2))^2)))*2i)/(f^2*(a^2*c*f^2 - b^2*c*e^2)^(1/2)) - (4*B*atan(
(67108864*B^5*a^16*c^7*f^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)
^(1/2) - a^(1/2))*(67108864*B^5*a^16*c^(15/2)*f^4 + 37748736*B^5*a^12*b^4*c
^(15/2)*e^4 - 100663296*B^5*a^14*b^2*c^(15/2)*e^2*f^2)) + (37748736*B^5*a^1
2*b^4*c^7*e^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1
/2))*(67108864*B^5*a^16*c^(15/2)*f^4 + 37748736*B^5*a^12*b^4*c^(15/2)*e^4 -
 100663296*B^5*a^14*b^2*c^(15/2)*e^2*f^2)) - (100663296*B^5*a^14*b^2*c^7*e^
2*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))*(67
108864*B^5*a^16*c^(15/2)*f^4 + 37748736*B^5*a^12*b^4*c^(15/2)*e^4 - 1006632
96*B^5*a^14*b^2*c^(15/2)*e^2*f^2))))/(b*c^(1/2)*f) - (A*a*atan((a*c*(a*c -
b*c*x)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)*2i - (a*c)^(3/2)*(a^4*c*f^2
- a^2*b^2*c*e^2)^(1/2)*1i + a*c*(a*c)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/
2)*1i + b*c*x*(a*c)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)*2i - a^(1/2)*c*
(a*c)^(1/2)*(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2)*(a + b*x)^(1/2)*2i)/(2*a^(5/2
)*b*c^2*e - 2*a^3*c^2*f*(a + b*x)^(1/2) - 2*a^2*b*c^2*e*(a + b*x)^(1/2) + 2
*a^(5/2)*b*c^2*f*x + 2*a^(5/2)*c*f*(a*c - b*c*x)^(1/2)*(a*c)^(1/2) - 2*a^(3
/2)*b*c*e*(a*c - b*c*x)^(1/2)*(a*c)^(1/2) + 2*a*b*c*e*(a*c - b*c*x)^(1/2)*(
a*c)^(1/2)*(a + b*x)^(1/2)))*2i)/(a^4*c*f^2 - a^2*b^2*c*e^2)^(1/2) + (4*C*e
*atan((67108864*C^5*a^8*c^7*f^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a +
 b*x)^(1/2) - a^(1/2))*(67108864*C^5*a^8*c^(15/2)*f^4 + 37748736*C^5*a^4*b^
4*c^(15/2)*e^4 - 100663296*C^5*a^6*b^2*c^(15/2)*e^2*f^2)) + (37748736*C^5*a
^4*b^4*c^7*e^4*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(
1/2))*(67108864*C^5*a^8*c^(15/2)*f^4 + 37748736*C^5*a^4*b^4*c^(15/2)*e^4 -
100663296*C^5*a^6*b^2*c^(15/2)*e^2*f^2)) - (100663296*C^5*a^6*b^2*c^7*e^2*f
^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))*(67108
864*C^5*a^8*c^(15/2)*f^4 + 37748736*C^5*a^4*b^4*c^(15/2)*e^4 - 100663296*C^
5*a^6*b^2*c^(15/2)*e^2*f^2))))/(b*c^(1/2)*f^2) - (8*C*a^(1/2)*(a*c)^(1/2)*(
```

```
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*f*((a + b*x)^(1/2) - a^(1/2))^2*
(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4/((a + b*x)^(1/2) - a^(1/2))^4 + c^2
+ (2*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2
))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
[Out] Timed out
```

3.32 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2}\,dx$

Optimal. Leaf size=322

$$\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{\sqrt{a+bx}\,(e+fx)\sqrt{ac-bcx}\,\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2c-b^2cx^2}\,\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\,(a\ \text{[cut off]}}{\sqrt{a^2c-b^2c\ \text{[cut off]}}}\right.}{\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(b^2e^2-a^2f^2\right)^{3/2}}$$

Rubi [A] time = 0.53, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {1610, 1651, 844, 217, 203, 725, 204}

$$\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{\sqrt{a+bx}\,(e+fx)\sqrt{ac-bcx}\,\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2c-b^2cx^2}\,\left(a^2f^2(2Ce-Bf)-b^2\left(Ce^3-Aef^2\right)\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\,\sqrt{b^2e^2-a^2f^2}}\right)}{\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}\,\left(b^2e^2-a^2f^2\right)^{3/2}}+\frac{C\sqrt{a^2c-b^2cx^2}\,\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2cx^2}}\right)}{b\sqrt{c}\,f^2\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x]
```

```
[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/((b^2*e^2 - a^2*f^2)*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + (C*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(b*S
qrt[c]*x)/Sqrt[a^2*c - b^2*c*x^2]])/(b*Sqrt[c]*f^2*Sqrt[a + b*x]*Sqrt[a*c -
 b*c*x]) + ((a^2*f^2*(2*C*e - B*f) - b^2*(C*e^3 - A*e*f^2))*Sqrt[a^2*c - b^
2*c*x^2]*ArcTan[(Sqrt[c]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a
^2*c - b^2*c*x^2])]])/(Sqrt[c]*f^2*(b^2*e^2 - a^2*f^2)^(3/2)*Sqrt[a + b*x]*S
qrt[a*c - b*c*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^2}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)^2\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\sqrt{a^2c-b^2cx^2}\int \frac{c\left(Ab^2e+a^2(Ce-Bf \ldots\right.}{(e+fx)\sqrt{a \ldots}}}{c\left(b^2e^2-a^2f^2\right)\sqrt{a+bx} \ldots}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a^2c-b^2cx^2}\right) \ldots}{f\left(b^2e^2-a^2f^2\right)\sqrt{a+bx} \ldots}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{\left(C\left(\frac{b^2e^2}{f}-a^2f\right)\sqrt{a^2c-b^2cx^2}\right) \ldots}{f\left(b^2e^2-a^2f^2\right) \ldots}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)} + \frac{C\sqrt{a^2c-b^2cx^2}\tan^{-1}\left(\frac{b\sqrt{c}\,x}{\sqrt{a^2c-b^2c \ldots}} \ldots\right.}{b\sqrt{c}\,f^2\sqrt{a+bx}\sqrt{ac-bcx} \ldots}$$

Mathematica [A] time = 0.79, size = 309, normalized size = 0.96

$$\frac{-\frac{2b^2e\sqrt{a-bx}\left(f(Af-Be)+Ce^2\right)\tanh^{-1}\left(\frac{\sqrt{a-bx}\sqrt{be-af}}{\sqrt{a+bx}\sqrt{-af-be}}\right)}{(-af-be)^{3/2}(be-af)^{3/2}}+\frac{f(bx-a)\sqrt{a+bx}\left(f(Af-Be)+Ce^2\right)}{(e+fx)(af-be)(af+be)}-\frac{2\sqrt{a-bx}(2Ce-Bf)\tanh^{-1}\left(\frac{\sqrt{a-bx}\sqrt{be-af}}{\sqrt{a+bx}\sqrt{-af-be}}\right)}{\sqrt{-af-be}\sqrt{be-af}}-\frac{2C\sqrt{a-bx}\tan^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{a+bx}}\right)}{b}}{f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2),x
]
```

```
[Out] ((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/((-(b*e) + a*f)*(b
*e + a*f)*(e + f*x)) - (2*C*Sqrt[a - b*x]*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x
]])/b - (2*(2*C*e - B*f)*Sqrt[a - b*x]*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[a - b*
x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])]])/(Sqrt[-(b*e) - a*f]*Sqrt[b*e - a*f
]) - (2*b^2*e*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a - b*x]*ArcTanh[(Sqrt[b*e -
a*f]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])]])/((-(b*e) - a*f)^(3
/2)*(b*e - a*f)^(3/2)))/(f^2*Sqrt[c*(a - b*x)])
```

IntegrateAlgebraic [A] time = 0.00, size = 282, normalized size = 0.88

$$-\frac{2\left(a^2Bf^3-2a^2Cef^2-Ab^2ef^2+b^2Ce^3\right)\tanh^{-1}\left(\frac{\sqrt{ac-bcx}\sqrt{af-be}}{\sqrt{c}\sqrt{a+bx}\sqrt{af+be}}\right)}{\sqrt{c}f^2(af-be)^{3/2}(af+be)^{3/2}}+\frac{2ab\sqrt{ac-bcx}\left(Af^2-Bef+Ce^2\right)}{f\sqrt{a+bx}(af-be)(af+be)\left(-\frac{be(ac-bcx)}{a+bx}+\frac{af(ac-bcx)}{a+bx}-acf-bce\right)}-\frac{2C\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\sqrt{a+bx}}\right)}{b\sqrt{c}f^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e +
f*x)^2),x]
```

```
[Out] (2*a*b*(C*e^2 - B*e*f + A*f^2)*Sqrt[a*c - b*c*x])/(f*(-(b*e) + a*f)*(b*e +
a*f)*Sqrt[a + b*x]*(-(b*c*e) - a*c*f - (b*e*(a*c - b*c*x))/(a + b*x) + (a*f
*(a*c - b*c*x))/(a + b*x))) - (2*C*ArcTan[Sqrt[a*c - b*c*x]/(Sqrt[c]*Sqrt[a
 + b*x])])/(b*Sqrt[c]*f^2) - (2*(b^2*C*e^3 - A*b^2*e*f^2 - 2*a^2*C*e*f^2 +
a^2*B*f^3)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[a*c - b*c*x])/(Sqrt[c]*Sqrt[b*e
 + a*f]*Sqrt[a + b*x])]])/(Sqrt[c]*f^2*(-(b*e) + a*f)^(3/2)*(b*e + a*f)^(3/2
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.00, size = 1200, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] ((b^2*c)^(1/2)*A*b^2*c*e*f^3*x*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c
/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))-(b^2*c)^(1/2)*B*a^2*c*f^4*
x*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c
)^(1/2)*f)/(f*x+e))+2*(b^2*c)^(1/2)*C*a^2*c*e*f^3*x*ln(2*(b^2*c*e*x+a^2*c*f
+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))+((a^2
*f^2-b^2*e^2)*c/f^2)^(1/2)*C*a^2*c*f^4*x*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^
2)*c)^(1/2)*x)-(b^2*c)^(1/2)*C*b^2*c*e^3*f*x*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*
f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))-((a^2*f^2-b^
2*e^2)*c/f^2)^(1/2)*C*b^2*c*e^2*f^2*x*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^2)*
c)^(1/2)*x)+(b^2*c)^(1/2)*A*b^2*c*e^2*f^2*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2
-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))-(b^2*c)^(1/2)*B
*a^2*c*e*f^3*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2
*x^2-a^2)*c)^(1/2)*f)/(f*x+e))+2*(b^2*c)^(1/2)*C*a^2*c*e^2*f^2*ln(2*(b^2*c*
e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*
x+e))+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*C*a^2*c*e*f^3*arctan((b^2*c)^(1/2)/(-
```

```
(b^2*x^2-a^2)*c)^(1/2)*x)-(b^2*c)^(1/2)*C*b^2*c*e^4*ln(2*(b^2*c*e*x+a^2*c*f
+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))-((a^2
*f^2-b^2*e^2)*c/f^2)^(1/2)*C*b^2*c*e^3*f*arctan((b^2*c)^(1/2)/(-(b^2*x^2-a^
2)*c)^(1/2)*x)-(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*((a^2*f^2-b^2*e^2)*c/
f^2)^(1/2)*A*f^4+(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*((a^2*f^2-b^2*e^2)*
c/f^2)^(1/2)*B*e*f^3-(b^2*c)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*((a^2*f^2-b^2*e
^2)*c/f^2)^(1/2)*C*e^2*f^2)*(-(b*x-a)*c)^(1/2)*(b*x+a)^(1/2)/(-(b^2*x^2-a^2
)*c)^(1/2)/(a*f-b*e)/(b^2*c)^(1/2)/(a*f+b*e)/(f*x+e)/((a^2*f^2-b^2*e^2)*c/f
^2)^(1/2)/c/f^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^2/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((4*b^2*c>0)', see `assume?` for mor
e details)Is (4*b^2*c     *(a^2*c-(b^2*c*e^2)
   /f^2))    /f^2    +(4*b^4*c^2*e^2)/f^4    zero or nonzero?
```

mupad [B] time = 19.40, size = 106511, normalized size = 330.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^2*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] ((4*B*a^2*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(((a + b*x)^(1/2) - a^(1
/2))^3*(b^3*e^3 - a^2*b*e*f^2)) + (8*B*a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^2)/((a^2*f^2 - b^2*e^2)*((a + b*x)^(1/2) - a^(1/2))^2)
- (4*B*a^2*c*f*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x)^(1/2) - a^(
1/2))*(b^3*e^3 - a^2*b*e*f^2)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4/((a
+ b*x)^(1/2) - a^(1/2))^4 + c^2 + (2*c*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
2)/((a + b*x)^(1/2) - a^(1/2))^2 - (4*a^(1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^
(1/2) - (a*c)^(1/2))^3)/(b*e*((a + b*x)^(1/2) - a^(1/2))^3) + (4*a^(1/2)*c*
f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b*e*((a + b*x)^(1/2) -
a^(1/2)))) - ((4*C*a^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((b^3*e^2 - a
^2*b*f^2)*((a + b*x)^(1/2) - a^(1/2))^3) - (4*C*a^2*c*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2)))/((b^3*e^2 - a^2*b*f^2)*((a + b*x)^(1/2) - a^(1/2))) + (8*C*
a^(1/2)*e*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a^2*f^3 - b^
2*e^2*f)*((a + b*x)^(1/2) - a^(1/2))^2))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^4/((a + b*x)^(1/2) - a^(1/2))^4 + c^2 + (2*c*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - (4*a^(1/2)*f*(a*c)^(1/2)*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*((a + b*x)^(1/2) - a^(1/2))^3) + (4*a
^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b*e*((a + b*x)
^(1/2) - a^(1/2)))) + ((4*A*a^2*c*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/
((b^3*e^4 - a^2*b*e^2*f^2)*((a + b*x)^(1/2) - a^(1/2))) - (4*A*a^2*f^2*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((b^3*e^4 - a^2*b*e^2*f^2)*((a + b*x)^(1
/2) - a^(1/2))^3) + (8*A*a^(1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2))^2)/((b^2*e^3 - a^2*e*f^2)*((a + b*x)^(1/2) - a^(1/2))^2))/(((a*c -
b*c*x)^(1/2) - (a*c)^(1/2))^4/((a + b*x)^(1/2) - a^(1/2))^4 + c^2 + (2*c*((
a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - (4*a^(
1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*((a + b*x)^(
1/2) - a^(1/2))^3) + (4*a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2)))) - (4*C*atan(((a*c - b*c*x)^(1/
2) - (a*c)^(1/2))/(c^(1/2)*((a + b*x)^(1/2) - a^(1/2)))))/(b*c^(1/2)*f^2) +
 (2*A*b^2*e*(atan((2*b^3*c^3*e^3 + 2*b*c^2*e*(a^2*c*f^2 - b^2*c*e^2) + 2*a^
```

2*b*c^3*e*f^2 + (3*a^(3/2)*f^3*(a*c)^(3/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 + (2*b^3*c^2*e^3*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - (3*a^(1/2)*f*(a*c)^(1/2)*
((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(
1/2) - a^(1/2))^3 - (a^(3/2)*c*f^3*(a*c)^(3/2)*((a*c - b*c*x)^(1/2) - (a*c)
^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + (2*b*c*e*((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))^2*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2))^2 + (a^(1/2
)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(a^2*c*f^2 - b^2*c*e^
2))/((a + b*x)^(1/2) - a^(1/2)) - (10*a^2*b*c^2*e*f^2*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (7*a^(1/2)*b^2*c^2*e^2*f*
(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)
) - (a^(1/2)*b^2*c*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)
/((a + b*x)^(1/2) - a^(1/2))^3)/(4*a^(1/2)*b*c^2*e*f*(a*c)^(1/2)*(b^2*c*e^2
 - a^2*c*f^2)^(1/2))) - atan(((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(a^2*c*f
^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (a^2*c*f^2*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + 2*a^(1/2)*b*c*e*f*(a*c)^(
1/2))/(2*b*c*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)))))/((a*f + b*e)*(a*f - b*e)*(
b^2*c*e^2 - a^2*c*f^2)^(1/2)) - (2*B*a^2*f*(atan((2*b^3*c^3*e^3 + 2*b*c^2*e
*(a^2*c*f^2 - b^2*c*e^2) + 2*a^2*b*c^3*e*f^2 + (3*a^(3/2)*f^3*(a*c)^(3/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 + (2*b^
3*c^2*e^3*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2)
)^2 - (3*a^(1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(a^2*c
*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2))^3 - (a^(3/2)*c*f^3*(a*c)^(3/
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + (2*b*
c*e*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(a^2*c*f^2 - b^2*c*e^2))/((a + b*
x)^(1/2) - a^(1/2))^2 + (a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (10*a^2*b*
c^2*e*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2)
)^2 + (7*a^(1/2)*b^2*c^2*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/
2)))/((a + b*x)^(1/2) - a^(1/2)) - (a^(1/2)*b^2*c*e^2*f*(a*c)^(1/2)*((a*c -
 b*c*x)^(1/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3)/(4*a^(1/2)*b
*c^2*e*f*(a*c)^(1/2)*(b^2*c*e^2 - a^2*c*f^2)^(1/2))) - atan(((((a*c - b*c*x
)^(1/2) - (a*c)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2))
 - (a^2*c*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/
2)) + 2*a^(1/2)*b*c*e*f*(a*c)^(1/2))/(2*b*c*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)
))))/((a*f + b*e)*(a*f - b*e)*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) - (C*e*(2*a^2*
f^2 - b^2*e^2)*(2*atan((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*((8*a^4*b^6*c
^4*e^6*f^4*((4096*C^3*e^3*(2*a^2*f^2 - b^2*e^2)^3*(136*C*a^(21/2)*b^2*c^3*e
f^15(a*c)^(5/2) - 90*C*a^(3/2)*b^12*c^4*e^11*f^5*(a*c)^(3/2) + 96*C*a^(5/
2)*b^10*c^3*e^9*f^7*(a*c)^(5/2) + 394*C*a^(7/2)*b^10*c^4*e^9*f^7*(a*c)^(3/2
) - 424*C*a^(9/2)*b^8*c^3*e^7*f^9*(a*c)^(5/2) - 642*C*a^(11/2)*b^8*c^4*e^7*
f^9*(a*c)^(3/2) + 696*C*a^(13/2)*b^6*c^3*e^5*f^11*(a*c)^(5/2) + 462*C*a^(15
/2)*b^6*c^4*e^5*f^11*(a*c)^(3/2) - 504*C*a^(17/2)*b^4*c^3*e^3*f^13*(a*c)^(5
/2) - 124*C*a^(19/2)*b^4*c^4*e^3*f^13*(a*c)^(3/2)))/(f^6*(a*f + b*e)^3*(a*f
 - b*e)^3*(b^2*c*e^2 - a^2*c*f^2)^(3/2)*(b^16*e^14*f^4 - 4*a^2*b^14*e^12*f^
6 + 6*a^4*b^12*e^10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12)) - (4096*
C*e*(2*a^2*f^2 - b^2*e^2)*(64*C^3*a^(21/2)*c^2*e*f^11*(a*c)^(5/2) + 32*C^3*
a^(5/2)*b^8*c^2*e^9*f^3*(a*c)^(5/2) + 600*C^3*a^(7/2)*b^8*c^3*e^9*f^3*(a*c)
^(3/2) - 160*C^3*a^(9/2)*b^6*c^2*e^7*f^5*(a*c)^(5/2) - 1376*C^3*a^(11/2)*b^
6*c^3*e^7*f^5*(a*c)^(3/2) + 288*C^3*a^(13/2)*b^4*c^2*e^5*f^7*(a*c)^(5/2) +
1368*C^3*a^(15/2)*b^4*c^3*e^5*f^7*(a*c)^(3/2) - 224*C^3*a^(17/2)*b^2*c^2*e^
3*f^9*(a*c)^(5/2) - 496*C^3*a^(19/2)*b^2*c^3*e^3*f^9*(a*c)^(3/2) - 96*C^3*a
^(3/2)*b^10*c^3*e^11*f*(a*c)^(3/2)))/(f^2*(a*f + b*e)*(a*f - b*e)*(b^2*c*e^
2 - a^2*c*f^2)^(1/2)*(b^16*e^14*f^4 - 4*a^2*b^14*e^12*f^6 + 6*a^4*b^12*e^10
*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12)))*(4*a^2*c*f^2 - 3*b^2*c*e^2
)*(4*a^6*c*f^6 - 3*b^6*c*e^6 + 8*a^2*b^4*c*e^4*f^2 - 8*a^4*b^2*c*e^2*f^4)^4
)/(164025*b^46*c^13*e^46 + 885735*b^44*c^12*e^44*(a^2*c*f^2 - b^2*c*e^2) +
117440512*a^30*c^5*f^30*(a^2*c*f^2 - b^2*c*e^2)^8 - 385875968*a^32*c^6*f^32
*(a^2*c*f^2 - b^2*c*e^2)^7 + 419430400*a^34*c^7*f^34*(a^2*c*f^2 - b^2*c*e^2

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)^6 - 150994944*a^36*c^8*f^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 236196*b^36*c^8*e
^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 1102248*b^38*c^9*e^38*(a^2*c*f^2 - b^2*c*e^
2)^4 + 2053593*b^40*c^10*e^40*(a^2*c*f^2 - b^2*c*e^2)^3 + 1909251*b^42*c^11
*e^42*(a^2*c*f^2 - b^2*c*e^2)^2 - 3937329*a^2*b^44*c^13*e^44*f^2 + 43893819
*a^4*b^42*c^13*e^42*f^4 - 301507155*a^6*b^40*c^13*e^40*f^6 + 1427514656*a^8
*b^38*c^13*e^38*f^8 - 4936911112*a^10*b^36*c^13*e^36*f^10 + 12893273616*a^1
2*b^34*c^13*e^34*f^12 - 25921630432*a^14*b^32*c^13*e^32*f^14 + 40519286096*
a^16*b^30*c^13*e^30*f^16 - 49376608256*a^18*b^28*c^13*e^28*f^18 + 467214018
56*a^20*b^26*c^13*e^26*f^20 - 33946324736*a^22*b^24*c^13*e^24*f^22 + 185565
79328*a^24*b^22*c^13*e^22*f^24 - 7375276032*a^26*b^20*c^13*e^20*f^26 + 2009
817088*a^28*b^18*c^13*e^18*f^28 - 335642624*a^30*b^16*c^13*e^16*f^30 + 2590
7200*a^32*b^14*c^13*e^14*f^32 - 21130794*a^2*b^42*c^12*e^42*f^2*(a^2*c*f^2
- b^2*c*e^2) + 234399015*a^4*b^40*c^12*e^40*f^4*(a^2*c*f^2 - b^2*c*e^2) - 1
604168280*a^6*b^38*c^12*e^38*f^6*(a^2*c*f^2 - b^2*c*e^2) + 7579098492*a^8*b
^36*c^12*e^36*f^8*(a^2*c*f^2 - b^2*c*e^2) - 26212380172*a^10*b^34*c^12*e^34
*f^10*(a^2*c*f^2 - b^2*c*e^2) + 68672994096*a^12*b^32*c^12*e^32*f^12*(a^2*c
*f^2 - b^2*c*e^2) - 139160589504*a^14*b^30*c^12*e^30*f^14*(a^2*c*f^2 - b^2*
c*e^2) + 220859191808*a^16*b^28*c^12*e^28*f^16*(a^2*c*f^2 - b^2*c*e^2) - 27
6344315328*a^18*b^26*c^12*e^26*f^18*(a^2*c*f^2 - b^2*c*e^2) + 273130561984*
a^20*b^24*c^12*e^24*f^20*(a^2*c*f^2 - b^2*c*e^2) - 212730002688*a^22*b^22*c
^12*e^22*f^22*(a^2*c*f^2 - b^2*c*e^2) + 129574234368*a^24*b^20*c^12*e^20*f^
24*(a^2*c*f^2 - b^2*c*e^2) - 60770569216*a^26*b^18*c^12*e^18*f^26*(a^2*c*f^
2 - b^2*c*e^2) + 21304706048*a^28*b^16*c^12*e^16*f^28*(a^2*c*f^2 - b^2*c*e^
2) - 5272965120*a^30*b^14*c^12*e^14*f^30*(a^2*c*f^2 - b^2*c*e^2) + 81944166
4*a^32*b^12*c^12*e^12*f^32*(a^2*c*f^2 - b^2*c*e^2) - 59392000*a^34*b^10*c^1
2*e^10*f^34*(a^2*c*f^2 - b^2*c*e^2) + 9289728*a^6*b^24*c^5*e^24*f^6*(a^2*c*
f^2 - b^2*c*e^2)^8 - 36884480*a^8*b^22*c^5*e^22*f^8*(a^2*c*f^2 - b^2*c*e^2)
^8 - 278604800*a^10*b^20*c^5*e^20*f^10*(a^2*c*f^2 - b^2*c*e^2)^8 + 27744832
00*a^12*b^18*c^5*e^18*f^12*(a^2*c*f^2 - b^2*c*e^2)^8 - 10869657600*a^14*b^1
6*c^5*e^16*f^14*(a^2*c*f^2 - b^2*c*e^2)^8 + 25237416960*a^16*b^14*c^5*e^14*
f^16*(a^2*c*f^2 - b^2*c*e^2)^8 - 38348909568*a^18*b^12*c^5*e^12*f^18*(a^2*c
*f^2 - b^2*c*e^2)^8 + 39084659712*a^20*b^10*c^5*e^10*f^20*(a^2*c*f^2 - b^2*
c*e^2)^8 - 26118635520*a^22*b^8*c^5*e^8*f^22*(a^2*c*f^2 - b^2*c*e^2)^8 + 10
414620672*a^24*b^6*c^5*e^6*f^24*(a^2*c*f^2 - b^2*c*e^2)^8 - 1708654592*a^26
*b^4*c^5*e^4*f^26*(a^2*c*f^2 - b^2*c*e^2)^8 - 276561920*a^28*b^2*c^5*e^2*f^
28*(a^2*c*f^2 - b^2*c*e^2)^8 - 9704448*a^4*b^28*c^6*e^28*f^4*(a^2*c*f^2 - b
^2*c*e^2)^7 + 260614656*a^6*b^26*c^6*e^26*f^6*(a^2*c*f^2 - b^2*c*e^2)^7 - 2
166022464*a^8*b^24*c^6*e^24*f^8*(a^2*c*f^2 - b^2*c*e^2)^7 + 8626147840*a^10
*b^22*c^6*e^22*f^10*(a^2*c*f^2 - b^2*c*e^2)^7 - 16771503616*a^12*b^20*c^6*e
^20*f^12*(a^2*c*f^2 - b^2*c*e^2)^7 + 3301800960*a^14*b^18*c^6*e^18*f^14*(a^
2*c*f^2 - b^2*c*e^2)^7 + 67337715968*a^16*b^16*c^6*e^16*f^16*(a^2*c*f^2 - b
^2*c*e^2)^7 - 189857873920*a^18*b^14*c^6*e^14*f^18*(a^2*c*f^2 - b^2*c*e^2)^
7 + 286100259840*a^20*b^12*c^6*e^12*f^20*(a^2*c*f^2 - b^2*c*e^2)^7 - 275789
894656*a^22*b^10*c^6*e^10*f^22*(a^2*c*f^2 - b^2*c*e^2)^7 + 173716537344*a^2
4*b^8*c^6*e^8*f^24*(a^2*c*f^2 - b^2*c*e^2)^7 - 67416424448*a^26*b^6*c^6*e^6
*f^26*(a^2*c*f^2 - b^2*c*e^2)^7 + 12831686656*a^28*b^4*c^6*e^4*f^28*(a^2*c*
f^2 - b^2*c*e^2)^7 + 222560256*a^30*b^2*c^6*e^2*f^30*(a^2*c*f^2 - b^2*c*e^2
)^7 + 2099520*a^2*b^32*c^7*e^32*f^2*(a^2*c*f^2 - b^2*c*e^2)^6 - 107014608*a
^4*b^30*c^7*e^30*f^4*(a^2*c*f^2 - b^2*c*e^2)^6 + 1848335616*a^6*b^28*c^7*e^
28*f^6*(a^2*c*f^2 - b^2*c*e^2)^6 - 15200005312*a^8*b^26*c^7*e^26*f^8*(a^2*c
*f^2 - b^2*c*e^2)^6 + 72612273792*a^10*b^24*c^7*e^24*f^10*(a^2*c*f^2 - b^2*
c*e^2)^6 - 221855779968*a^12*b^22*c^7*e^22*f^12*(a^2*c*f^2 - b^2*c*e^2)^6 +
 450717857536*a^14*b^20*c^7*e^20*f^14*(a^2*c*f^2 - b^2*c*e^2)^6 - 600578910
208*a^16*b^18*c^7*e^18*f^16*(a^2*c*f^2 - b^2*c*e^2)^6 + 459464530688*a^18*b
^16*c^7*e^16*f^18*(a^2*c*f^2 - b^2*c*e^2)^6 - 33638947840*a^20*b^14*c^7*e^1
4*f^20*(a^2*c*f^2 - b^2*c*e^2)^6 - 376299926528*a^22*b^12*c^7*e^12*f^22*(a^
2*c*f^2 - b^2*c*e^2)^6 + 488874068992*a^24*b^10*c^7*e^10*f^24*(a^2*c*f^2 -
b^2*c*e^2)^6 - 333407809536*a^26*b^8*c^7*e^8*f^26*(a^2*c*f^2 - b^2*c*e^2)^6
 + 134140313600*a^28*b^6*c^7*e^6*f^28*(a^2*c*f^2 - b^2*c*e^2)^6 - 282209157
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12*a^30*b^4*c^7*e^4*f^30*(a^2*c*f^2 - b^2*c*e^2)^6 + 1230503936*a^32*b^2*c^
7*e^2*f^32*(a^2*c*f^2 - b^2*c*e^2)^6 + 3335904*a^2*b^34*c^8*e^34*f^2*(a^2*c
*f^2 - b^2*c*e^2)^5 - 290521728*a^4*b^32*c^8*e^32*f^4*(a^2*c*f^2 - b^2*c*e^
2)^5 + 4865684544*a^6*b^30*c^8*e^30*f^6*(a^2*c*f^2 - b^2*c*e^2)^5 - 4043739
4528*a^8*b^28*c^8*e^28*f^8*(a^2*c*f^2 - b^2*c*e^2)^5 + 205602254656*a^10*b^
26*c^8*e^26*f^10*(a^2*c*f^2 - b^2*c*e^2)^5 - 703885344192*a^12*b^24*c^8*e^2
4*f^12*(a^2*c*f^2 - b^2*c*e^2)^5 + 1709253482624*a^14*b^22*c^8*e^22*f^14*(a
^2*c*f^2 - b^2*c*e^2)^5 - 3029282695168*a^16*b^20*c^8*e^20*f^16*(a^2*c*f^2
- b^2*c*e^2)^5 + 3966230827520*a^18*b^18*c^8*e^18*f^18*(a^2*c*f^2 - b^2*c*e
^2)^5 - 3822339813632*a^20*b^16*c^8*e^16*f^20*(a^2*c*f^2 - b^2*c*e^2)^5 + 2
640438056960*a^22*b^14*c^8*e^14*f^22*(a^2*c*f^2 - b^2*c*e^2)^5 - 1208501415
936*a^24*b^12*c^8*e^12*f^24*(a^2*c*f^2 - b^2*c*e^2)^5 + 269338092544*a^26*b
^10*c^8*e^10*f^26*(a^2*c*f^2 - b^2*c*e^2)^5 + 53783212032*a^28*b^8*c^8*e^8*
f^28*(a^2*c*f^2 - b^2*c*e^2)^5 - 60985360384*a^30*b^6*c^8*e^6*f^30*(a^2*c*f
^2 - b^2*c*e^2)^5 + 17917083648*a^32*b^4*c^8*e^4*f^32*(a^2*c*f^2 - b^2*c*e^
2)^5 - 1558708224*a^34*b^2*c^8*e^2*f^34*(a^2*c*f^2 - b^2*c*e^2)^5 - 1191769
2*a^2*b^36*c^9*e^36*f^2*(a^2*c*f^2 - b^2*c*e^2)^4 - 224907516*a^4*b^34*c^9*
e^34*f^4*(a^2*c*f^2 - b^2*c*e^2)^4 + 5303932560*a^6*b^32*c^9*e^32*f^6*(a^2*
c*f^2 - b^2*c*e^2)^4 - 48206418480*a^8*b^30*c^9*e^30*f^8*(a^2*c*f^2 - b^2*c
*e^2)^4 + 261450609120*a^10*b^28*c^9*e^28*f^10*(a^2*c*f^2 - b^2*c*e^2)^4 -
962361040256*a^12*b^26*c^9*e^26*f^12*(a^2*c*f^2 - b^2*c*e^2)^4 + 2558559358
080*a^14*b^24*c^9*e^24*f^14*(a^2*c*f^2 - b^2*c*e^2)^4 - 5091804150656*a^16*
b^22*c^9*e^22*f^16*(a^2*c*f^2 - b^2*c*e^2)^4 + 7750806514944*a^18*b^20*c^9*
e^20*f^18*(a^2*c*f^2 - b^2*c*e^2)^4 - 9137207485952*a^20*b^18*c^9*e^18*f^20
*(a^2*c*f^2 - b^2*c*e^2)^4 + 8384563280128*a^22*b^16*c^9*e^16*f^22*(a^2*c*f
^2 - b^2*c*e^2)^4 - 5975281259520*a^24*b^14*c^9*e^14*f^24*(a^2*c*f^2 - b^2*
c*e^2)^4 + 3269297268736*a^26*b^12*c^9*e^12*f^26*(a^2*c*f^2 - b^2*c*e^2)^4
- 1339171540992*a^28*b^10*c^9*e^10*f^28*(a^2*c*f^2 - b^2*c*e^2)^4 + 3912501
94432*a^30*b^8*c^9*e^8*f^30*(a^2*c*f^2 - b^2*c*e^2)^4 - 74114154496*a^32*b^
6*c^9*e^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^4 + 7299203072*a^34*b^4*c^9*e^4*f^34
*(a^2*c*f^2 - b^2*c*e^2)^4 - 148635648*a^36*b^2*c^9*e^2*f^36*(a^2*c*f^2 - b
^2*c*e^2)^4 - 38704068*a^2*b^38*c^10*e^38*f^2*(a^2*c*f^2 - b^2*c*e^2)^3 + 1
88845992*a^4*b^36*c^10*e^36*f^4*(a^2*c*f^2 - b^2*c*e^2)^3 + 1157124204*a^6*
b^34*c^10*e^34*f^6*(a^2*c*f^2 - b^2*c*e^2)^3 - 20586361424*a^8*b^32*c^10*e^
32*f^8*(a^2*c*f^2 - b^2*c*e^2)^3 + 135395499200*a^10*b^30*c^10*e^30*f^10*(a
^2*c*f^2 - b^2*c*e^2)^3 - 555513858464*a^12*b^28*c^10*e^28*f^12*(a^2*c*f^2
- b^2*c*e^2)^3 + 1608776388864*a^14*b^26*c^10*e^26*f^14*(a^2*c*f^2 - b^2*c*
e^2)^3 - 3473989271488*a^16*b^24*c^10*e^24*f^16*(a^2*c*f^2 - b^2*c*e^2)^3 +
 5766181411456*a^18*b^22*c^10*e^22*f^18*(a^2*c*f^2 - b^2*c*e^2)^3 - 7493983
209472*a^20*b^20*c^10*e^20*f^20*(a^2*c*f^2 - b^2*c*e^2)^3 + 7713917084672*a
^22*b^18*c^10*e^18*f^22*(a^2*c*f^2 - b^2*c*e^2)^3 - 6328467293184*a^24*b^16
*c^10*e^16*f^24*(a^2*c*f^2 - b^2*c*e^2)^3 + 4142950034432*a^26*b^14*c^10*e^
14*f^26*(a^2*c*f^2 - b^2*c*e^2)^3 - 2152681536512*a^28*b^12*c^10*e^12*f^28*
(a^2*c*f^2 - b^2*c*e^2)^3 + 874199511040*a^30*b^10*c^10*e^10*f^30*(a^2*c*f^
2 - b^2*c*e^2)^3 - 268759150592*a^32*b^8*c^10*e^8*f^32*(a^2*c*f^2 - b^2*c*e
^2)^3 + 58872545280*a^34*b^6*c^10*e^6*f^34*(a^2*c*f^2 - b^2*c*e^2)^3 - 8151
957504*a^36*b^4*c^10*e^4*f^36*(a^2*c*f^2 - b^2*c*e^2)^3 + 530841600*a^38*b^
2*c^10*e^2*f^38*(a^2*c*f^2 - b^2*c*e^2)^3 - 42743457*a^2*b^40*c^11*e^40*f^2
*(a^2*c*f^2 - b^2*c*e^2)^2 + 411055884*a^4*b^38*c^11*e^38*f^4*(a^2*c*f^2 -
b^2*c*e^2)^2 - 2180887236*a^6*b^36*c^11*e^36*f^6*(a^2*c*f^2 - b^2*c*e^2)^2
+ 6404946508*a^8*b^34*c^11*e^34*f^8*(a^2*c*f^2 - b^2*c*e^2)^2 - 5434005264*
a^10*b^32*c^11*e^32*f^10*(a^2*c*f^2 - b^2*c*e^2)^2 - 38868373520*a^12*b^30*
c^11*e^30*f^12*(a^2*c*f^2 - b^2*c*e^2)^2 + 208447613600*a^14*b^28*c^11*e^28
*f^14*(a^2*c*f^2 - b^2*c*e^2)^2 - 579674999104*a^16*b^26*c^11*e^26*f^16*(a^
2*c*f^2 - b^2*c*e^2)^2 + 1104967566592*a^18*b^24*c^11*e^24*f^18*(a^2*c*f^2
- b^2*c*e^2)^2 - 1554566531328*a^20*b^22*c^11*e^22*f^20*(a^2*c*f^2 - b^2*c*
e^2)^2 + 1659734381312*a^22*b^20*c^11*e^20*f^22*(a^2*c*f^2 - b^2*c*e^2)^2 -
 1356361512192*a^24*b^18*c^11*e^18*f^24*(a^2*c*f^2 - b^2*c*e^2)^2 + 8453313
59744*a^26*b^16*c^11*e^16*f^26*(a^2*c*f^2 - b^2*c*e^2)^2 - 395676895232*a^2
```

8*b^14*c^11*e^14*f^28*(a^2*c*f^2 - b^2*c*e^2)^2 + 134902689792*a^30*b^12*c^
11*e^12*f^30*(a^2*c*f^2 - b^2*c*e^2)^2 - 31670587392*a^32*b^10*c^11*e^10*f^
32*(a^2*c*f^2 - b^2*c*e^2)^2 + 4584669184*a^34*b^8*c^11*e^8*f^34*(a^2*c*f^2
 - b^2*c*e^2)^2 - 309657600*a^36*b^6*c^11*e^6*f^36*(a^2*c*f^2 - b^2*c*e^2)^
2) + (2*a^4*b^5*c^3*e^5*f^4*(4*a^2*c*f^2 - 3*b^2*c*e^2)^2*((16384*(12*C^4*a
^(7/2)*b^4*c^3*e^7*(a*c)^(3/2) + 48*C^4*a^(15/2)*c^3*e^3*f^4*(a*c)^(3/2) -
48*C^4*a^(11/2)*b^2*c^3*e^5*f^2*(a*c)^(3/2)))/(b^13*e^12*f^3 - 3*a^2*b^11*e
^10*f^5 + 3*a^4*b^9*e^8*f^7 - a^6*b^7*e^6*f^9) + (16384*C^4*e^4*(2*a^2*f^2
- b^2*e^2)^4*(5*a^(17/2)*b^2*c^4*e*f^14*(a*c)^(5/2) + 6*a^(3/2)*b^10*c^5*e^
9*f^6*(a*c)^(3/2) - 5*a^(5/2)*b^8*c^4*e^7*f^8*(a*c)^(5/2) - 18*a^(7/2)*b^8*
c^5*e^7*f^8*(a*c)^(3/2) + 15*a^(9/2)*b^6*c^4*e^5*f^10*(a*c)^(5/2) + 18*a^(1
1/2)*b^6*c^5*e^5*f^10*(a*c)^(3/2) - 15*a^(13/2)*b^4*c^4*e^3*f^12*(a*c)^(5/2
) - 6*a^(15/2)*b^4*c^5*e^3*f^12*(a*c)^(3/2)))/(f^8*(a*f + b*e)^4*(a*f - b*e
)^4*(a^2*c*f^2 - b^2*c*e^2)^2*(b^13*e^12*f^3 - 3*a^2*b^11*e^10*f^5 + 3*a^4*
b^9*e^8*f^7 - a^6*b^7*e^6*f^9)) - (16384*C^2*e^2*(2*a^2*f^2 - b^2*e^2)^2*(2
0*C^2*a^(17/2)*c^3*e*f^10*(a*c)^(5/2) - 3*C^2*a^(3/2)*b^8*c^4*e^9*f^2*(a*c)
^(3/2) - 8*C^2*a^(5/2)*b^6*c^3*e^7*f^4*(a*c)^(5/2) + 11*C^2*a^(7/2)*b^6*c^4
*e^7*f^4*(a*c)^(3/2) + 36*C^2*a^(9/2)*b^4*c^3*e^5*f^6*(a*c)^(5/2) - 20*C^2*
a^(11/2)*b^4*c^4*e^5*f^6*(a*c)^(3/2) - 48*C^2*a^(13/2)*b^2*c^3*e^3*f^8*(a*c
)^(5/2) + 12*C^2*a^(15/2)*b^2*c^4*e^3*f^8*(a*c)^(3/2)))/(f^4*(a*f + b*e)^2*
(a*f - b*e)^2*(a^2*c*f^2 - b^2*c*e^2)*(b^13*e^12*f^3 - 3*a^2*b^11*e^10*f^5
+ 3*a^4*b^9*e^8*f^7 - a^6*b^7*e^6*f^9)))*(4*a^6*c*f^6 - 3*b^6*c*e^6 + 8*a^2
*b^4*c*e^4*f^2 - 8*a^4*b^2*c*e^2*f^4)^4)/((b^2*c*e^2 - a^2*c*f^2)^(1/2)*(16
4025*b^46*c^13*e^46 + 885735*b^44*c^12*e^44*(a^2*c*f^2 - b^2*c*e^2) + 11744
0512*a^30*c^5*f^30*(a^2*c*f^2 - b^2*c*e^2)^8 - 385875968*a^32*c^6*f^32*(a^2
*c*f^2 - b^2*c*e^2)^7 + 419430400*a^34*c^7*f^34*(a^2*c*f^2 - b^2*c*e^2)^6 -
 150994944*a^36*c^8*f^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 236196*b^36*c^8*e^36*(
a^2*c*f^2 - b^2*c*e^2)^5 + 1102248*b^38*c^9*e^38*(a^2*c*f^2 - b^2*c*e^2)^4
+ 2053593*b^40*c^10*e^40*(a^2*c*f^2 - b^2*c*e^2)^3 + 1909251*b^42*c^11*e^42
*(a^2*c*f^2 - b^2*c*e^2)^2 - 3937329*a^2*b^44*c^13*e^44*f^2 + 43893819*a^4*
b^42*c^13*e^42*f^4 - 301507155*a^6*b^40*c^13*e^40*f^6 + 1427514656*a^8*b^38
*c^13*e^38*f^8 - 4936911112*a^10*b^36*c^13*e^36*f^10 + 12893273616*a^12*b^3
4*c^13*e^34*f^12 - 25921630432*a^14*b^32*c^13*e^32*f^14 + 40519286096*a^16*
b^30*c^13*e^30*f^16 - 49376608256*a^18*b^28*c^13*e^28*f^18 + 46721401856*a^
20*b^26*c^13*e^26*f^20 - 33946324736*a^22*b^24*c^13*e^24*f^22 + 18556579328
*a^24*b^22*c^13*e^22*f^24 - 7375276032*a^26*b^20*c^13*e^20*f^26 + 200981708
8*a^28*b^18*c^13*e^18*f^28 - 335642624*a^30*b^16*c^13*e^16*f^30 + 25907200*
a^32*b^14*c^13*e^14*f^32 - 21130794*a^2*b^42*c^12*e^42*f^2*(a^2*c*f^2 - b^2
*c*e^2) + 234399015*a^4*b^40*c^12*e^40*f^4*(a^2*c*f^2 - b^2*c*e^2) - 160416
8280*a^6*b^38*c^12*e^38*f^6*(a^2*c*f^2 - b^2*c*e^2) + 7579098492*a^8*b^36*c
^12*e^36*f^8*(a^2*c*f^2 - b^2*c*e^2) - 26212380172*a^10*b^34*c^12*e^34*f^10
*(a^2*c*f^2 - b^2*c*e^2) + 68672994096*a^12*b^32*c^12*e^32*f^12*(a^2*c*f^2
- b^2*c*e^2) - 139160589504*a^14*b^30*c^12*e^30*f^14*(a^2*c*f^2 - b^2*c*e^2
) + 220859191808*a^16*b^28*c^12*e^28*f^16*(a^2*c*f^2 - b^2*c*e^2) - 2763443
15328*a^18*b^26*c^12*e^26*f^18*(a^2*c*f^2 - b^2*c*e^2) + 273130561984*a^20*
b^24*c^12*e^24*f^20*(a^2*c*f^2 - b^2*c*e^2) - 212730002688*a^22*b^22*c^12*e
^22*f^22*(a^2*c*f^2 - b^2*c*e^2) + 129574234368*a^24*b^20*c^12*e^20*f^24*(a
^2*c*f^2 - b^2*c*e^2) - 60770569216*a^26*b^18*c^12*e^18*f^26*(a^2*c*f^2 - b
^2*c*e^2) + 21304706048*a^28*b^16*c^12*e^16*f^28*(a^2*c*f^2 - b^2*c*e^2) -
5272965120*a^30*b^14*c^12*e^14*f^30*(a^2*c*f^2 - b^2*c*e^2) + 819441664*a^3
2*b^12*c^12*e^12*f^32*(a^2*c*f^2 - b^2*c*e^2) - 59392000*a^34*b^10*c^12*e^1
0*f^34*(a^2*c*f^2 - b^2*c*e^2) + 9289728*a^6*b^24*c^5*e^24*f^6*(a^2*c*f^2 -
 b^2*c*e^2)^8 - 36884480*a^8*b^22*c^5*e^22*f^8*(a^2*c*f^2 - b^2*c*e^2)^8 -
278604800*a^10*b^20*c^5*e^20*f^10*(a^2*c*f^2 - b^2*c*e^2)^8 + 2774483200*a^
12*b^18*c^5*e^18*f^12*(a^2*c*f^2 - b^2*c*e^2)^8 - 10869657600*a^14*b^16*c^5
*e^16*f^14*(a^2*c*f^2 - b^2*c*e^2)^8 + 25237416960*a^16*b^14*c^5*e^14*f^16*
(a^2*c*f^2 - b^2*c*e^2)^8 - 38348909568*a^18*b^12*c^5*e^12*f^18*(a^2*c*f^2
- b^2*c*e^2)^8 + 39084659712*a^20*b^10*c^5*e^10*f^20*(a^2*c*f^2 - b^2*c*e^2
)^8 - 26118635520*a^22*b^8*c^5*e^8*f^22*(a^2*c*f^2 - b^2*c*e^2)^8 + 1041462

```
0672*a^24*b^6*c^5*e^6*f^24*(a^2*c*f^2 - b^2*c*e^2)^8 - 1708654592*a^26*b^4*
c^5*e^4*f^26*(a^2*c*f^2 - b^2*c*e^2)^8 - 276561920*a^28*b^2*c^5*e^2*f^28*(a
^2*c*f^2 - b^2*c*e^2)^8 - 9704448*a^4*b^28*c^6*e^28*f^4*(a^2*c*f^2 - b^2*c*
e^2)^7 + 260614656*a^6*b^26*c^6*e^26*f^6*(a^2*c*f^2 - b^2*c*e^2)^7 - 216602
2464*a^8*b^24*c^6*e^24*f^8*(a^2*c*f^2 - b^2*c*e^2)^7 + 8626147840*a^10*b^22
*c^6*e^22*f^10*(a^2*c*f^2 - b^2*c*e^2)^7 - 16771503616*a^12*b^20*c^6*e^20*f
^12*(a^2*c*f^2 - b^2*c*e^2)^7 + 3301800960*a^14*b^18*c^6*e^18*f^14*(a^2*c*f
^2 - b^2*c*e^2)^7 + 67337715968*a^16*b^16*c^6*e^16*f^16*(a^2*c*f^2 - b^2*c*
e^2)^7 - 189857873920*a^18*b^14*c^6*e^14*f^18*(a^2*c*f^2 - b^2*c*e^2)^7 + 2
86100259840*a^20*b^12*c^6*e^12*f^20*(a^2*c*f^2 - b^2*c*e^2)^7 - 27578989465
6*a^22*b^10*c^6*e^10*f^22*(a^2*c*f^2 - b^2*c*e^2)^7 + 173716537344*a^24*b^8
*c^6*e^8*f^24*(a^2*c*f^2 - b^2*c*e^2)^7 - 67416424448*a^26*b^6*c^6*e^6*f^26
*(a^2*c*f^2 - b^2*c*e^2)^7 + 12831686656*a^28*b^4*c^6*e^4*f^28*(a^2*c*f^2 -
 b^2*c*e^2)^7 + 222560256*a^30*b^2*c^6*e^2*f^30*(a^2*c*f^2 - b^2*c*e^2)^7 +
 2099520*a^2*b^32*c^7*e^32*f^2*(a^2*c*f^2 - b^2*c*e^2)^6 - 107014608*a^4*b^
30*c^7*e^30*f^4*(a^2*c*f^2 - b^2*c*e^2)^6 + 1848335616*a^6*b^28*c^7*e^28*f^
6*(a^2*c*f^2 - b^2*c*e^2)^6 - 15200005312*a^8*b^26*c^7*e^26*f^8*(a^2*c*f^2
- b^2*c*e^2)^6 + 72612273792*a^10*b^24*c^7*e^24*f^10*(a^2*c*f^2 - b^2*c*e^2
)^6 - 221855779968*a^12*b^22*c^7*e^22*f^12*(a^2*c*f^2 - b^2*c*e^2)^6 + 4507
17857536*a^14*b^20*c^7*e^20*f^14*(a^2*c*f^2 - b^2*c*e^2)^6 - 600578910208*a
^16*b^18*c^7*e^18*f^16*(a^2*c*f^2 - b^2*c*e^2)^6 + 459464530688*a^18*b^16*c
^7*e^16*f^18*(a^2*c*f^2 - b^2*c*e^2)^6 - 33638947840*a^20*b^14*c^7*e^14*f^2
0*(a^2*c*f^2 - b^2*c*e^2)^6 - 376299926528*a^22*b^12*c^7*e^12*f^22*(a^2*c*f
^2 - b^2*c*e^2)^6 + 488874068992*a^24*b^10*c^7*e^10*f^24*(a^2*c*f^2 - b^2*c
*e^2)^6 - 333407809536*a^26*b^8*c^7*e^8*f^26*(a^2*c*f^2 - b^2*c*e^2)^6 + 13
4140313600*a^28*b^6*c^7*e^6*f^28*(a^2*c*f^2 - b^2*c*e^2)^6 - 28220915712*a^
30*b^4*c^7*e^4*f^30*(a^2*c*f^2 - b^2*c*e^2)^6 + 1230503936*a^32*b^2*c^7*e^2
*f^32*(a^2*c*f^2 - b^2*c*e^2)^6 + 3335904*a^2*b^34*c^8*e^34*f^2*(a^2*c*f^2
- b^2*c*e^2)^5 - 290521728*a^4*b^32*c^8*e^32*f^4*(a^2*c*f^2 - b^2*c*e^2)^5
+ 4865684544*a^6*b^30*c^8*e^30*f^6*(a^2*c*f^2 - b^2*c*e^2)^5 - 40437394528*
a^8*b^28*c^8*e^28*f^8*(a^2*c*f^2 - b^2*c*e^2)^5 + 205602254656*a^10*b^26*c^
8*e^26*f^10*(a^2*c*f^2 - b^2*c*e^2)^5 - 703885344192*a^12*b^24*c^8*e^24*f^1
2*(a^2*c*f^2 - b^2*c*e^2)^5 + 1709253482624*a^14*b^22*c^8*e^22*f^14*(a^2*c*
f^2 - b^2*c*e^2)^5 - 3029282695168*a^16*b^20*c^8*e^20*f^16*(a^2*c*f^2 - b^2
*c*e^2)^5 + 3966230827520*a^18*b^18*c^8*e^18*f^18*(a^2*c*f^2 - b^2*c*e^2)^5
 - 3822339813632*a^20*b^16*c^8*e^16*f^20*(a^2*c*f^2 - b^2*c*e^2)^5 + 264043
8056960*a^22*b^14*c^8*e^14*f^22*(a^2*c*f^2 - b^2*c*e^2)^5 - 1208501415936*a
^24*b^12*c^8*e^12*f^24*(a^2*c*f^2 - b^2*c*e^2)^5 + 269338092544*a^26*b^10*c
^8*e^10*f^26*(a^2*c*f^2 - b^2*c*e^2)^5 + 53783212032*a^28*b^8*c^8*e^8*f^28*
(a^2*c*f^2 - b^2*c*e^2)^5 - 60985360384*a^30*b^6*c^8*e^6*f^30*(a^2*c*f^2 -
b^2*c*e^2)^5 + 17917083648*a^32*b^4*c^8*e^4*f^32*(a^2*c*f^2 - b^2*c*e^2)^5
- 1558708224*a^34*b^2*c^8*e^2*f^34*(a^2*c*f^2 - b^2*c*e^2)^5 - 11917692*a^2
*b^36*c^9*e^36*f^2*(a^2*c*f^2 - b^2*c*e^2)^4 - 224907516*a^4*b^34*c^9*e^34*
f^4*(a^2*c*f^2 - b^2*c*e^2)^4 + 5303932560*a^6*b^32*c^9*e^32*f^6*(a^2*c*f^2
 - b^2*c*e^2)^4 - 48206418480*a^8*b^30*c^9*e^30*f^8*(a^2*c*f^2 - b^2*c*e^2)
^4 + 261450609120*a^10*b^28*c^9*e^28*f^10*(a^2*c*f^2 - b^2*c*e^2)^4 - 96236
1040256*a^12*b^26*c^9*e^26*f^12*(a^2*c*f^2 - b^2*c*e^2)^4 + 2558559358080*a
^14*b^24*c^9*e^24*f^14*(a^2*c*f^2 - b^2*c*e^2)^4 - 5091804150656*a^16*b^22*
c^9*e^22*f^16*(a^2*c*f^2 - b^2*c*e^2)^4 + 7750806514944*a^18*b^20*c^9*e^20*
f^18*(a^2*c*f^2 - b^2*c*e^2)^4 - 9137207485952*a^20*b^18*c^9*e^18*f^20*(a^2
*c*f^2 - b^2*c*e^2)^4 + 8384563280128*a^22*b^16*c^9*e^16*f^22*(a^2*c*f^2 -
b^2*c*e^2)^4 - 5975281259520*a^24*b^14*c^9*e^14*f^24*(a^2*c*f^2 - b^2*c*e^2
)^4 + 3269297268736*a^26*b^12*c^9*e^12*f^26*(a^2*c*f^2 - b^2*c*e^2)^4 - 133
9171540992*a^28*b^10*c^9*e^10*f^28*(a^2*c*f^2 - b^2*c*e^2)^4 + 391250194432
*a^30*b^8*c^9*e^8*f^30*(a^2*c*f^2 - b^2*c*e^2)^4 - 74114154496*a^32*b^6*c^9
*e^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^4 + 7299203072*a^34*b^4*c^9*e^4*f^34*(a^2
*c*f^2 - b^2*c*e^2)^4 - 148635648*a^36*b^2*c^9*e^2*f^36*(a^2*c*f^2 - b^2*c*
e^2)^4 - 38704068*a^2*b^38*c^10*e^38*f^2*(a^2*c*f^2 - b^2*c*e^2)^3 + 188845
992*a^4*b^36*c^10*e^36*f^4*(a^2*c*f^2 - b^2*c*e^2)^3 + 1157124204*a^6*b^34*
```

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c^10*e^34*f^6*(a^2*c*f^2 - b^2*c*e^2)^3 - 20586361424*a^8*b^32*c^10*e^32*f^
8*(a^2*c*f^2 - b^2*c*e^2)^3 + 135395499200*a^10*b^30*c^10*e^30*f^10*(a^2*c*
f^2 - b^2*c*e^2)^3 - 555513858464*a^12*b^28*c^10*e^28*f^12*(a^2*c*f^2 - b^2
*c*e^2)^3 + 1608776388864*a^14*b^26*c^10*e^26*f^14*(a^2*c*f^2 - b^2*c*e^2)^
3 - 3473989271488*a^16*b^24*c^10*e^24*f^16*(a^2*c*f^2 - b^2*c*e^2)^3 + 5766
181411456*a^18*b^22*c^10*e^22*f^18*(a^2*c*f^2 - b^2*c*e^2)^3 - 749398320947
2*a^20*b^20*c^10*e^20*f^20*(a^2*c*f^2 - b^2*c*e^2)^3 + 7713917084672*a^22*b
^18*c^10*e^18*f^22*(a^2*c*f^2 - b^2*c*e^2)^3 - 6328467293184*a^24*b^16*c^10
*e^16*f^24*(a^2*c*f^2 - b^2*c*e^2)^3 + 4142950034432*a^26*b^14*c^10*e^14*f^
26*(a^2*c*f^2 - b^2*c*e^2)^3 - 2152681536512*a^28*b^12*c^10*e^12*f^28*(a^2*
c*f^2 - b^2*c*e^2)^3 + 874199511040*a^30*b^10*c^10*e^10*f^30*(a^2*c*f^2 - b
^2*c*e^2)^3 - 268759150592*a^32*b^8*c^10*e^8*f^32*(a^2*c*f^2 - b^2*c*e^2)^3
 + 58872545280*a^34*b^6*c^10*e^6*f^34*(a^2*c*f^2 - b^2*c*e^2)^3 - 815195750
4*a^36*b^4*c^10*e^4*f^36*(a^2*c*f^2 - b^2*c*e^2)^3 + 530841600*a^38*b^2*c^1
0*e^2*f^38*(a^2*c*f^2 - b^2*c*e^2)^3 - 42743457*a^2*b^40*c^11*e^40*f^2*(a^2
*c*f^2 - b^2*c*e^2)^2 + 411055884*a^4*b^38*c^11*e^38*f^4*(a^2*c*f^2 - b^2*c
*e^2)^2 - 2180887236*a^6*b^36*c^11*e^36*f^6*(a^2*c*f^2 - b^2*c*e^2)^2 + 640
4946508*a^8*b^34*c^11*e^34*f^8*(a^2*c*f^2 - b^2*c*e^2)^2 - 5434005264*a^10*
b^32*c^11*e^32*f^10*(a^2*c*f^2 - b^2*c*e^2)^2 - 38868373520*a^12*b^30*c^11*
e^30*f^12*(a^2*c*f^2 - b^2*c*e^2)^2 + 208447613600*a^14*b^28*c^11*e^28*f^14
*(a^2*c*f^2 - b^2*c*e^2)^2 - 579674999104*a^16*b^26*c^11*e^26*f^16*(a^2*c*f
^2 - b^2*c*e^2)^2 + 1104967566592*a^18*b^24*c^11*e^24*f^18*(a^2*c*f^2 - b^2
*c*e^2)^2 - 1554566531328*a^20*b^22*c^11*e^22*f^20*(a^2*c*f^2 - b^2*c*e^2)^
2 + 1659734381312*a^22*b^20*c^11*e^20*f^22*(a^2*c*f^2 - b^2*c*e^2)^2 - 1356
361512192*a^24*b^18*c^11*e^18*f^24*(a^2*c*f^2 - b^2*c*e^2)^2 + 845331359744
*a^26*b^16*c^11*e^16*f^26*(a^2*c*f^2 - b^2*c*e^2)^2 - 395676895232*a^28*b^1
4*c^11*e^14*f^28*(a^2*c*f^2 - b^2*c*e^2)^2 + 134902689792*a^30*b^12*c^11*e^
12*f^30*(a^2*c*f^2 - b^2*c*e^2)^2 - 31670587392*a^32*b^10*c^11*e^10*f^32*(a
^2*c*f^2 - b^2*c*e^2)^2 + 4584669184*a^34*b^8*c^11*e^8*f^34*(a^2*c*f^2 - b^
2*c*e^2)^2 - 309657600*a^36*b^6*c^11*e^6*f^36*(a^2*c*f^2 - b^2*c*e^2)^2)) +
 (2*a^(3/2)*b^5*c*e^5*f^3*((16384*C^3*e^3*(2*a^2*f^2 - b^2*e^2)^3*(20*C*a^1
2*c^6*f^13 + 22*C*a^4*b^8*c^6*e^8*f^5 - 88*C*a^6*b^6*c^6*e^6*f^7 + 130*C*a^
8*b^4*c^6*e^4*f^9 - 84*C*a^10*b^2*c^6*e^2*f^11))/(f^6*(a*f + b*e)^3*(a*f -
b*e)^3*(b^2*c*e^2 - a^2*c*f^2)^(3/2)*(b^13*e^12*f^3 - 3*a^2*b^11*e^10*f^5 +
 3*a^4*b^9*e^8*f^7 - a^6*b^7*e^6*f^9)) + (16384*C*e*(2*a^2*f^2 - b^2*e^2)*(
96*C^3*a^10*c^5*e^2*f^7 - 28*C^3*a^4*b^6*c^5*e^8*f + 132*C^3*a^6*b^4*c^5*e^
6*f^3 - 200*C^3*a^8*b^2*c^5*e^4*f^5))/(f^2*(a*f + b*e)*(a*f - b*e)*(b^2*c*e
^2 - a^2*c*f^2)^(1/2)*(b^13*e^12*f^3 - 3*a^2*b^11*e^10*f^5 + 3*a^4*b^9*e^8*
f^7 - a^6*b^7*e^6*f^9)))*(a*c)^(3/2)*(4*a^2*c*f^2 - b^2*c*e^2)*(4*a^2*c*f^2
 - 3*b^2*c*e^2)*(4*a^6*c*f^6 - 3*b^6*c*e^6 + 8*a^2*b^4*c*e^4*f^2 - 8*a^4*b^
2*c*e^2*f^4)^4)/(164025*b^46*c^13*e^46 + 885735*b^44*c^12*e^44*(a^2*c*f^2 -
 b^2*c*e^2) + 117440512*a^30*c^5*f^30*(a^2*c*f^2 - b^2*c*e^2)^8 - 385875968
*a^32*c^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^7 + 419430400*a^34*c^7*f^34*(a^2*c*f
^2 - b^2*c*e^2)^6 - 150994944*a^36*c^8*f^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 236
196*b^36*c^8*e^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 1102248*b^38*c^9*e^38*(a^2*c*
f^2 - b^2*c*e^2)^4 + 2053593*b^40*c^10*e^40*(a^2*c*f^2 - b^2*c*e^2)^3 + 190
9251*b^42*c^11*e^42*(a^2*c*f^2 - b^2*c*e^2)^2 - 3937329*a^2*b^44*c^13*e^44*
f^2 + 43893819*a^4*b^42*c^13*e^42*f^4 - 301507155*a^6*b^40*c^13*e^40*f^6 +
1427514656*a^8*b^38*c^13*e^38*f^8 - 4936911112*a^10*b^36*c^13*e^36*f^10 + 1
2893273616*a^12*b^34*c^13*e^34*f^12 - 25921630432*a^14*b^32*c^13*e^32*f^14
+ 40519286096*a^16*b^30*c^13*e^30*f^16 - 49376608256*a^18*b^28*c^13*e^28*f^
18 + 46721401856*a^20*b^26*c^13*e^26*f^20 - 33946324736*a^22*b^24*c^13*e^24
*f^22 + 18556579328*a^24*b^22*c^13*e^22*f^24 - 7375276032*a^26*b^20*c^13*e^
20*f^26 + 2009817088*a^28*b^18*c^13*e^18*f^28 - 335642624*a^30*b^16*c^13*e^
16*f^30 + 25907200*a^32*b^14*c^13*e^14*f^32 - 21130794*a^2*b^42*c^12*e^42*f
^2*(a^2*c*f^2 - b^2*c*e^2) + 234399015*a^4*b^40*c^12*e^40*f^4*(a^2*c*f^2 -
b^2*c*e^2) - 1604168280*a^6*b^38*c^12*e^38*f^6*(a^2*c*f^2 - b^2*c*e^2) + 75
79098492*a^8*b^36*c^12*e^36*f^8*(a^2*c*f^2 - b^2*c*e^2) - 26212380172*a^10*
b^34*c^12*e^34*f^10*(a^2*c*f^2 - b^2*c*e^2) + 68672994096*a^12*b^32*c^12*e^
```

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32*f^12*(a^2*c*f^2 - b^2*c*e^2) - 139160589504*a^14*b^30*c^12*e^30*f^14*(a^
2*c*f^2 - b^2*c*e^2) + 220859191808*a^16*b^28*c^12*e^28*f^16*(a^2*c*f^2 - b
^2*c*e^2) - 276344315328*a^18*b^26*c^12*e^26*f^18*(a^2*c*f^2 - b^2*c*e^2) +
 273130561984*a^20*b^24*c^12*e^24*f^20*(a^2*c*f^2 - b^2*c*e^2) - 2127300026
88*a^22*b^22*c^12*e^22*f^22*(a^2*c*f^2 - b^2*c*e^2) + 129574234368*a^24*b^2
0*c^12*e^20*f^24*(a^2*c*f^2 - b^2*c*e^2) - 60770569216*a^26*b^18*c^12*e^18*
f^26*(a^2*c*f^2 - b^2*c*e^2) + 21304706048*a^28*b^16*c^12*e^16*f^28*(a^2*c*
f^2 - b^2*c*e^2) - 5272965120*a^30*b^14*c^12*e^14*f^30*(a^2*c*f^2 - b^2*c*e
^2) + 819441664*a^32*b^12*c^12*e^12*f^32*(a^2*c*f^2 - b^2*c*e^2) - 59392000
*a^34*b^10*c^12*e^10*f^34*(a^2*c*f^2 - b^2*c*e^2) + 9289728*a^6*b^24*c^5*e^
24*f^6*(a^2*c*f^2 - b^2*c*e^2)^8 - 36884480*a^8*b^22*c^5*e^22*f^8*(a^2*c*f^
2 - b^2*c*e^2)^8 - 278604800*a^10*b^20*c^5*e^20*f^10*(a^2*c*f^2 - b^2*c*e^2
)^8 + 2774483200*a^12*b^18*c^5*e^18*f^12*(a^2*c*f^2 - b^2*c*e^2)^8 - 108696
57600*a^14*b^16*c^5*e^16*f^14*(a^2*c*f^2 - b^2*c*e^2)^8 + 25237416960*a^16*
b^14*c^5*e^14*f^16*(a^2*c*f^2 - b^2*c*e^2)^8 - 38348909568*a^18*b^12*c^5*e^
12*f^18*(a^2*c*f^2 - b^2*c*e^2)^8 + 39084659712*a^20*b^10*c^5*e^10*f^20*(a^
2*c*f^2 - b^2*c*e^2)^8 - 26118635520*a^22*b^8*c^5*e^8*f^22*(a^2*c*f^2 - b^2
*c*e^2)^8 + 10414620672*a^24*b^6*c^5*e^6*f^24*(a^2*c*f^2 - b^2*c*e^2)^8 - 1
708654592*a^26*b^4*c^5*e^4*f^26*(a^2*c*f^2 - b^2*c*e^2)^8 - 276561920*a^28*
b^2*c^5*e^2*f^28*(a^2*c*f^2 - b^2*c*e^2)^8 - 9704448*a^4*b^28*c^6*e^28*f^4*
(a^2*c*f^2 - b^2*c*e^2)^7 + 260614656*a^6*b^26*c^6*e^26*f^6*(a^2*c*f^2 - b^
2*c*e^2)^7 - 2166022464*a^8*b^24*c^6*e^24*f^8*(a^2*c*f^2 - b^2*c*e^2)^7 + 8
626147840*a^10*b^22*c^6*e^22*f^10*(a^2*c*f^2 - b^2*c*e^2)^7 - 16771503616*a
^12*b^20*c^6*e^20*f^12*(a^2*c*f^2 - b^2*c*e^2)^7 + 3301800960*a^14*b^18*c^6
*e^18*f^14*(a^2*c*f^2 - b^2*c*e^2)^7 + 67337715968*a^16*b^16*c^6*e^16*f^16*
(a^2*c*f^2 - b^2*c*e^2)^7 - 189857873920*a^18*b^14*c^6*e^14*f^18*(a^2*c*f^2
 - b^2*c*e^2)^7 + 286100259840*a^20*b^12*c^6*e^12*f^20*(a^2*c*f^2 - b^2*c*e
^2)^7 - 275789894656*a^22*b^10*c^6*e^10*f^22*(a^2*c*f^2 - b^2*c*e^2)^7 + 17
3716537344*a^24*b^8*c^6*e^8*f^24*(a^2*c*f^2 - b^2*c*e^2)^7 - 67416424448*a^
26*b^6*c^6*e^6*f^26*(a^2*c*f^2 - b^2*c*e^2)^7 + 12831686656*a^28*b^4*c^6*e^
4*f^28*(a^2*c*f^2 - b^2*c*e^2)^7 + 222560256*a^30*b^2*c^6*e^2*f^30*(a^2*c*f
^2 - b^2*c*e^2)^7 + 2099520*a^2*b^32*c^7*e^32*f^2*(a^2*c*f^2 - b^2*c*e^2)^6
 - 107014608*a^4*b^30*c^7*e^30*f^4*(a^2*c*f^2 - b^2*c*e^2)^6 + 1848335616*a
^6*b^28*c^7*e^28*f^6*(a^2*c*f^2 - b^2*c*e^2)^6 - 15200005312*a^8*b^26*c^7*e
^26*f^8*(a^2*c*f^2 - b^2*c*e^2)^6 + 72612273792*a^10*b^24*c^7*e^24*f^10*(a^
2*c*f^2 - b^2*c*e^2)^6 - 221855779968*a^12*b^22*c^7*e^22*f^12*(a^2*c*f^2 -
b^2*c*e^2)^6 + 450717857536*a^14*b^20*c^7*e^20*f^14*(a^2*c*f^2 - b^2*c*e^2)
^6 - 600578910208*a^16*b^18*c^7*e^18*f^16*(a^2*c*f^2 - b^2*c*e^2)^6 + 45946
4530688*a^18*b^16*c^7*e^16*f^18*(a^2*c*f^2 - b^2*c*e^2)^6 - 33638947840*a^2
0*b^14*c^7*e^14*f^20*(a^2*c*f^2 - b^2*c*e^2)^6 - 376299926528*a^22*b^12*c^7
*e^12*f^22*(a^2*c*f^2 - b^2*c*e^2)^6 + 488874068992*a^24*b^10*c^7*e^10*f^24
*(a^2*c*f^2 - b^2*c*e^2)^6 - 333407809536*a^26*b^8*c^7*e^8*f^26*(a^2*c*f^2
- b^2*c*e^2)^6 + 134140313600*a^28*b^6*c^7*e^6*f^28*(a^2*c*f^2 - b^2*c*e^2)
^6 - 28220915712*a^30*b^4*c^7*e^4*f^30*(a^2*c*f^2 - b^2*c*e^2)^6 + 12305039
36*a^32*b^2*c^7*e^2*f^32*(a^2*c*f^2 - b^2*c*e^2)^6 + 3335904*a^2*b^34*c^8*e
^34*f^2*(a^2*c*f^2 - b^2*c*e^2)^5 - 290521728*a^4*b^32*c^8*e^32*f^4*(a^2*c*
f^2 - b^2*c*e^2)^5 + 4865684544*a^6*b^30*c^8*e^30*f^6*(a^2*c*f^2 - b^2*c*e^
2)^5 - 40437394528*a^8*b^28*c^8*e^28*f^8*(a^2*c*f^2 - b^2*c*e^2)^5 + 205602
254656*a^10*b^26*c^8*e^26*f^10*(a^2*c*f^2 - b^2*c*e^2)^5 - 703885344192*a^1
2*b^24*c^8*e^24*f^12*(a^2*c*f^2 - b^2*c*e^2)^5 + 1709253482624*a^14*b^22*c^
8*e^22*f^14*(a^2*c*f^2 - b^2*c*e^2)^5 - 3029282695168*a^16*b^20*c^8*e^20*f^
16*(a^2*c*f^2 - b^2*c*e^2)^5 + 3966230827520*a^18*b^18*c^8*e^18*f^18*(a^2*c
*f^2 - b^2*c*e^2)^5 - 3822339813632*a^20*b^16*c^8*e^16*f^20*(a^2*c*f^2 - b^
2*c*e^2)^5 + 2640438056960*a^22*b^14*c^8*e^14*f^22*(a^2*c*f^2 - b^2*c*e^2)^
5 - 1208501415936*a^24*b^12*c^8*e^12*f^24*(a^2*c*f^2 - b^2*c*e^2)^5 + 26933
8092544*a^26*b^10*c^8*e^10*f^26*(a^2*c*f^2 - b^2*c*e^2)^5 + 53783212032*a^2
8*b^8*c^8*e^8*f^28*(a^2*c*f^2 - b^2*c*e^2)^5 - 60985360384*a^30*b^6*c^8*e^6
*f^30*(a^2*c*f^2 - b^2*c*e^2)^5 + 17917083648*a^32*b^4*c^8*e^4*f^32*(a^2*c*
f^2 - b^2*c*e^2)^5 - 1558708224*a^34*b^2*c^8*e^2*f^34*(a^2*c*f^2 - b^2*c*e^
```

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2)^5 - 11917692*a^2*b^36*c^9*e^36*f^2*(a^2*c*f^2 - b^2*c*e^2)^4 - 224907516
*a^4*b^34*c^9*e^34*f^4*(a^2*c*f^2 - b^2*c*e^2)^4 + 5303932560*a^6*b^32*c^9*
e^32*f^6*(a^2*c*f^2 - b^2*c*e^2)^4 - 48206418480*a^8*b^30*c^9*e^30*f^8*(a^2
*c*f^2 - b^2*c*e^2)^4 + 261450609120*a^10*b^28*c^9*e^28*f^10*(a^2*c*f^2 - b
^2*c*e^2)^4 - 962361040256*a^12*b^26*c^9*e^26*f^12*(a^2*c*f^2 - b^2*c*e^2)^
4 + 2558559358080*a^14*b^24*c^9*e^24*f^14*(a^2*c*f^2 - b^2*c*e^2)^4 - 50918
04150656*a^16*b^22*c^9*e^22*f^16*(a^2*c*f^2 - b^2*c*e^2)^4 + 7750806514944*
a^18*b^20*c^9*e^20*f^18*(a^2*c*f^2 - b^2*c*e^2)^4 - 9137207485952*a^20*b^18
*c^9*e^18*f^20*(a^2*c*f^2 - b^2*c*e^2)^4 + 8384563280128*a^22*b^16*c^9*e^16
*f^22*(a^2*c*f^2 - b^2*c*e^2)^4 - 5975281259520*a^24*b^14*c^9*e^14*f^24*(a^
2*c*f^2 - b^2*c*e^2)^4 + 3269297268736*a^26*b^12*c^9*e^12*f^26*(a^2*c*f^2 -
 b^2*c*e^2)^4 - 1339171540992*a^28*b^10*c^9*e^10*f^28*(a^2*c*f^2 - b^2*c*e^
2)^4 + 391250194432*a^30*b^8*c^9*e^8*f^30*(a^2*c*f^2 - b^2*c*e^2)^4 - 74114
154496*a^32*b^6*c^9*e^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^4 + 7299203072*a^34*b^
4*c^9*e^4*f^34*(a^2*c*f^2 - b^2*c*e^2)^4 - 148635648*a^36*b^2*c^9*e^2*f^36*
(a^2*c*f^2 - b^2*c*e^2)^4 - 38704068*a^2*b^38*c^10*e^38*f^2*(a^2*c*f^2 - b^
2*c*e^2)^3 + 188845992*a^4*b^36*c^10*e^36*f^4*(a^2*c*f^2 - b^2*c*e^2)^3 + 1
157124204*a^6*b^34*c^10*e^34*f^6*(a^2*c*f^2 - b^2*c*e^2)^3 - 20586361424*a^
8*b^32*c^10*e^32*f^8*(a^2*c*f^2 - b^2*c*e^2)^3 + 135395499200*a^10*b^30*c^1
0*e^30*f^10*(a^2*c*f^2 - b^2*c*e^2)^3 - 555513858464*a^12*b^28*c^10*e^28*f^
12*(a^2*c*f^2 - b^2*c*e^2)^3 + 1608776388864*a^14*b^26*c^10*e^26*f^14*(a^2*
c*f^2 - b^2*c*e^2)^3 - 3473989271488*a^16*b^24*c^10*e^24*f^16*(a^2*c*f^2 -
b^2*c*e^2)^3 + 5766181411456*a^18*b^22*c^10*e^22*f^18*(a^2*c*f^2 - b^2*c*e^
2)^3 - 7493983209472*a^20*b^20*c^10*e^20*f^20*(a^2*c*f^2 - b^2*c*e^2)^3 + 7
713917084672*a^22*b^18*c^10*e^18*f^22*(a^2*c*f^2 - b^2*c*e^2)^3 - 632846729
3184*a^24*b^16*c^10*e^16*f^24*(a^2*c*f^2 - b^2*c*e^2)^3 + 4142950034432*a^2
6*b^14*c^10*e^14*f^26*(a^2*c*f^2 - b^2*c*e^2)^3 - 2152681536512*a^28*b^12*c
^10*e^12*f^28*(a^2*c*f^2 - b^2*c*e^2)^3 + 874199511040*a^30*b^10*c^10*e^10*
f^30*(a^2*c*f^2 - b^2*c*e^2)^3 - 268759150592*a^32*b^8*c^10*e^8*f^32*(a^2*c
*f^2 - b^2*c*e^2)^3 + 58872545280*a^34*b^6*c^10*e^6*f^34*(a^2*c*f^2 - b^2*c
*e^2)^3 - 8151957504*a^36*b^4*c^10*e^4*f^36*(a^2*c*f^2 - b^2*c*e^2)^3 + 530
841600*a^38*b^2*c^10*e^2*f^38*(a^2*c*f^2 - b^2*c*e^2)^3 - 42743457*a^2*b^40
*c^11*e^40*f^2*(a^2*c*f^2 - b^2*c*e^2)^2 + 411055884*a^4*b^38*c^11*e^38*f^4
*(a^2*c*f^2 - b^2*c*e^2)^2 - 2180887236*a^6*b^36*c^11*e^36*f^6*(a^2*c*f^2 -
 b^2*c*e^2)^2 + 6404946508*a^8*b^34*c^11*e^34*f^8*(a^2*c*f^2 - b^2*c*e^2)^2
 - 5434005264*a^10*b^32*c^11*e^32*f^10*(a^2*c*f^2 - b^2*c*e^2)^2 - 38868373
520*a^12*b^30*c^11*e^30*f^12*(a^2*c*f^2 - b^2*c*e^2)^2 + 208447613600*a^14*
b^28*c^11*e^28*f^14*(a^2*c*f^2 - b^2*c*e^2)^2 - 579674999104*a^16*b^26*c^11
*e^26*f^16*(a^2*c*f^2 - b^2*c*e^2)^2 + 1104967566592*a^18*b^24*c^11*e^24*f^
18*(a^2*c*f^2 - b^2*c*e^2)^2 - 1554566531328*a^20*b^22*c^11*e^22*f^20*(a^2*
c*f^2 - b^2*c*e^2)^2 + 1659734381312*a^22*b^20*c^11*e^20*f^22*(a^2*c*f^2 -
b^2*c*e^2)^2 - 1356361512192*a^24*b^18*c^11*e^18*f^24*(a^2*c*f^2 - b^2*c*e^
2)^2 + 845331359744*a^26*b^16*c^11*e^16*f^26*(a^2*c*f^2 - b^2*c*e^2)^2 - 39
5676895232*a^28*b^14*c^11*e^14*f^28*(a^2*c*f^2 - b^2*c*e^2)^2 + 13490268979
2*a^30*b^12*c^11*e^12*f^30*(a^2*c*f^2 - b^2*c*e^2)^2 - 31670587392*a^32*b^1
0*c^11*e^10*f^32*(a^2*c*f^2 - b^2*c*e^2)^2 + 4584669184*a^34*b^8*c^11*e^8*f
^34*(a^2*c*f^2 - b^2*c*e^2)^2 - 309657600*a^36*b^6*c^11*e^6*f^36*(a^2*c*f^2
 - b^2*c*e^2)^2) - (4*a^(3/2)*b^6*c^2*e^6*f^3*(a*c)^(3/2)*(2*a^2*c*f^2 - b^
2*c*e^2)*(4*a^2*c*f^2 - 3*b^2*c*e^2)*((4096*(112*C^4*a^4*b^8*c^4*e^10 + 448
*C^4*a^12*c^4*e^2*f^8 - 668*C^4*a^6*b^6*c^4*e^8*f^2 + 1440*C^4*a^8*b^4*c^4*
e^6*f^4 - 1328*C^4*a^10*b^2*c^4*e^4*f^6))/(b^16*e^14*f^4 - 4*a^2*b^14*e^12*
f^6 + 6*a^4*b^12*e^10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12) + (4096
*C^4*e^4*(2*a^2*f^2 - b^2*e^2)^4*(9*a^2*b^14*c^6*e^12*f^6 - 47*a^4*b^12*c^6
*e^10*f^8 + 98*a^6*b^10*c^6*e^8*f^10 - 102*a^8*b^8*c^6*e^6*f^12 + 53*a^10*b
^6*c^6*e^4*f^14 - 11*a^12*b^4*c^6*e^2*f^16))/(f^8*(a*f + b*e)^4*(a*f - b*e)
^4*(a^2*c*f^2 - b^2*c*e^2)^2*(b^16*e^14*f^4 - 4*a^2*b^14*e^12*f^6 + 6*a^4*b
^12*e^10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12)) + (4096*C^2*e^2*(2*
a^2*f^2 - b^2*e^2)^2*(9*C^2*a^2*b^12*c^5*e^12*f^2 - 144*C^2*a^14*c^5*f^14 +
 74*C^2*a^4*b^10*c^5*e^10*f^4 - 519*C^2*a^6*b^8*c^5*e^8*f^6 + 1168*C^2*a^8*
```

b^6*c^5*e^6*f^8 - 1264*C^2*a^10*b^4*c^5*e^4*f^10 + 676*C^2*a^12*b^2*c^5*e^2*f^12))/(f^4*(a*f + b*e)^2*(a*f - b*e)^2*(a^2*c*f^2 - b^2*c*e^2)*(b^16*e^14*f^4 - 4*a^2*b^14*e^12*f^6 + 6*a^4*b^12*e^10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12)))*(4*a^6*c*f^6 - 3*b^6*c*e^6 + 8*a^2*b^4*c*e^4*f^2 - 8*a^4*b^2*c*e^2*f^4)^4)/((b^2*c*e^2 - a^2*c*f^2)^(1/2)*(164025*b^46*c^13*e^46 + 885735*b^44*c^12*e^44*(a^2*c*f^2 - b^2*c*e^2) + 117440512*a^30*c^5*f^30*(a^2*c*f^2 - b^2*c*e^2)^8 - 385875968*a^32*c^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^7 + 419430400*a^34*c^7*f^34*(a^2*c*f^2 - b^2*c*e^2)^6 - 150994944*a^36*c^8*f^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 236196*b^36*c^8*e^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 1102248*b^38*c^9*e^38*(a^2*c*f^2 - b^2*c*e^2)^4 + 2053593*b^40*c^10*e^40*(a^2*c*f^2 - b^2*c*e^2)^3 + 1909251*b^42*c^11*e^42*(a^2*c*f^2 - b^2*c*e^2)^2 - 3937329*a^2*b^44*c^13*e^44*f^2 + 43893819*a^4*b^42*c^13*e^42*f^4 - 301507155*a^6*b^40*c^13*e^40*f^6 + 1427514656*a^8*b^38*c^13*e^38*f^8 - 4936911112*a^10*b^36*c^13*e^36*f^10 + 12893273616*a^12*b^34*c^13*e^34*f^12 - 25921630432*a^14*b^32*c^13*e^32*f^14 + 40519286096*a^16*b^30*c^13*e^30*f^16 - 49376608256*a^18*b^28*c^13*e^28*f^18 + 46721401856*a^20*b^26*c^13*e^26*f^20 - 33946324736*a^22*b^24*c^13*e^24*f^22 + 18556579328*a^24*b^22*c^13*e^22*f^24 - 7375276032*a^26*b^20*c^13*e^20*f^26 + 2009817088*a^28*b^18*c^13*e^18*f^28 - 335642624*a^30*b^16*c^13*e^16*f^30 + 25907200*a^32*b^14*c^13*e^14*f^32 - 21130794*a^2*b^42*c^12*e^42*f^2*(a^2*c*f^2 - b^2*c*e^2) + 234399015*a^4*b^40*c^12*e^40*f^4*(a^2*c*f^2 - b^2*c*e^2) - 1604168280*a^6*b^38*c^12*e^38*f^6*(a^2*c*f^2 - b^2*c*e^2) + 7579098492*a^8*b^36*c^12*e^36*f^8*(a^2*c*f^2 - b^2*c*e^2) - 26212380172*a^10*b^34*c^12*e^34*f^10*(a^2*c*f^2 - b^2*c*e^2) + 68672994096*a^12*b^32*c^12*e^32*f^12*(a^2*c*f^2 - b^2*c*e^2) - 139160589504*a^14*b^30*c^12*e^30*f^14*(a^2*c*f^2 - b^2*c*e^2) + 220859191808*a^16*b^28*c^12*e^28*f^16*(a^2*c*f^2 - b^2*c*e^2) - 276344315328*a^18*b^26*c^12*e^26*f^18*(a^2*c*f^2 - b^2*c*e^2) + 273130561984*a^20*b^24*c^12*e^24*f^20*(a^2*c*f^2 - b^2*c*e^2) - 212730002688*a^22*b^22*c^12*e^22*f^22*(a^2*c*f^2 - b^2*c*e^2) + 129574234368*a^24*b^20*c^12*e^20*f^24*(a^2*c*f^2 - b^2*c*e^2) - 60770569216*a^26*b^18*c^12*e^18*f^26*(a^2*c*f^2 - b^2*c*e^2) + 21304706048*a^28*b^16*c^12*e^16*f^28*(a^2*c*f^2 - b^2*c*e^2) - 5272965120*a^30*b^14*c^12*e^14*f^30*(a^2*c*f^2 - b^2*c*e^2) + 819441664*a^32*b^12*c^12*e^12*f^32*(a^2*c*f^2 - b^2*c*e^2) - 59392000*a^34*b^10*c^12*e^10*f^34*(a^2*c*f^2 - b^2*c*e^2) + 9289728*a^6*b^24*c^5*e^24*f^6*(a^2*c*f^2 - b^2*c*e^2)^8 - 36884480*a^8*b^22*c^5*e^22*f^8*(a^2*c*f^2 - b^2*c*e^2)^8 - 278604800*a^10*b^20*c^5*e^20*f^10*(a^2*c*f^2 - b^2*c*e^2)^8 + 2774483200*a^12*b^18*c^5*e^18*f^12*(a^2*c*f^2 - b^2*c*e^2)^8 - 10869657600*a^14*b^16*c^5*e^16*f^14*(a^2*c*f^2 - b^2*c*e^2)^8 + 25237416960*a^16*b^14*c^5*e^14*f^16*(a^2*c*f^2 - b^2*c*e^2)^8 - 38348909568*a^18*b^12*c^5*e^12*f^18*(a^2*c*f^2 - b^2*c*e^2)^8 + 39084659712*a^20*b^10*c^5*e^10*f^20*(a^2*c*f^2 - b^2*c*e^2)^8 - 26118635520*a^22*b^8*c^5*e^8*f^22*(a^2*c*f^2 - b^2*c*e^2)^8 + 10414620672*a^24*b^6*c^5*e^6*f^24*(a^2*c*f^2 - b^2*c*e^2)^8 - 1708654592*a^26*b^4*c^5*e^4*f^26*(a^2*c*f^2 - b^2*c*e^2)^8 - 276561920*a^28*b^2*c^5*e^2*f^28*(a^2*c*f^2 - b^2*c*e^2)^8 - 9704448*a^4*b^28*c^6*e^28*f^4*(a^2*c*f^2 - b^2*c*e^2)^7 + 260614656*a^6*b^26*c^6*e^26*f^6*(a^2*c*f^2 - b^2*c*e^2)^7 - 2166022464*a^8*b^24*c^6*e^24*f^8*(a^2*c*f^2 - b^2*c*e^2)^7 + 8626147840*a^10*b^22*c^6*e^22*f^10*(a^2*c*f^2 - b^2*c*e^2)^7 - 16771503616*a^12*b^20*c^6*e^20*f^12*(a^2*c*f^2 - b^2*c*e^2)^7 + 3301800960*a^14*b^18*c^6*e^18*f^14*(a^2*c*f^2 - b^2*c*e^2)^7 + 67337715968*a^16*b^16*c^6*e^16*f^16*(a^2*c*f^2 - b^2*c*e^2)^7 - 189857873920*a^18*b^14*c^6*e^14*f^18*(a^2*c*f^2 - b^2*c*e^2)^7 + 286100259840*a^20*b^12*c^6*e^12*f^20*(a^2*c*f^2 - b^2*c*e^2)^7 - 275789894656*a^22*b^10*c^6*e^10*f^22*(a^2*c*f^2 - b^2*c*e^2)^7 + 173716537344*a^24*b^8*c^6*e^8*f^24*(a^2*c*f^2 - b^2*c*e^2)^7 - 67416424448*a^26*b^6*c^6*e^6*f^26*(a^2*c*f^2 - b^2*c*e^2)^7 + 12831686656*a^28*b^4*c^6*e^4*f^28*(a^2*c*f^2 - b^2*c*e^2)^7 + 222560256*a^30*b^2*c^6*e^2*f^30*(a^2*c*f^2 - b^2*c*e^2)^7 + 2099520*a^2*b^32*c^7*e^32*f^2*(a^2*c*f^2 - b^2*c*e^2)^6 - 107014608*a^4*b^30*c^7*e^30*f^4*(a^2*c*f^2 - b^2*c*e^2)^6 + 1848335616*a^6*b^28*c^7*e^28*f^6*(a^2*c*f^2 - b^2*c*e^2)^6 - 15200005312*a^8*b^26*c^7*e^26*f^8*(a^2*c*f^2 - b^2*c*e^2)^6 + 72612273792*a^10*b^24*c^7*e^24*f^10*(a^2*c*f^2 - b^2*c*e^2)^6 - 221855779968*a^12

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*b^22*c^7*e^22*f^12*(a^2*c*f^2 - b^2*c*e^2)^6 + 450717857536*a^14*b^20*c^7*
e^20*f^14*(a^2*c*f^2 - b^2*c*e^2)^6 - 600578910208*a^16*b^18*c^7*e^18*f^16*
(a^2*c*f^2 - b^2*c*e^2)^6 + 459464530688*a^18*b^16*c^7*e^16*f^18*(a^2*c*f^2
 - b^2*c*e^2)^6 - 33638947840*a^20*b^14*c^7*e^14*f^20*(a^2*c*f^2 - b^2*c*e^
2)^6 - 376299926528*a^22*b^12*c^7*e^12*f^22*(a^2*c*f^2 - b^2*c*e^2)^6 + 488
874068992*a^24*b^10*c^7*e^10*f^24*(a^2*c*f^2 - b^2*c*e^2)^6 - 333407809536*
a^26*b^8*c^7*e^8*f^26*(a^2*c*f^2 - b^2*c*e^2)^6 + 134140313600*a^28*b^6*c^7
*e^6*f^28*(a^2*c*f^2 - b^2*c*e^2)^6 - 28220915712*a^30*b^4*c^7*e^4*f^30*(a^
2*c*f^2 - b^2*c*e^2)^6 + 1230503936*a^32*b^2*c^7*e^2*f^32*(a^2*c*f^2 - b^2*
c*e^2)^6 + 3335904*a^2*b^34*c^8*e^34*f^2*(a^2*c*f^2 - b^2*c*e^2)^5 - 290521
728*a^4*b^32*c^8*e^32*f^4*(a^2*c*f^2 - b^2*c*e^2)^5 + 4865684544*a^6*b^30*c
^8*e^30*f^6*(a^2*c*f^2 - b^2*c*e^2)^5 - 40437394528*a^8*b^28*c^8*e^28*f^8*(
a^2*c*f^2 - b^2*c*e^2)^5 + 205602254656*a^10*b^26*c^8*e^26*f^10*(a^2*c*f^2
 - b^2*c*e^2)^5 - 703885344192*a^12*b^24*c^8*e^24*f^12*(a^2*c*f^2 - b^2*c*e^
2)^5 + 1709253482624*a^14*b^22*c^8*e^22*f^14*(a^2*c*f^2 - b^2*c*e^2)^5 - 30
29282695168*a^16*b^20*c^8*e^20*f^16*(a^2*c*f^2 - b^2*c*e^2)^5 + 39662308275
20*a^18*b^18*c^8*e^18*f^18*(a^2*c*f^2 - b^2*c*e^2)^5 - 3822339813632*a^20*b
^16*c^8*e^16*f^20*(a^2*c*f^2 - b^2*c*e^2)^5 + 2640438056960*a^22*b^14*c^8*e
^14*f^22*(a^2*c*f^2 - b^2*c*e^2)^5 - 1208501415936*a^24*b^12*c^8*e^12*f^24*
(a^2*c*f^2 - b^2*c*e^2)^5 + 269338092544*a^26*b^10*c^8*e^10*f^26*(a^2*c*f^2
 - b^2*c*e^2)^5 + 53783212032*a^28*b^8*c^8*e^8*f^28*(a^2*c*f^2 - b^2*c*e^2)
^5 - 60985360384*a^30*b^6*c^8*e^6*f^30*(a^2*c*f^2 - b^2*c*e^2)^5 + 17917083
648*a^32*b^4*c^8*e^4*f^32*(a^2*c*f^2 - b^2*c*e^2)^5 - 1558708224*a^34*b^2*c
^8*e^2*f^34*(a^2*c*f^2 - b^2*c*e^2)^5 - 11917692*a^2*b^36*c^9*e^36*f^2*(a^2
*c*f^2 - b^2*c*e^2)^4 - 224907516*a^4*b^34*c^9*e^34*f^4*(a^2*c*f^2 - b^2*c*
e^2)^4 + 5303932560*a^6*b^32*c^9*e^32*f^6*(a^2*c*f^2 - b^2*c*e^2)^4 - 48206
418480*a^8*b^30*c^9*e^30*f^8*(a^2*c*f^2 - b^2*c*e^2)^4 + 261450609120*a^10*
b^28*c^9*e^28*f^10*(a^2*c*f^2 - b^2*c*e^2)^4 - 962361040256*a^12*b^26*c^9*e
^26*f^12*(a^2*c*f^2 - b^2*c*e^2)^4 + 2558559358080*a^14*b^24*c^9*e^24*f^14*
(a^2*c*f^2 - b^2*c*e^2)^4 - 5091804150656*a^16*b^22*c^9*e^22*f^16*(a^2*c*f^
2 - b^2*c*e^2)^4 + 7750806514944*a^18*b^20*c^9*e^20*f^18*(a^2*c*f^2 - b^2*c
*e^2)^4 - 9137207485952*a^20*b^18*c^9*e^18*f^20*(a^2*c*f^2 - b^2*c*e^2)^4 +
 8384563280128*a^22*b^16*c^9*e^16*f^22*(a^2*c*f^2 - b^2*c*e^2)^4 - 59752812
59520*a^24*b^14*c^9*e^14*f^24*(a^2*c*f^2 - b^2*c*e^2)^4 + 3269297268736*a^2
6*b^12*c^9*e^12*f^26*(a^2*c*f^2 - b^2*c*e^2)^4 - 1339171540992*a^28*b^10*c^
9*e^10*f^28*(a^2*c*f^2 - b^2*c*e^2)^4 + 391250194432*a^30*b^8*c^9*e^8*f^30*
(a^2*c*f^2 - b^2*c*e^2)^4 - 74114154496*a^32*b^6*c^9*e^6*f^32*(a^2*c*f^2 -
b^2*c*e^2)^4 + 7299203072*a^34*b^4*c^9*e^4*f^34*(a^2*c*f^2 - b^2*c*e^2)^4 -
 148635648*a^36*b^2*c^9*e^2*f^36*(a^2*c*f^2 - b^2*c*e^2)^4 - 38704068*a^2*b
^38*c^10*e^38*f^2*(a^2*c*f^2 - b^2*c*e^2)^3 + 188845992*a^4*b^36*c^10*e^36*
f^4*(a^2*c*f^2 - b^2*c*e^2)^3 + 1157124204*a^6*b^34*c^10*e^34*f^6*(a^2*c*f^
2 - b^2*c*e^2)^3 - 20586361424*a^8*b^32*c^10*e^32*f^8*(a^2*c*f^2 - b^2*c*e^
2)^3 + 135395499200*a^10*b^30*c^10*e^30*f^10*(a^2*c*f^2 - b^2*c*e^2)^3 - 55
5513858464*a^12*b^28*c^10*e^28*f^12*(a^2*c*f^2 - b^2*c*e^2)^3 + 16087763888
64*a^14*b^26*c^10*e^26*f^14*(a^2*c*f^2 - b^2*c*e^2)^3 - 3473989271488*a^16*
b^24*c^10*e^24*f^16*(a^2*c*f^2 - b^2*c*e^2)^3 + 5766181411456*a^18*b^22*c^1
0*e^22*f^18*(a^2*c*f^2 - b^2*c*e^2)^3 - 7493983209472*a^20*b^20*c^10*e^20*f
^20*(a^2*c*f^2 - b^2*c*e^2)^3 + 7713917084672*a^22*b^18*c^10*e^18*f^22*(a^2
*c*f^2 - b^2*c*e^2)^3 - 6328467293184*a^24*b^16*c^10*e^16*f^24*(a^2*c*f^2 -
 b^2*c*e^2)^3 + 4142950034432*a^26*b^14*c^10*e^14*f^26*(a^2*c*f^2 - b^2*c*e
^2)^3 - 2152681536512*a^28*b^12*c^10*e^12*f^28*(a^2*c*f^2 - b^2*c*e^2)^3 +
874199511040*a^30*b^10*c^10*e^10*f^30*(a^2*c*f^2 - b^2*c*e^2)^3 - 268759150
592*a^32*b^8*c^10*e^8*f^32*(a^2*c*f^2 - b^2*c*e^2)^3 + 58872545280*a^34*b^6
*c^10*e^6*f^34*(a^2*c*f^2 - b^2*c*e^2)^3 - 8151957504*a^36*b^4*c^10*e^4*f^3
6*(a^2*c*f^2 - b^2*c*e^2)^3 + 530841600*a^38*b^2*c^10*e^2*f^38*(a^2*c*f^2 -
 b^2*c*e^2)^3 - 42743457*a^2*b^40*c^11*e^40*f^2*(a^2*c*f^2 - b^2*c*e^2)^2 +
 411055884*a^4*b^38*c^11*e^38*f^4*(a^2*c*f^2 - b^2*c*e^2)^2 - 2180887236*a^
6*b^36*c^11*e^36*f^6*(a^2*c*f^2 - b^2*c*e^2)^2 + 6404946508*a^8*b^34*c^11*e
^34*f^8*(a^2*c*f^2 - b^2*c*e^2)^2 - 5434005264*a^10*b^32*c^11*e^32*f^10*(a^
```

2*c*f^2 - b^2*c*e^2)^2 - 38868373520*a^12*b^30*c^11*e^30*f^12*(a^2*c*f^2 -
b^2*c*e^2)^2 + 208447613600*a^14*b^28*c^11*e^28*f^14*(a^2*c*f^2 - b^2*c*e^2
)^2 - 579674999104*a^16*b^26*c^11*e^26*f^16*(a^2*c*f^2 - b^2*c*e^2)^2 + 110
4967566592*a^18*b^24*c^11*e^24*f^18*(a^2*c*f^2 - b^2*c*e^2)^2 - 15545665313
28*a^20*b^22*c^11*e^22*f^20*(a^2*c*f^2 - b^2*c*e^2)^2 + 1659734381312*a^22*
b^20*c^11*e^20*f^22*(a^2*c*f^2 - b^2*c*e^2)^2 - 1356361512192*a^24*b^18*c^1
1*e^18*f^24*(a^2*c*f^2 - b^2*c*e^2)^2 + 845331359744*a^26*b^16*c^11*e^16*f^
26*(a^2*c*f^2 - b^2*c*e^2)^2 - 395676895232*a^28*b^14*c^11*e^14*f^28*(a^2*c
*f^2 - b^2*c*e^2)^2 + 134902689792*a^30*b^12*c^11*e^12*f^30*(a^2*c*f^2 - b^
2*c*e^2)^2 - 31670587392*a^32*b^10*c^11*e^10*f^32*(a^2*c*f^2 - b^2*c*e^2)^2
 + 4584669184*a^34*b^8*c^11*e^8*f^34*(a^2*c*f^2 - b^2*c*e^2)^2 - 309657600*
a^36*b^6*c^11*e^6*f^36*(a^2*c*f^2 - b^2*c*e^2)^2)))*(b^16*e^12*f^6*(a^2*c*f
^2 - b^2*c*e^2)^2 - 4*a^2*b^14*e^10*f^8*(a^2*c*f^2 - b^2*c*e^2)^2 + 6*a^4*b
^12*e^8*f^10*(a^2*c*f^2 - b^2*c*e^2)^2 - 4*a^6*b^10*e^6*f^12*(a^2*c*f^2 - b
^2*c*e^2)^2 + a^8*b^8*e^4*f^14*(a^2*c*f^2 - b^2*c*e^2)^2))/(((a + b*x)^(1/2
) - a^(1/2))^2*(16384*C^4*a^6*c^3*f^4 + 4096*C^4*a^2*b^4*c^3*e^4 - 16384*C^
4*a^4*b^2*c^3*e^2*f^2)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*((2*a^4*b^
5*c^3*e^5*f^4*(4*a^2*c*f^2 - 3*b^2*c*e^2)^2*((4096*(112*C^4*a^4*b^8*c^4*e^1
0 + 448*C^4*a^12*c^4*e^2*f^8 - 668*C^4*a^6*b^6*c^4*e^8*f^2 + 1440*C^4*a^8*b
^4*c^4*e^6*f^4 - 1328*C^4*a^10*b^2*c^4*e^4*f^6))/(b^16*e^14*f^4 - 4*a^2*b^1
4*e^12*f^6 + 6*a^4*b^12*e^10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12)
+ (4096*C^4*e^4*(2*a^2*f^2 - b^2*e^2)^4*(9*a^2*b^14*c^6*e^12*f^6 - 47*a^4*b
^12*c^6*e^10*f^8 + 98*a^6*b^10*c^6*e^8*f^10 - 102*a^8*b^8*c^6*e^6*f^12 + 53
*a^10*b^6*c^6*e^4*f^14 - 11*a^12*b^4*c^6*e^2*f^16))/(f^8*(a*f + b*e)^4*(a*f
 - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)^2*(b^16*e^14*f^4 - 4*a^2*b^14*e^12*f^6 +
6*a^4*b^12*e^10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12)) + (4096*C^2*
e^2*(2*a^2*f^2 - b^2*e^2)^2*(9*C^2*a^2*b^12*c^5*e^12*f^2 - 144*C^2*a^14*c^5
*f^14 + 74*C^2*a^4*b^10*c^5*e^10*f^4 - 519*C^2*a^6*b^8*c^5*e^8*f^6 + 1168*C
^2*a^8*b^6*c^5*e^6*f^8 - 1264*C^2*a^10*b^4*c^5*e^4*f^10 + 676*C^2*a^12*b^2*
c^5*e^2*f^12))/(f^4*(a*f + b*e)^2*(a*f - b*e)^2*(a^2*c*f^2 - b^2*c*e^2)*(b^
16*e^14*f^4 - 4*a^2*b^14*e^12*f^6 + 6*a^4*b^12*e^10*f^8 - 4*a^6*b^10*e^8*f^
10 + a^8*b^8*e^6*f^12)))*(4*a^6*c*f^6 - 3*b^6*c*e^6 + 8*a^2*b^4*c*e^4*f^2 -
 8*a^4*b^2*c*e^2*f^4)^4)/((b^2*c*e^2 - a^2*c*f^2)^(1/2)*(164025*b^46*c^13*e
^46 + 885735*b^44*c^12*e^44*(a^2*c*f^2 - b^2*c*e^2) + 117440512*a^30*c^5*f^
30*(a^2*c*f^2 - b^2*c*e^2)^8 - 385875968*a^32*c^6*f^32*(a^2*c*f^2 - b^2*c*e
^2)^7 + 419430400*a^34*c^7*f^34*(a^2*c*f^2 - b^2*c*e^2)^6 - 150994944*a^36*
c^8*f^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 236196*b^36*c^8*e^36*(a^2*c*f^2 - b^2*
c*e^2)^5 + 1102248*b^38*c^9*e^38*(a^2*c*f^2 - b^2*c*e^2)^4 + 2053593*b^40*c
^10*e^40*(a^2*c*f^2 - b^2*c*e^2)^3 + 1909251*b^42*c^11*e^42*(a^2*c*f^2 - b^
2*c*e^2)^2 - 3937329*a^2*b^44*c^13*e^44*f^2 + 43893819*a^4*b^42*c^13*e^42*f
^4 - 301507155*a^6*b^40*c^13*e^40*f^6 + 1427514656*a^8*b^38*c^13*e^38*f^8 -
 4936911112*a^10*b^36*c^13*e^36*f^10 + 12893273616*a^12*b^34*c^13*e^34*f^12
 - 25921630432*a^14*b^32*c^13*e^32*f^14 + 40519286096*a^16*b^30*c^13*e^30*f
^16 - 49376608256*a^18*b^28*c^13*e^28*f^18 + 46721401856*a^20*b^26*c^13*e^2
6*f^20 - 33946324736*a^22*b^24*c^13*e^24*f^22 + 18556579328*a^24*b^22*c^13*
e^22*f^24 - 7375276032*a^26*b^20*c^13*e^20*f^26 + 2009817088*a^28*b^18*c^13
*e^18*f^28 - 335642624*a^30*b^16*c^13*e^16*f^30 + 25907200*a^32*b^14*c^13*e
^14*f^32 - 21130794*a^2*b^42*c^12*e^42*f^2*(a^2*c*f^2 - b^2*c*e^2) + 234399
015*a^4*b^40*c^12*e^40*f^4*(a^2*c*f^2 - b^2*c*e^2) - 1604168280*a^6*b^38*c^
12*e^38*f^6*(a^2*c*f^2 - b^2*c*e^2) + 7579098492*a^8*b^36*c^12*e^36*f^8*(a^
2*c*f^2 - b^2*c*e^2) - 26212380172*a^10*b^34*c^12*e^34*f^10*(a^2*c*f^2 - b^
2*c*e^2) + 68672994096*a^12*b^32*c^12*e^32*f^12*(a^2*c*f^2 - b^2*c*e^2) - 1
39160589504*a^14*b^30*c^12*e^30*f^14*(a^2*c*f^2 - b^2*c*e^2) + 220859191808
*a^16*b^28*c^12*e^28*f^16*(a^2*c*f^2 - b^2*c*e^2) - 276344315328*a^18*b^26*
c^12*e^26*f^18*(a^2*c*f^2 - b^2*c*e^2) + 273130561984*a^20*b^24*c^12*e^24*f
^20*(a^2*c*f^2 - b^2*c*e^2) - 212730002688*a^22*b^22*c^12*e^22*f^22*(a^2*c*
f^2 - b^2*c*e^2) + 129574234368*a^24*b^20*c^12*e^20*f^24*(a^2*c*f^2 - b^2*c
*e^2) - 60770569216*a^26*b^18*c^12*e^18*f^26*(a^2*c*f^2 - b^2*c*e^2) + 2130
4706048*a^28*b^16*c^12*e^16*f^28*(a^2*c*f^2 - b^2*c*e^2) - 5272965120*a^30*

```
b^14*c^12*e^14*f^30*(a^2*c*f^2 - b^2*c*e^2) + 819441664*a^32*b^12*c^12*e^12
*f^32*(a^2*c*f^2 - b^2*c*e^2) - 59392000*a^34*b^10*c^12*e^10*f^34*(a^2*c*f^
2 - b^2*c*e^2) + 9289728*a^6*b^24*c^5*e^24*f^6*(a^2*c*f^2 - b^2*c*e^2)^8 -
36884480*a^8*b^22*c^5*e^22*f^8*(a^2*c*f^2 - b^2*c*e^2)^8 - 278604800*a^10*b
^20*c^5*e^20*f^10*(a^2*c*f^2 - b^2*c*e^2)^8 + 2774483200*a^12*b^18*c^5*e^18
*f^12*(a^2*c*f^2 - b^2*c*e^2)^8 - 10869657600*a^14*b^16*c^5*e^16*f^14*(a^2*
c*f^2 - b^2*c*e^2)^8 + 25237416960*a^16*b^14*c^5*e^14*f^16*(a^2*c*f^2 - b^2
*c*e^2)^8 - 38348909568*a^18*b^12*c^5*e^12*f^18*(a^2*c*f^2 - b^2*c*e^2)^8 +
 39084659712*a^20*b^10*c^5*e^10*f^20*(a^2*c*f^2 - b^2*c*e^2)^8 - 2611863552
0*a^22*b^8*c^5*e^8*f^22*(a^2*c*f^2 - b^2*c*e^2)^8 + 10414620672*a^24*b^6*c^
5*e^6*f^24*(a^2*c*f^2 - b^2*c*e^2)^8 - 1708654592*a^26*b^4*c^5*e^4*f^26*(a^
2*c*f^2 - b^2*c*e^2)^8 - 276561920*a^28*b^2*c^5*e^2*f^28*(a^2*c*f^2 - b^2*c
*e^2)^8 - 9704448*a^4*b^28*c^6*e^28*f^4*(a^2*c*f^2 - b^2*c*e^2)^7 + 2606146
56*a^6*b^26*c^6*e^26*f^6*(a^2*c*f^2 - b^2*c*e^2)^7 - 2166022464*a^8*b^24*c^
6*e^24*f^8*(a^2*c*f^2 - b^2*c*e^2)^7 + 8626147840*a^10*b^22*c^6*e^22*f^10*(
a^2*c*f^2 - b^2*c*e^2)^7 - 16771503616*a^12*b^20*c^6*e^20*f^12*(a^2*c*f^2 -
 b^2*c*e^2)^7 + 3301800960*a^14*b^18*c^6*e^18*f^14*(a^2*c*f^2 - b^2*c*e^2)^
7 + 67337715968*a^16*b^16*c^6*e^16*f^16*(a^2*c*f^2 - b^2*c*e^2)^7 - 1898578
73920*a^18*b^14*c^6*e^14*f^18*(a^2*c*f^2 - b^2*c*e^2)^7 + 286100259840*a^20
*b^12*c^6*e^12*f^20*(a^2*c*f^2 - b^2*c*e^2)^7 - 275789894656*a^22*b^10*c^6*
e^10*f^22*(a^2*c*f^2 - b^2*c*e^2)^7 + 173716537344*a^24*b^8*c^6*e^8*f^24*(a
^2*c*f^2 - b^2*c*e^2)^7 - 67416424448*a^26*b^6*c^6*e^6*f^26*(a^2*c*f^2 - b^
2*c*e^2)^7 + 12831686656*a^28*b^4*c^6*e^4*f^28*(a^2*c*f^2 - b^2*c*e^2)^7 +
222560256*a^30*b^2*c^6*e^2*f^30*(a^2*c*f^2 - b^2*c*e^2)^7 + 2099520*a^2*b^3
2*c^7*e^32*f^2*(a^2*c*f^2 - b^2*c*e^2)^6 - 107014608*a^4*b^30*c^7*e^30*f^4*
(a^2*c*f^2 - b^2*c*e^2)^6 + 1848335616*a^6*b^28*c^7*e^28*f^6*(a^2*c*f^2 - b
^2*c*e^2)^6 - 15200005312*a^8*b^26*c^7*e^26*f^8*(a^2*c*f^2 - b^2*c*e^2)^6 +
 72612273792*a^10*b^24*c^7*e^24*f^10*(a^2*c*f^2 - b^2*c*e^2)^6 - 2218557799
68*a^12*b^22*c^7*e^22*f^12*(a^2*c*f^2 - b^2*c*e^2)^6 + 450717857536*a^14*b^
20*c^7*e^20*f^14*(a^2*c*f^2 - b^2*c*e^2)^6 - 600578910208*a^16*b^18*c^7*e^1
8*f^16*(a^2*c*f^2 - b^2*c*e^2)^6 + 459464530688*a^18*b^16*c^7*e^16*f^18*(a^
2*c*f^2 - b^2*c*e^2)^6 - 33638947840*a^20*b^14*c^7*e^14*f^20*(a^2*c*f^2 - b
^2*c*e^2)^6 - 376299926528*a^22*b^12*c^7*e^12*f^22*(a^2*c*f^2 - b^2*c*e^2)^
6 + 488874068992*a^24*b^10*c^7*e^10*f^24*(a^2*c*f^2 - b^2*c*e^2)^6 - 333407
809536*a^26*b^8*c^7*e^8*f^26*(a^2*c*f^2 - b^2*c*e^2)^6 + 134140313600*a^28*
b^6*c^7*e^6*f^28*(a^2*c*f^2 - b^2*c*e^2)^6 - 28220915712*a^30*b^4*c^7*e^4*f
^30*(a^2*c*f^2 - b^2*c*e^2)^6 + 1230503936*a^32*b^2*c^7*e^2*f^32*(a^2*c*f^2
 - b^2*c*e^2)^6 + 3335904*a^2*b^34*c^8*e^34*f^2*(a^2*c*f^2 - b^2*c*e^2)^5 -
 290521728*a^4*b^32*c^8*e^32*f^4*(a^2*c*f^2 - b^2*c*e^2)^5 + 4865684544*a^6
*b^30*c^8*e^30*f^6*(a^2*c*f^2 - b^2*c*e^2)^5 - 40437394528*a^8*b^28*c^8*e^2
8*f^8*(a^2*c*f^2 - b^2*c*e^2)^5 + 205602254656*a^10*b^26*c^8*e^26*f^10*(a^2
*c*f^2 - b^2*c*e^2)^5 - 703885344192*a^12*b^24*c^8*e^24*f^12*(a^2*c*f^2 - b
^2*c*e^2)^5 + 1709253482624*a^14*b^22*c^8*e^22*f^14*(a^2*c*f^2 - b^2*c*e^2)
^5 - 3029282695168*a^16*b^20*c^8*e^20*f^16*(a^2*c*f^2 - b^2*c*e^2)^5 + 3966
230827520*a^18*b^18*c^8*e^18*f^18*(a^2*c*f^2 - b^2*c*e^2)^5 - 3822339813632
*a^20*b^16*c^8*e^16*f^20*(a^2*c*f^2 - b^2*c*e^2)^5 + 2640438056960*a^22*b^1
4*c^8*e^14*f^22*(a^2*c*f^2 - b^2*c*e^2)^5 - 1208501415936*a^24*b^12*c^8*e^1
2*f^24*(a^2*c*f^2 - b^2*c*e^2)^5 + 269338092544*a^26*b^10*c^8*e^10*f^26*(a^
2*c*f^2 - b^2*c*e^2)^5 + 53783212032*a^28*b^8*c^8*e^8*f^28*(a^2*c*f^2 - b^2
*c*e^2)^5 - 60985360384*a^30*b^6*c^8*e^6*f^30*(a^2*c*f^2 - b^2*c*e^2)^5 + 1
7917083648*a^32*b^4*c^8*e^4*f^32*(a^2*c*f^2 - b^2*c*e^2)^5 - 1558708224*a^3
4*b^2*c^8*e^2*f^34*(a^2*c*f^2 - b^2*c*e^2)^5 - 11917692*a^2*b^36*c^9*e^36*f
^2*(a^2*c*f^2 - b^2*c*e^2)^4 - 224907516*a^4*b^34*c^9*e^34*f^4*(a^2*c*f^2 -
 b^2*c*e^2)^4 + 5303932560*a^6*b^32*c^9*e^32*f^6*(a^2*c*f^2 - b^2*c*e^2)^4
- 48206418480*a^8*b^30*c^9*e^30*f^8*(a^2*c*f^2 - b^2*c*e^2)^4 + 26145060912
0*a^10*b^28*c^9*e^28*f^10*(a^2*c*f^2 - b^2*c*e^2)^4 - 962361040256*a^12*b^2
6*c^9*e^26*f^12*(a^2*c*f^2 - b^2*c*e^2)^4 + 2558559358080*a^14*b^24*c^9*e^2
4*f^14*(a^2*c*f^2 - b^2*c*e^2)^4 - 5091804150656*a^16*b^22*c^9*e^22*f^16*(a
^2*c*f^2 - b^2*c*e^2)^4 + 7750806514944*a^18*b^20*c^9*e^20*f^18*(a^2*c*f^2
```

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- b^2*c*e^2)^4 - 9137207485952*a^20*b^18*c^9*e^18*f^20*(a^2*c*f^2 - b^2*c*e
^2)^4 + 8384563280128*a^22*b^16*c^9*e^16*f^22*(a^2*c*f^2 - b^2*c*e^2)^4 - 5
975281259520*a^24*b^14*c^9*e^14*f^24*(a^2*c*f^2 - b^2*c*e^2)^4 + 3269297268
736*a^26*b^12*c^9*e^12*f^26*(a^2*c*f^2 - b^2*c*e^2)^4 - 1339171540992*a^28*
b^10*c^9*e^10*f^28*(a^2*c*f^2 - b^2*c*e^2)^4 + 391250194432*a^30*b^8*c^9*e^
8*f^30*(a^2*c*f^2 - b^2*c*e^2)^4 - 74114154496*a^32*b^6*c^9*e^6*f^32*(a^2*c
*f^2 - b^2*c*e^2)^4 + 7299203072*a^34*b^4*c^9*e^4*f^34*(a^2*c*f^2 - b^2*c*e
^2)^4 - 148635648*a^36*b^2*c^9*e^2*f^36*(a^2*c*f^2 - b^2*c*e^2)^4 - 3870406
8*a^2*b^38*c^10*e^38*f^2*(a^2*c*f^2 - b^2*c*e^2)^3 + 188845992*a^4*b^36*c^1
0*e^36*f^4*(a^2*c*f^2 - b^2*c*e^2)^3 + 1157124204*a^6*b^34*c^10*e^34*f^6*(a
^2*c*f^2 - b^2*c*e^2)^3 - 20586361424*a^8*b^32*c^10*e^32*f^8*(a^2*c*f^2 - b
^2*c*e^2)^3 + 135395499200*a^10*b^30*c^10*e^30*f^10*(a^2*c*f^2 - b^2*c*e^2)
^3 - 555513858464*a^12*b^28*c^10*e^28*f^12*(a^2*c*f^2 - b^2*c*e^2)^3 + 1608
776388864*a^14*b^26*c^10*e^26*f^14*(a^2*c*f^2 - b^2*c*e^2)^3 - 347398927148
8*a^16*b^24*c^10*e^24*f^16*(a^2*c*f^2 - b^2*c*e^2)^3 + 5766181411456*a^18*b
^22*c^10*e^22*f^18*(a^2*c*f^2 - b^2*c*e^2)^3 - 7493983209472*a^20*b^20*c^10
*e^20*f^20*(a^2*c*f^2 - b^2*c*e^2)^3 + 7713917084672*a^22*b^18*c^10*e^18*f^
22*(a^2*c*f^2 - b^2*c*e^2)^3 - 6328467293184*a^24*b^16*c^10*e^16*f^24*(a^2*
c*f^2 - b^2*c*e^2)^3 + 4142950034432*a^26*b^14*c^10*e^14*f^26*(a^2*c*f^2 -
b^2*c*e^2)^3 - 2152681536512*a^28*b^12*c^10*e^12*f^28*(a^2*c*f^2 - b^2*c*e^
2)^3 + 874199511040*a^30*b^10*c^10*e^10*f^30*(a^2*c*f^2 - b^2*c*e^2)^3 - 26
8759150592*a^32*b^8*c^10*e^8*f^32*(a^2*c*f^2 - b^2*c*e^2)^3 + 58872545280*a
^34*b^6*c^10*e^6*f^34*(a^2*c*f^2 - b^2*c*e^2)^3 - 8151957504*a^36*b^4*c^10*
e^4*f^36*(a^2*c*f^2 - b^2*c*e^2)^3 + 530841600*a^38*b^2*c^10*e^2*f^38*(a^2*
c*f^2 - b^2*c*e^2)^3 - 42743457*a^2*b^40*c^11*e^40*f^2*(a^2*c*f^2 - b^2*c*e
^2)^2 + 411055884*a^4*b^38*c^11*e^38*f^4*(a^2*c*f^2 - b^2*c*e^2)^2 - 218088
7236*a^6*b^36*c^11*e^36*f^6*(a^2*c*f^2 - b^2*c*e^2)^2 + 6404946508*a^8*b^34
*c^11*e^34*f^8*(a^2*c*f^2 - b^2*c*e^2)^2 - 5434005264*a^10*b^32*c^11*e^32*f
^10*(a^2*c*f^2 - b^2*c*e^2)^2 - 38868373520*a^12*b^30*c^11*e^30*f^12*(a^2*c
*f^2 - b^2*c*e^2)^2 + 208447613600*a^14*b^28*c^11*e^28*f^14*(a^2*c*f^2 - b^
2*c*e^2)^2 - 579674999104*a^16*b^26*c^11*e^26*f^16*(a^2*c*f^2 - b^2*c*e^2)^
2 + 1104967566592*a^18*b^24*c^11*e^24*f^18*(a^2*c*f^2 - b^2*c*e^2)^2 - 1554
566531328*a^20*b^22*c^11*e^22*f^20*(a^2*c*f^2 - b^2*c*e^2)^2 + 165973438131
2*a^22*b^20*c^11*e^20*f^22*(a^2*c*f^2 - b^2*c*e^2)^2 - 1356361512192*a^24*b
^18*c^11*e^18*f^24*(a^2*c*f^2 - b^2*c*e^2)^2 + 845331359744*a^26*b^16*c^11*
e^16*f^26*(a^2*c*f^2 - b^2*c*e^2)^2 - 395676895232*a^28*b^14*c^11*e^14*f^28
*(a^2*c*f^2 - b^2*c*e^2)^2 + 134902689792*a^30*b^12*c^11*e^12*f^30*(a^2*c*f
^2 - b^2*c*e^2)^2 - 31670587392*a^32*b^10*c^11*e^10*f^32*(a^2*c*f^2 - b^2*c
*e^2)^2 + 4584669184*a^34*b^8*c^11*e^8*f^34*(a^2*c*f^2 - b^2*c*e^2)^2 - 309
657600*a^36*b^6*c^11*e^6*f^36*(a^2*c*f^2 - b^2*c*e^2)^2)) - (2*a^(3/2)*b^5*
c*e^5*f^3*((4096*C^3*e^3*(2*a^2*f^2 - b^2*e^2)^3*(136*C*a^(21/2)*b^2*c^3*e*
f^15*(a*c)^(5/2) - 90*C*a^(3/2)*b^12*c^4*e^11*f^5*(a*c)^(3/2) + 96*C*a^(5/2
)*b^10*c^3*e^9*f^7*(a*c)^(5/2) + 394*C*a^(7/2)*b^10*c^4*e^9*f^7*(a*c)^(3/2)
 - 424*C*a^(9/2)*b^8*c^3*e^7*f^9*(a*c)^(5/2) - 642*C*a^(11/2)*b^8*c^4*e^7*f
^9*(a*c)^(3/2) + 696*C*a^(13/2)*b^6*c^3*e^5*f^11*(a*c)^(5/2) + 462*C*a^(15/
2)*b^6*c^4*e^5*f^11*(a*c)^(3/2) - 504*C*a^(17/2)*b^4*c^3*e^3*f^13*(a*c)^(5/
2) - 124*C*a^(19/2)*b^4*c^4*e^3*f^13*(a*c)^(3/2)))/(f^6*(a*f + b*e)^3*(a*f
- b*e)^3*(b^2*c*e^2 - a^2*c*f^2)^(3/2)*(b^16*e^14*f^4 - 4*a^2*b^14*e^12*f^6
 + 6*a^4*b^12*e^10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12)) - (4096*C
*e*(2*a^2*f^2 - b^2*e^2)*(64*C^3*a^(21/2)*c^2*e*f^11*(a*c)^(5/2) + 32*C^3*a
^(5/2)*b^8*c^2*e^9*f^3*(a*c)^(5/2) + 600*C^3*a^(7/2)*b^8*c^3*e^9*f^3*(a*c)^
(3/2) - 160*C^3*a^(9/2)*b^6*c^2*e^7*f^5*(a*c)^(5/2) - 1376*C^3*a^(11/2)*b^6
*c^3*e^7*f^5*(a*c)^(3/2) + 288*C^3*a^(13/2)*b^4*c^2*e^5*f^7*(a*c)^(5/2) + 1
368*C^3*a^(15/2)*b^4*c^3*e^5*f^7*(a*c)^(3/2) - 224*C^3*a^(17/2)*b^2*c^2*e^3
*f^9*(a*c)^(5/2) - 496*C^3*a^(19/2)*b^2*c^3*e^3*f^9*(a*c)^(3/2) - 96*C^3*a^
(3/2)*b^10*c^3*e^11*f*(a*c)^(3/2)))/(f^2*(a*f + b*e)*(a*f - b*e)*(b^2*c*e^2
 - a^2*c*f^2)^(1/2)*(b^16*e^14*f^4 - 4*a^2*b^14*e^12*f^6 + 6*a^4*b^12*e^10*
f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12)))*(a*c)^(3/2)*(4*a^2*c*f^2 -
b^2*c*e^2)*(4*a^2*c*f^2 - 3*b^2*c*e^2)*(4*a^6*c*f^6 - 3*b^6*c*e^6 + 8*a^2*b
```

^4*c*e^4*f^2 - 8*a^4*b^2*c*e^2*f^4)^4)/(164025*b^46*c^13*e^46 + 885735*b^44
*c^12*e^44*(a^2*c*f^2 - b^2*c*e^2) + 117440512*a^30*c^5*f^30*(a^2*c*f^2 - b
^2*c*e^2)^8 - 385875968*a^32*c^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^7 + 419430400
*a^34*c^7*f^34*(a^2*c*f^2 - b^2*c*e^2)^6 - 150994944*a^36*c^8*f^36*(a^2*c*f
^2 - b^2*c*e^2)^5 + 236196*b^36*c^8*e^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 110224
8*b^38*c^9*e^38*(a^2*c*f^2 - b^2*c*e^2)^4 + 2053593*b^40*c^10*e^40*(a^2*c*f
^2 - b^2*c*e^2)^3 + 1909251*b^42*c^11*e^42*(a^2*c*f^2 - b^2*c*e^2)^2 - 3937
329*a^2*b^44*c^13*e^44*f^2 + 43893819*a^4*b^42*c^13*e^42*f^4 - 301507155*a^
6*b^40*c^13*e^40*f^6 + 1427514656*a^8*b^38*c^13*e^38*f^8 - 4936911112*a^10*
b^36*c^13*e^36*f^10 + 12893273616*a^12*b^34*c^13*e^34*f^12 - 25921630432*a^
14*b^32*c^13*e^32*f^14 + 40519286096*a^16*b^30*c^13*e^30*f^16 - 49376608256
*a^18*b^28*c^13*e^28*f^18 + 46721401856*a^20*b^26*c^13*e^26*f^20 - 33946324
736*a^22*b^24*c^13*e^24*f^22 + 18556579328*a^24*b^22*c^13*e^22*f^24 - 73752
76032*a^26*b^20*c^13*e^20*f^26 + 2009817088*a^28*b^18*c^13*e^18*f^28 - 3356
42624*a^30*b^16*c^13*e^16*f^30 + 25907200*a^32*b^14*c^13*e^14*f^32 - 211307
94*a^2*b^42*c^12*e^42*f^2*(a^2*c*f^2 - b^2*c*e^2) + 234399015*a^4*b^40*c^12
*e^40*f^4*(a^2*c*f^2 - b^2*c*e^2) - 1604168280*a^6*b^38*c^12*e^38*f^6*(a^2*
c*f^2 - b^2*c*e^2) + 7579098492*a^8*b^36*c^12*e^36*f^8*(a^2*c*f^2 - b^2*c*e
^2) - 26212380172*a^10*b^34*c^12*e^34*f^10*(a^2*c*f^2 - b^2*c*e^2) + 686729
94096*a^12*b^32*c^12*e^32*f^12*(a^2*c*f^2 - b^2*c*e^2) - 139160589504*a^14*
b^30*c^12*e^30*f^14*(a^2*c*f^2 - b^2*c*e^2) + 220859191808*a^16*b^28*c^12*e
^28*f^16*(a^2*c*f^2 - b^2*c*e^2) - 276344315328*a^18*b^26*c^12*e^26*f^18*(a
^2*c*f^2 - b^2*c*e^2) + 273130561984*a^20*b^24*c^12*e^24*f^20*(a^2*c*f^2 -
b^2*c*e^2) - 212730002688*a^22*b^22*c^12*e^22*f^22*(a^2*c*f^2 - b^2*c*e^2)
+ 129574234368*a^24*b^20*c^12*e^20*f^24*(a^2*c*f^2 - b^2*c*e^2) - 607705692
16*a^26*b^18*c^12*e^18*f^26*(a^2*c*f^2 - b^2*c*e^2) + 21304706048*a^28*b^16
*c^12*e^16*f^28*(a^2*c*f^2 - b^2*c*e^2) - 5272965120*a^30*b^14*c^12*e^14*f^
30*(a^2*c*f^2 - b^2*c*e^2) + 819441664*a^32*b^12*c^12*e^12*f^32*(a^2*c*f^2
- b^2*c*e^2) - 59392000*a^34*b^10*c^12*e^10*f^34*(a^2*c*f^2 - b^2*c*e^2) +
9289728*a^6*b^24*c^5*e^24*f^6*(a^2*c*f^2 - b^2*c*e^2)^8 - 36884480*a^8*b^22
*c^5*e^22*f^8*(a^2*c*f^2 - b^2*c*e^2)^8 - 278604800*a^10*b^20*c^5*e^20*f^10
*(a^2*c*f^2 - b^2*c*e^2)^8 + 2774483200*a^12*b^18*c^5*e^18*f^12*(a^2*c*f^2
- b^2*c*e^2)^8 - 10869657600*a^14*b^16*c^5*e^16*f^14*(a^2*c*f^2 - b^2*c*e^2
)^8 + 25237416960*a^16*b^14*c^5*e^14*f^16*(a^2*c*f^2 - b^2*c*e^2)^8 - 38348
909568*a^18*b^12*c^5*e^12*f^18*(a^2*c*f^2 - b^2*c*e^2)^8 + 39084659712*a^20
*b^10*c^5*e^10*f^20*(a^2*c*f^2 - b^2*c*e^2)^8 - 26118635520*a^22*b^8*c^5*e^
8*f^22*(a^2*c*f^2 - b^2*c*e^2)^8 + 10414620672*a^24*b^6*c^5*e^6*f^24*(a^2*c
*f^2 - b^2*c*e^2)^8 - 1708654592*a^26*b^4*c^5*e^4*f^26*(a^2*c*f^2 - b^2*c*e
^2)^8 - 276561920*a^28*b^2*c^5*e^2*f^28*(a^2*c*f^2 - b^2*c*e^2)^8 - 9704448
*a^4*b^28*c^6*e^28*f^4*(a^2*c*f^2 - b^2*c*e^2)^7 + 260614656*a^6*b^26*c^6*e
^26*f^6*(a^2*c*f^2 - b^2*c*e^2)^7 - 2166022464*a^8*b^24*c^6*e^24*f^8*(a^2*c
*f^2 - b^2*c*e^2)^7 + 8626147840*a^10*b^22*c^6*e^22*f^10*(a^2*c*f^2 - b^2*c
*e^2)^7 - 16771503616*a^12*b^20*c^6*e^20*f^12*(a^2*c*f^2 - b^2*c*e^2)^7 + 3
301800960*a^14*b^18*c^6*e^18*f^14*(a^2*c*f^2 - b^2*c*e^2)^7 + 67337715968*a
^16*b^16*c^6*e^16*f^16*(a^2*c*f^2 - b^2*c*e^2)^7 - 189857873920*a^18*b^14*c
^6*e^14*f^18*(a^2*c*f^2 - b^2*c*e^2)^7 + 286100259840*a^20*b^12*c^6*e^12*f^
20*(a^2*c*f^2 - b^2*c*e^2)^7 - 275789894656*a^22*b^10*c^6*e^10*f^22*(a^2*c*
f^2 - b^2*c*e^2)^7 + 173716537344*a^24*b^8*c^6*e^8*f^24*(a^2*c*f^2 - b^2*c*
e^2)^7 - 67416424448*a^26*b^6*c^6*e^6*f^26*(a^2*c*f^2 - b^2*c*e^2)^7 + 1283
1686656*a^28*b^4*c^6*e^4*f^28*(a^2*c*f^2 - b^2*c*e^2)^7 + 222560256*a^30*b^
2*c^6*e^2*f^30*(a^2*c*f^2 - b^2*c*e^2)^7 + 2099520*a^2*b^32*c^7*e^32*f^2*(a
^2*c*f^2 - b^2*c*e^2)^6 - 107014608*a^4*b^30*c^7*e^30*f^4*(a^2*c*f^2 - b^2*
c*e^2)^6 + 1848335616*a^6*b^28*c^7*e^28*f^6*(a^2*c*f^2 - b^2*c*e^2)^6 - 152
00005312*a^8*b^26*c^7*e^26*f^8*(a^2*c*f^2 - b^2*c*e^2)^6 + 72612273792*a^10
*b^24*c^7*e^24*f^10*(a^2*c*f^2 - b^2*c*e^2)^6 - 221855779968*a^12*b^22*c^7*
e^22*f^12*(a^2*c*f^2 - b^2*c*e^2)^6 + 450717857536*a^14*b^20*c^7*e^20*f^14*
(a^2*c*f^2 - b^2*c*e^2)^6 - 600578910208*a^16*b^18*c^7*e^18*f^16*(a^2*c*f^2
 - b^2*c*e^2)^6 + 459464530688*a^18*b^16*c^7*e^16*f^18*(a^2*c*f^2 - b^2*c*e
^2)^6 - 33638947840*a^20*b^14*c^7*e^14*f^20*(a^2*c*f^2 - b^2*c*e^2)^6 - 376

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299926528*a^22*b^12*c^7*e^12*f^22*(a^2*c*f^2 - b^2*c*e^2)^6 + 488874068992*
a^24*b^10*c^7*e^10*f^24*(a^2*c*f^2 - b^2*c*e^2)^6 - 333407809536*a^26*b^8*c
^7*e^8*f^26*(a^2*c*f^2 - b^2*c*e^2)^6 + 134140313600*a^28*b^6*c^7*e^6*f^28*
(a^2*c*f^2 - b^2*c*e^2)^6 - 28220915712*a^30*b^4*c^7*e^4*f^30*(a^2*c*f^2 -
b^2*c*e^2)^6 + 1230503936*a^32*b^2*c^7*e^2*f^32*(a^2*c*f^2 - b^2*c*e^2)^6 +
 3335904*a^2*b^34*c^8*e^34*f^2*(a^2*c*f^2 - b^2*c*e^2)^5 - 290521728*a^4*b^
32*c^8*e^32*f^4*(a^2*c*f^2 - b^2*c*e^2)^5 + 4865684544*a^6*b^30*c^8*e^30*f^
6*(a^2*c*f^2 - b^2*c*e^2)^5 - 40437394528*a^8*b^28*c^8*e^28*f^8*(a^2*c*f^2
- b^2*c*e^2)^5 + 205602254656*a^10*b^26*c^8*e^26*f^10*(a^2*c*f^2 - b^2*c*e^
2)^5 - 703885344192*a^12*b^24*c^8*e^24*f^12*(a^2*c*f^2 - b^2*c*e^2)^5 + 170
9253482624*a^14*b^22*c^8*e^22*f^14*(a^2*c*f^2 - b^2*c*e^2)^5 - 302928269516
8*a^16*b^20*c^8*e^20*f^16*(a^2*c*f^2 - b^2*c*e^2)^5 + 3966230827520*a^18*b^
18*c^8*e^18*f^18*(a^2*c*f^2 - b^2*c*e^2)^5 - 3822339813632*a^20*b^16*c^8*e^
16*f^20*(a^2*c*f^2 - b^2*c*e^2)^5 + 2640438056960*a^22*b^14*c^8*e^14*f^22*(
a^2*c*f^2 - b^2*c*e^2)^5 - 1208501415936*a^24*b^12*c^8*e^12*f^24*(a^2*c*f^2
 - b^2*c*e^2)^5 + 269338092544*a^26*b^10*c^8*e^10*f^26*(a^2*c*f^2 - b^2*c*e
^2)^5 + 53783212032*a^28*b^8*c^8*e^8*f^28*(a^2*c*f^2 - b^2*c*e^2)^5 - 60985
360384*a^30*b^6*c^8*e^6*f^30*(a^2*c*f^2 - b^2*c*e^2)^5 + 17917083648*a^32*b
^4*c^8*e^4*f^32*(a^2*c*f^2 - b^2*c*e^2)^5 - 1558708224*a^34*b^2*c^8*e^2*f^3
4*(a^2*c*f^2 - b^2*c*e^2)^5 - 11917692*a^2*b^36*c^9*e^36*f^2*(a^2*c*f^2 - b
^2*c*e^2)^4 - 224907516*a^4*b^34*c^9*e^34*f^4*(a^2*c*f^2 - b^2*c*e^2)^4 + 5
303932560*a^6*b^32*c^9*e^32*f^6*(a^2*c*f^2 - b^2*c*e^2)^4 - 48206418480*a^8
*b^30*c^9*e^30*f^8*(a^2*c*f^2 - b^2*c*e^2)^4 + 261450609120*a^10*b^28*c^9*e
^28*f^10*(a^2*c*f^2 - b^2*c*e^2)^4 - 962361040256*a^12*b^26*c^9*e^26*f^12*(
a^2*c*f^2 - b^2*c*e^2)^4 + 2558559358080*a^14*b^24*c^9*e^24*f^14*(a^2*c*f^2
 - b^2*c*e^2)^4 - 5091804150656*a^16*b^22*c^9*e^22*f^16*(a^2*c*f^2 - b^2*c*
e^2)^4 + 7750806514944*a^18*b^20*c^9*e^20*f^18*(a^2*c*f^2 - b^2*c*e^2)^4 -
9137207485952*a^20*b^18*c^9*e^18*f^20*(a^2*c*f^2 - b^2*c*e^2)^4 + 838456328
0128*a^22*b^16*c^9*e^16*f^22*(a^2*c*f^2 - b^2*c*e^2)^4 - 5975281259520*a^24
*b^14*c^9*e^14*f^24*(a^2*c*f^2 - b^2*c*e^2)^4 + 3269297268736*a^26*b^12*c^9
*e^12*f^26*(a^2*c*f^2 - b^2*c*e^2)^4 - 1339171540992*a^28*b^10*c^9*e^10*f^2
8*(a^2*c*f^2 - b^2*c*e^2)^4 + 391250194432*a^30*b^8*c^9*e^8*f^30*(a^2*c*f^2
 - b^2*c*e^2)^4 - 74114154496*a^32*b^6*c^9*e^6*f^32*(a^2*c*f^2 - b^2*c*e^2)
^4 + 7299203072*a^34*b^4*c^9*e^4*f^34*(a^2*c*f^2 - b^2*c*e^2)^4 - 148635648
*a^36*b^2*c^9*e^2*f^36*(a^2*c*f^2 - b^2*c*e^2)^4 - 38704068*a^2*b^38*c^10*e
^38*f^2*(a^2*c*f^2 - b^2*c*e^2)^3 + 188845992*a^4*b^36*c^10*e^36*f^4*(a^2*c
*f^2 - b^2*c*e^2)^3 + 1157124204*a^6*b^34*c^10*e^34*f^6*(a^2*c*f^2 - b^2*c*
e^2)^3 - 20586361424*a^8*b^32*c^10*e^32*f^8*(a^2*c*f^2 - b^2*c*e^2)^3 + 135
395499200*a^10*b^30*c^10*e^30*f^10*(a^2*c*f^2 - b^2*c*e^2)^3 - 555513858464
*a^12*b^28*c^10*e^28*f^12*(a^2*c*f^2 - b^2*c*e^2)^3 + 1608776388864*a^14*b^
26*c^10*e^26*f^14*(a^2*c*f^2 - b^2*c*e^2)^3 - 3473989271488*a^16*b^24*c^10*
e^24*f^16*(a^2*c*f^2 - b^2*c*e^2)^3 + 5766181411456*a^18*b^22*c^10*e^22*f^1
8*(a^2*c*f^2 - b^2*c*e^2)^3 - 7493983209472*a^20*b^20*c^10*e^20*f^20*(a^2*c
*f^2 - b^2*c*e^2)^3 + 7713917084672*a^22*b^18*c^10*e^18*f^22*(a^2*c*f^2 - b
^2*c*e^2)^3 - 6328467293184*a^24*b^16*c^10*e^16*f^24*(a^2*c*f^2 - b^2*c*e^2
)^3 + 4142950034432*a^26*b^14*c^10*e^14*f^26*(a^2*c*f^2 - b^2*c*e^2)^3 - 21
52681536512*a^28*b^12*c^10*e^12*f^28*(a^2*c*f^2 - b^2*c*e^2)^3 + 8741995110
40*a^30*b^10*c^10*e^10*f^30*(a^2*c*f^2 - b^2*c*e^2)^3 - 268759150592*a^32*b
^8*c^10*e^8*f^32*(a^2*c*f^2 - b^2*c*e^2)^3 + 58872545280*a^34*b^6*c^10*e^6*
f^34*(a^2*c*f^2 - b^2*c*e^2)^3 - 8151957504*a^36*b^4*c^10*e^4*f^36*(a^2*c*f
^2 - b^2*c*e^2)^3 + 530841600*a^38*b^2*c^10*e^2*f^38*(a^2*c*f^2 - b^2*c*e^2
)^3 - 42743457*a^2*b^40*c^11*e^40*f^2*(a^2*c*f^2 - b^2*c*e^2)^2 + 411055884
*a^4*b^38*c^11*e^38*f^4*(a^2*c*f^2 - b^2*c*e^2)^2 - 2180887236*a^6*b^36*c^1
1*e^36*f^6*(a^2*c*f^2 - b^2*c*e^2)^2 + 6404946508*a^8*b^34*c^11*e^34*f^8*(a
^2*c*f^2 - b^2*c*e^2)^2 - 5434005264*a^10*b^32*c^11*e^32*f^10*(a^2*c*f^2 -
b^2*c*e^2)^2 - 38868373520*a^12*b^30*c^11*e^30*f^12*(a^2*c*f^2 - b^2*c*e^2)
^2 + 208447613600*a^14*b^28*c^11*e^28*f^14*(a^2*c*f^2 - b^2*c*e^2)^2 - 5796
74999104*a^16*b^26*c^11*e^26*f^16*(a^2*c*f^2 - b^2*c*e^2)^2 + 1104967566592
*a^18*b^24*c^11*e^24*f^18*(a^2*c*f^2 - b^2*c*e^2)^2 - 1554566531328*a^20*b^
```

22*c^11*e^22*f^20*(a^2*c*f^2 - b^2*c*e^2)^2 + 1659734381312*a^22*b^20*c^11*
e^20*f^22*(a^2*c*f^2 - b^2*c*e^2)^2 - 1356361512192*a^24*b^18*c^11*e^18*f^2
4*(a^2*c*f^2 - b^2*c*e^2)^2 + 845331359744*a^26*b^16*c^11*e^16*f^26*(a^2*c*
f^2 - b^2*c*e^2)^2 - 395676895232*a^28*b^14*c^11*e^14*f^28*(a^2*c*f^2 - b^2
*c*e^2)^2 + 134902689792*a^30*b^12*c^11*e^12*f^30*(a^2*c*f^2 - b^2*c*e^2)^2
 - 31670587392*a^32*b^10*c^11*e^10*f^32*(a^2*c*f^2 - b^2*c*e^2)^2 + 4584669
184*a^34*b^8*c^11*e^8*f^34*(a^2*c*f^2 - b^2*c*e^2)^2 - 309657600*a^36*b^6*c
^11*e^6*f^36*(a^2*c*f^2 - b^2*c*e^2)^2))*(b^16*e^12*f^6*(a^2*c*f^2 - b^2*c*
e^2)^2 - 4*a^2*b^14*e^10*f^8*(a^2*c*f^2 - b^2*c*e^2)^2 + 6*a^4*b^12*e^8*f^1
0*(a^2*c*f^2 - b^2*c*e^2)^2 - 4*a^6*b^10*e^6*f^12*(a^2*c*f^2 - b^2*c*e^2)^2
 + a^8*b^8*e^4*f^14*(a^2*c*f^2 - b^2*c*e^2)^2))/(((a + b*x)^(1/2) - a^(1/2)
)^3*(16384*C^4*a^6*c^3*f^4 + 4096*C^4*a^2*b^4*c^3*e^4 - 16384*C^4*a^4*b^2*c
^3*e^2*f^2)) - (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*((8*a^4*b^6*c^4*e^6*f^4
*((16384*C^3*e^3*(2*a^2*f^2 - b^2*e^2)^3*(20*C*a^12*c^6*f^13 + 22*C*a^4*b^8
*c^6*e^8*f^5 - 88*C*a^6*b^6*c^6*e^6*f^7 + 130*C*a^8*b^4*c^6*e^4*f^9 - 84*C*
a^10*b^2*c^6*e^2*f^11))/(f^6*(a*f + b*e)^3*(a*f - b*e)^3*(b^2*c*e^2 - a^2*c
f^2)^(3/2)(b^13*e^12*f^3 - 3*a^2*b^11*e^10*f^5 + 3*a^4*b^9*e^8*f^7 - a^6*
b^7*e^6*f^9)) + (16384*C*e*(2*a^2*f^2 - b^2*e^2)*(96*C^3*a^10*c^5*e^2*f^7 -
 28*C^3*a^4*b^6*c^5*e^8*f + 132*C^3*a^6*b^4*c^5*e^6*f^3 - 200*C^3*a^8*b^2*c
^5*e^4*f^5))/(f^2*(a*f + b*e)*(a*f - b*e)*(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^
13*e^12*f^3 - 3*a^2*b^11*e^10*f^5 + 3*a^4*b^9*e^8*f^7 - a^6*b^7*e^6*f^9)))*
(4*a^2*c*f^2 - 3*b^2*c*e^2)*(4*a^6*c*f^6 - 3*b^6*c*e^6 + 8*a^2*b^4*c*e^4*f^
2 - 8*a^4*b^2*c*e^2*f^4)^4)/(164025*b^46*c^13*e^46 + 885735*b^44*c^12*e^44*
(a^2*c*f^2 - b^2*c*e^2) + 117440512*a^30*c^5*f^30*(a^2*c*f^2 - b^2*c*e^2)^8
 - 385875968*a^32*c^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^7 + 419430400*a^34*c^7*f
^34*(a^2*c*f^2 - b^2*c*e^2)^6 - 150994944*a^36*c^8*f^36*(a^2*c*f^2 - b^2*c*
e^2)^5 + 236196*b^36*c^8*e^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 1102248*b^38*c^9*
e^38*(a^2*c*f^2 - b^2*c*e^2)^4 + 2053593*b^40*c^10*e^40*(a^2*c*f^2 - b^2*c*
e^2)^3 + 1909251*b^42*c^11*e^42*(a^2*c*f^2 - b^2*c*e^2)^2 - 3937329*a^2*b^4
4*c^13*e^44*f^2 + 43893819*a^4*b^42*c^13*e^42*f^4 - 301507155*a^6*b^40*c^13
*e^40*f^6 + 1427514656*a^8*b^38*c^13*e^38*f^8 - 4936911112*a^10*b^36*c^13*e
^36*f^10 + 12893273616*a^12*b^34*c^13*e^34*f^12 - 25921630432*a^14*b^32*c^1
3*e^32*f^14 + 40519286096*a^16*b^30*c^13*e^30*f^16 - 49376608256*a^18*b^28*
c^13*e^28*f^18 + 46721401856*a^20*b^26*c^13*e^26*f^20 - 33946324736*a^22*b^
24*c^13*e^24*f^22 + 18556579328*a^24*b^22*c^13*e^22*f^24 - 7375276032*a^26*
b^20*c^13*e^20*f^26 + 2009817088*a^28*b^18*c^13*e^18*f^28 - 335642624*a^30*
b^16*c^13*e^16*f^30 + 25907200*a^32*b^14*c^13*e^14*f^32 - 21130794*a^2*b^42
*c^12*e^42*f^2*(a^2*c*f^2 - b^2*c*e^2) + 234399015*a^4*b^40*c^12*e^40*f^4*(
a^2*c*f^2 - b^2*c*e^2) - 1604168280*a^6*b^38*c^12*e^38*f^6*(a^2*c*f^2 - b^2
*c*e^2) + 7579098492*a^8*b^36*c^12*e^36*f^8*(a^2*c*f^2 - b^2*c*e^2) - 26212
380172*a^10*b^34*c^12*e^34*f^10*(a^2*c*f^2 - b^2*c*e^2) + 68672994096*a^12*
b^32*c^12*e^32*f^12*(a^2*c*f^2 - b^2*c*e^2) - 139160589504*a^14*b^30*c^12*e
^30*f^14*(a^2*c*f^2 - b^2*c*e^2) + 220859191808*a^16*b^28*c^12*e^28*f^16*(a
^2*c*f^2 - b^2*c*e^2) - 276344315328*a^18*b^26*c^12*e^26*f^18*(a^2*c*f^2 -
b^2*c*e^2) + 273130561984*a^20*b^24*c^12*e^24*f^20*(a^2*c*f^2 - b^2*c*e^2)
 - 212730002688*a^22*b^22*c^12*e^22*f^22*(a^2*c*f^2 - b^2*c*e^2) + 129574234
368*a^24*b^20*c^12*e^20*f^24*(a^2*c*f^2 - b^2*c*e^2) - 60770569216*a^26*b^1
8*c^12*e^18*f^26*(a^2*c*f^2 - b^2*c*e^2) + 21304706048*a^28*b^16*c^12*e^16*
f^28*(a^2*c*f^2 - b^2*c*e^2) - 5272965120*a^30*b^14*c^12*e^14*f^30*(a^2*c*f
^2 - b^2*c*e^2) + 819441664*a^32*b^12*c^12*e^12*f^32*(a^2*c*f^2 - b^2*c*e^2
) - 59392000*a^34*b^10*c^12*e^10*f^34*(a^2*c*f^2 - b^2*c*e^2) + 9289728*a^6
*b^24*c^5*e^24*f^6*(a^2*c*f^2 - b^2*c*e^2)^8 - 36884480*a^8*b^22*c^5*e^22*f
^8*(a^2*c*f^2 - b^2*c*e^2)^8 - 278604800*a^10*b^20*c^5*e^20*f^10*(a^2*c*f^2
 - b^2*c*e^2)^8 + 2774483200*a^12*b^18*c^5*e^18*f^12*(a^2*c*f^2 - b^2*c*e^2
)^8 - 10869657600*a^14*b^16*c^5*e^16*f^14*(a^2*c*f^2 - b^2*c*e^2)^8 + 25237
416960*a^16*b^14*c^5*e^14*f^16*(a^2*c*f^2 - b^2*c*e^2)^8 - 38348909568*a^18
*b^12*c^5*e^12*f^18*(a^2*c*f^2 - b^2*c*e^2)^8 + 39084659712*a^20*b^10*c^5*e
^10*f^20*(a^2*c*f^2 - b^2*c*e^2)^8 - 26118635520*a^22*b^8*c^5*e^8*f^22*(a^2
*c*f^2 - b^2*c*e^2)^8 + 10414620672*a^24*b^6*c^5*e^6*f^24*(a^2*c*f^2 - b^2*

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c*e^2)^8 - 1708654592*a^26*b^4*c^5*e^4*f^26*(a^2*c*f^2 - b^2*c*e^2)^8 - 276
561920*a^28*b^2*c^5*e^2*f^28*(a^2*c*f^2 - b^2*c*e^2)^8 - 9704448*a^4*b^28*c
^6*e^28*f^4*(a^2*c*f^2 - b^2*c*e^2)^7 + 260614656*a^6*b^26*c^6*e^26*f^6*(a^
2*c*f^2 - b^2*c*e^2)^7 - 2166022464*a^8*b^24*c^6*e^24*f^8*(a^2*c*f^2 - b^2*
c*e^2)^7 + 8626147840*a^10*b^22*c^6*e^22*f^10*(a^2*c*f^2 - b^2*c*e^2)^7 - 1
6771503616*a^12*b^20*c^6*e^20*f^12*(a^2*c*f^2 - b^2*c*e^2)^7 + 3301800960*a
^14*b^18*c^6*e^18*f^14*(a^2*c*f^2 - b^2*c*e^2)^7 + 67337715968*a^16*b^16*c^
6*e^16*f^16*(a^2*c*f^2 - b^2*c*e^2)^7 - 189857873920*a^18*b^14*c^6*e^14*f^1
8*(a^2*c*f^2 - b^2*c*e^2)^7 + 286100259840*a^20*b^12*c^6*e^12*f^20*(a^2*c*f
^2 - b^2*c*e^2)^7 - 275789894656*a^22*b^10*c^6*e^10*f^22*(a^2*c*f^2 - b^2*c
*e^2)^7 + 173716537344*a^24*b^8*c^6*e^8*f^24*(a^2*c*f^2 - b^2*c*e^2)^7 - 67
416424448*a^26*b^6*c^6*e^6*f^26*(a^2*c*f^2 - b^2*c*e^2)^7 + 12831686656*a^2
8*b^4*c^6*e^4*f^28*(a^2*c*f^2 - b^2*c*e^2)^7 + 222560256*a^30*b^2*c^6*e^2*f
^30*(a^2*c*f^2 - b^2*c*e^2)^7 + 2099520*a^2*b^32*c^7*e^32*f^2*(a^2*c*f^2 -
b^2*c*e^2)^6 - 107014608*a^4*b^30*c^7*e^30*f^4*(a^2*c*f^2 - b^2*c*e^2)^6 +
1848335616*a^6*b^28*c^7*e^28*f^6*(a^2*c*f^2 - b^2*c*e^2)^6 - 15200005312*a^
8*b^26*c^7*e^26*f^8*(a^2*c*f^2 - b^2*c*e^2)^6 + 72612273792*a^10*b^24*c^7*e
^24*f^10*(a^2*c*f^2 - b^2*c*e^2)^6 - 221855779968*a^12*b^22*c^7*e^22*f^12*(
a^2*c*f^2 - b^2*c*e^2)^6 + 450717857536*a^14*b^20*c^7*e^20*f^14*(a^2*c*f^2
- b^2*c*e^2)^6 - 600578910208*a^16*b^18*c^7*e^18*f^16*(a^2*c*f^2 - b^2*c*e^
2)^6 + 459464530688*a^18*b^16*c^7*e^16*f^18*(a^2*c*f^2 - b^2*c*e^2)^6 - 336
38947840*a^20*b^14*c^7*e^14*f^20*(a^2*c*f^2 - b^2*c*e^2)^6 - 376299926528*a
^22*b^12*c^7*e^12*f^22*(a^2*c*f^2 - b^2*c*e^2)^6 + 488874068992*a^24*b^10*c
^7*e^10*f^24*(a^2*c*f^2 - b^2*c*e^2)^6 - 333407809536*a^26*b^8*c^7*e^8*f^26
*(a^2*c*f^2 - b^2*c*e^2)^6 + 134140313600*a^28*b^6*c^7*e^6*f^28*(a^2*c*f^2
- b^2*c*e^2)^6 - 28220915712*a^30*b^4*c^7*e^4*f^30*(a^2*c*f^2 - b^2*c*e^2)^
6 + 1230503936*a^32*b^2*c^7*e^2*f^32*(a^2*c*f^2 - b^2*c*e^2)^6 + 3335904*a^
2*b^34*c^8*e^34*f^2*(a^2*c*f^2 - b^2*c*e^2)^5 - 290521728*a^4*b^32*c^8*e^32
*f^4*(a^2*c*f^2 - b^2*c*e^2)^5 + 4865684544*a^6*b^30*c^8*e^30*f^6*(a^2*c*f^
2 - b^2*c*e^2)^5 - 40437394528*a^8*b^28*c^8*e^28*f^8*(a^2*c*f^2 - b^2*c*e^2
)^5 + 205602254656*a^10*b^26*c^8*e^26*f^10*(a^2*c*f^2 - b^2*c*e^2)^5 - 7038
85344192*a^12*b^24*c^8*e^24*f^12*(a^2*c*f^2 - b^2*c*e^2)^5 + 1709253482624*
a^14*b^22*c^8*e^22*f^14*(a^2*c*f^2 - b^2*c*e^2)^5 - 3029282695168*a^16*b^20
*c^8*e^20*f^16*(a^2*c*f^2 - b^2*c*e^2)^5 + 3966230827520*a^18*b^18*c^8*e^18
*f^18*(a^2*c*f^2 - b^2*c*e^2)^5 - 3822339813632*a^20*b^16*c^8*e^16*f^20*(a^
2*c*f^2 - b^2*c*e^2)^5 + 2640438056960*a^22*b^14*c^8*e^14*f^22*(a^2*c*f^2 -
 b^2*c*e^2)^5 - 1208501415936*a^24*b^12*c^8*e^12*f^24*(a^2*c*f^2 - b^2*c*e^
2)^5 + 269338092544*a^26*b^10*c^8*e^10*f^26*(a^2*c*f^2 - b^2*c*e^2)^5 + 537
83212032*a^28*b^8*c^8*e^8*f^28*(a^2*c*f^2 - b^2*c*e^2)^5 - 60985360384*a^30
*b^6*c^8*e^6*f^30*(a^2*c*f^2 - b^2*c*e^2)^5 + 17917083648*a^32*b^4*c^8*e^4*
f^32*(a^2*c*f^2 - b^2*c*e^2)^5 - 1558708224*a^34*b^2*c^8*e^2*f^34*(a^2*c*f^
2 - b^2*c*e^2)^5 - 11917692*a^2*b^36*c^9*e^36*f^2*(a^2*c*f^2 - b^2*c*e^2)^4
 - 224907516*a^4*b^34*c^9*e^34*f^4*(a^2*c*f^2 - b^2*c*e^2)^4 + 5303932560*a
^6*b^32*c^9*e^32*f^6*(a^2*c*f^2 - b^2*c*e^2)^4 - 48206418480*a^8*b^30*c^9*e
^30*f^8*(a^2*c*f^2 - b^2*c*e^2)^4 + 261450609120*a^10*b^28*c^9*e^28*f^10*(a
^2*c*f^2 - b^2*c*e^2)^4 - 962361040256*a^12*b^26*c^9*e^26*f^12*(a^2*c*f^2 -
 b^2*c*e^2)^4 + 2558559358080*a^14*b^24*c^9*e^24*f^14*(a^2*c*f^2 - b^2*c*e^
2)^4 - 5091804150656*a^16*b^22*c^9*e^22*f^16*(a^2*c*f^2 - b^2*c*e^2)^4 + 77
50806514944*a^18*b^20*c^9*e^20*f^18*(a^2*c*f^2 - b^2*c*e^2)^4 - 91372074859
52*a^20*b^18*c^9*e^18*f^20*(a^2*c*f^2 - b^2*c*e^2)^4 + 8384563280128*a^22*b
^16*c^9*e^16*f^22*(a^2*c*f^2 - b^2*c*e^2)^4 - 5975281259520*a^24*b^14*c^9*e
^14*f^24*(a^2*c*f^2 - b^2*c*e^2)^4 + 3269297268736*a^26*b^12*c^9*e^12*f^26*
(a^2*c*f^2 - b^2*c*e^2)^4 - 1339171540992*a^28*b^10*c^9*e^10*f^28*(a^2*c*f^
2 - b^2*c*e^2)^4 + 391250194432*a^30*b^8*c^9*e^8*f^30*(a^2*c*f^2 - b^2*c*e^
2)^4 - 74114154496*a^32*b^6*c^9*e^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^4 + 729920
3072*a^34*b^4*c^9*e^4*f^34*(a^2*c*f^2 - b^2*c*e^2)^4 - 148635648*a^36*b^2*c
^9*e^2*f^36*(a^2*c*f^2 - b^2*c*e^2)^4 - 38704068*a^2*b^38*c^10*e^38*f^2*(a^
2*c*f^2 - b^2*c*e^2)^3 + 188845992*a^4*b^36*c^10*e^36*f^4*(a^2*c*f^2 - b^2*
c*e^2)^3 + 1157124204*a^6*b^34*c^10*e^34*f^6*(a^2*c*f^2 - b^2*c*e^2)^3 - 20
```

586361424*a^8*b^32*c^10*e^32*f^8*(a^2*c*f^2 - b^2*c*e^2)^3 + 135395499200*a
^10*b^30*c^10*e^30*f^10*(a^2*c*f^2 - b^2*c*e^2)^3 - 555513858464*a^12*b^28*
c^10*e^28*f^12*(a^2*c*f^2 - b^2*c*e^2)^3 + 1608776388864*a^14*b^26*c^10*e^2
6*f^14*(a^2*c*f^2 - b^2*c*e^2)^3 - 3473989271488*a^16*b^24*c^10*e^24*f^16*(
a^2*c*f^2 - b^2*c*e^2)^3 + 5766181411456*a^18*b^22*c^10*e^22*f^18*(a^2*c*f^
2 - b^2*c*e^2)^3 - 7493983209472*a^20*b^20*c^10*e^20*f^20*(a^2*c*f^2 - b^2*
c*e^2)^3 + 7713917084672*a^22*b^18*c^10*e^18*f^22*(a^2*c*f^2 - b^2*c*e^2)^3
 - 6328467293184*a^24*b^16*c^10*e^16*f^24*(a^2*c*f^2 - b^2*c*e^2)^3 + 41429
50034432*a^26*b^14*c^10*e^14*f^26*(a^2*c*f^2 - b^2*c*e^2)^3 - 2152681536512
*a^28*b^12*c^10*e^12*f^28*(a^2*c*f^2 - b^2*c*e^2)^3 + 874199511040*a^30*b^1
0*c^10*e^10*f^30*(a^2*c*f^2 - b^2*c*e^2)^3 - 268759150592*a^32*b^8*c^10*e^8
f^32(a^2*c*f^2 - b^2*c*e^2)^3 + 58872545280*a^34*b^6*c^10*e^6*f^34*(a^2*c
*f^2 - b^2*c*e^2)^3 - 8151957504*a^36*b^4*c^10*e^4*f^36*(a^2*c*f^2 - b^2*c*
e^2)^3 + 530841600*a^38*b^2*c^10*e^2*f^38*(a^2*c*f^2 - b^2*c*e^2)^3 - 42743
457*a^2*b^40*c^11*e^40*f^2*(a^2*c*f^2 - b^2*c*e^2)^2 + 411055884*a^4*b^38*c
^11*e^38*f^4*(a^2*c*f^2 - b^2*c*e^2)^2 - 2180887236*a^6*b^36*c^11*e^36*f^6*
(a^2*c*f^2 - b^2*c*e^2)^2 + 6404946508*a^8*b^34*c^11*e^34*f^8*(a^2*c*f^2 -
b^2*c*e^2)^2 - 5434005264*a^10*b^32*c^11*e^32*f^10*(a^2*c*f^2 - b^2*c*e^2)^
2 - 38868373520*a^12*b^30*c^11*e^30*f^12*(a^2*c*f^2 - b^2*c*e^2)^2 + 208447
613600*a^14*b^28*c^11*e^28*f^14*(a^2*c*f^2 - b^2*c*e^2)^2 - 579674999104*a^
16*b^26*c^11*e^26*f^16*(a^2*c*f^2 - b^2*c*e^2)^2 + 1104967566592*a^18*b^24*
c^11*e^24*f^18*(a^2*c*f^2 - b^2*c*e^2)^2 - 1554566531328*a^20*b^22*c^11*e^2
2*f^20*(a^2*c*f^2 - b^2*c*e^2)^2 + 1659734381312*a^22*b^20*c^11*e^20*f^22*(
a^2*c*f^2 - b^2*c*e^2)^2 - 1356361512192*a^24*b^18*c^11*e^18*f^24*(a^2*c*f^
2 - b^2*c*e^2)^2 + 845331359744*a^26*b^16*c^11*e^16*f^26*(a^2*c*f^2 - b^2*c
*e^2)^2 - 395676895232*a^28*b^14*c^11*e^14*f^28*(a^2*c*f^2 - b^2*c*e^2)^2 +
 134902689792*a^30*b^12*c^11*e^12*f^30*(a^2*c*f^2 - b^2*c*e^2)^2 - 31670587
392*a^32*b^10*c^11*e^10*f^32*(a^2*c*f^2 - b^2*c*e^2)^2 + 4584669184*a^34*b^
8*c^11*e^8*f^34*(a^2*c*f^2 - b^2*c*e^2)^2 - 309657600*a^36*b^6*c^11*e^6*f^3
6*(a^2*c*f^2 - b^2*c*e^2)^2) - (2*a^4*b^5*c^3*e^5*f^4*(4*a^2*c*f^2 - 3*b^2*
c*e^2)^2*((4096*(16*C^4*a^4*b^8*c^5*e^10 + 64*C^4*a^12*c^5*e^2*f^8 - 92*C^4
*a^6*b^6*c^5*e^8*f^2 + 192*C^4*a^8*b^4*c^5*e^6*f^4 - 176*C^4*a^10*b^2*c^5*e
^4*f^6))/(b^16*e^14*f^4 - 4*a^2*b^14*e^12*f^6 + 6*a^4*b^12*e^10*f^8 - 4*a^6
*b^10*e^8*f^10 + a^8*b^8*e^6*f^12) + (4096*C^4*e^4*(2*a^2*f^2 - b^2*e^2)^4*
(9*a^2*b^14*c^7*e^12*f^6 - 43*a^4*b^12*c^7*e^10*f^8 + 82*a^6*b^10*c^7*e^8*f
^10 - 78*a^8*b^8*c^7*e^6*f^12 + 37*a^10*b^6*c^7*e^4*f^14 - 7*a^12*b^4*c^7*e
^2*f^16))/(f^8*(a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)^2*(b^16*
e^14*f^4 - 4*a^2*b^14*e^12*f^6 + 6*a^4*b^12*e^10*f^8 - 4*a^6*b^10*e^8*f^10
 + a^8*b^8*e^6*f^12)) + (4096*C^2*e^2*(2*a^2*f^2 - b^2*e^2)^2*(16*C^2*a^14*c
^6*f^14 + 9*C^2*a^2*b^12*c^6*e^12*f^2 - 54*C^2*a^4*b^10*c^6*e^10*f^4 + 121*
C^2*a^6*b^8*c^6*e^8*f^6 - 128*C^2*a^8*b^6*c^6*e^6*f^8 + 80*C^2*a^10*b^4*c^6
*e^4*f^10 - 44*C^2*a^12*b^2*c^6*e^2*f^12))/(f^4*(a*f + b*e)^2*(a*f - b*e)^2
*(a^2*c*f^2 - b^2*c*e^2)*(b^16*e^14*f^4 - 4*a^2*b^14*e^12*f^6 + 6*a^4*b^12*
e^10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12)))*(4*a^6*c*f^6 - 3*b^6*c
*e^6 + 8*a^2*b^4*c*e^4*f^2 - 8*a^4*b^2*c*e^2*f^4)^4)/((b^2*c*e^2 - a^2*c*f^
2)^(1/2)*(164025*b^46*c^13*e^46 + 885735*b^44*c^12*e^44*(a^2*c*f^2 - b^2*c*
e^2) + 117440512*a^30*c^5*f^30*(a^2*c*f^2 - b^2*c*e^2)^8 - 385875968*a^32*c
^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^7 + 419430400*a^34*c^7*f^34*(a^2*c*f^2 - b^
2*c*e^2)^6 - 150994944*a^36*c^8*f^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 236196*b^3
6*c^8*e^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 1102248*b^38*c^9*e^38*(a^2*c*f^2 - b
^2*c*e^2)^4 + 2053593*b^40*c^10*e^40*(a^2*c*f^2 - b^2*c*e^2)^3 + 1909251*b^
42*c^11*e^42*(a^2*c*f^2 - b^2*c*e^2)^2 - 3937329*a^2*b^44*c^13*e^44*f^2 + 4
3893819*a^4*b^42*c^13*e^42*f^4 - 301507155*a^6*b^40*c^13*e^40*f^6 + 1427514
656*a^8*b^38*c^13*e^38*f^8 - 4936911112*a^10*b^36*c^13*e^36*f^10 + 12893273
616*a^12*b^34*c^13*e^34*f^12 - 25921630432*a^14*b^32*c^13*e^32*f^14 + 40519
286096*a^16*b^30*c^13*e^30*f^16 - 49376608256*a^18*b^28*c^13*e^28*f^18 + 46
721401856*a^20*b^26*c^13*e^26*f^20 - 33946324736*a^22*b^24*c^13*e^24*f^22 +
 18556579328*a^24*b^22*c^13*e^22*f^24 - 7375276032*a^26*b^20*c^13*e^20*f^26
 + 2009817088*a^28*b^18*c^13*e^18*f^28 - 335642624*a^30*b^16*c^13*e^16*f^30

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 + 25907200*a^32*b^14*c^13*e^14*f^32 - 21130794*a^2*b^42*c^12*e^42*f^2*(a^2
*c*f^2 - b^2*c*e^2) + 234399015*a^4*b^40*c^12*e^40*f^4*(a^2*c*f^2 - b^2*c*e
^2) - 1604168280*a^6*b^38*c^12*e^38*f^6*(a^2*c*f^2 - b^2*c*e^2) + 757909849
2*a^8*b^36*c^12*e^36*f^8*(a^2*c*f^2 - b^2*c*e^2) - 26212380172*a^10*b^34*c^
12*e^34*f^10*(a^2*c*f^2 - b^2*c*e^2) + 68672994096*a^12*b^32*c^12*e^32*f^12
*(a^2*c*f^2 - b^2*c*e^2) - 139160589504*a^14*b^30*c^12*e^30*f^14*(a^2*c*f^2
 - b^2*c*e^2) + 220859191808*a^16*b^28*c^12*e^28*f^16*(a^2*c*f^2 - b^2*c*e^
2) - 276344315328*a^18*b^26*c^12*e^26*f^18*(a^2*c*f^2 - b^2*c*e^2) + 273130
561984*a^20*b^24*c^12*e^24*f^20*(a^2*c*f^2 - b^2*c*e^2) - 212730002688*a^22
*b^22*c^12*e^22*f^22*(a^2*c*f^2 - b^2*c*e^2) + 129574234368*a^24*b^20*c^12*
e^20*f^24*(a^2*c*f^2 - b^2*c*e^2) - 60770569216*a^26*b^18*c^12*e^18*f^26*(a
^2*c*f^2 - b^2*c*e^2) + 21304706048*a^28*b^16*c^12*e^16*f^28*(a^2*c*f^2 - b
^2*c*e^2) - 5272965120*a^30*b^14*c^12*e^14*f^30*(a^2*c*f^2 - b^2*c*e^2) + 8
19441664*a^32*b^12*c^12*e^12*f^32*(a^2*c*f^2 - b^2*c*e^2) - 59392000*a^34*b
^10*c^12*e^10*f^34*(a^2*c*f^2 - b^2*c*e^2) + 9289728*a^6*b^24*c^5*e^24*f^6*
(a^2*c*f^2 - b^2*c*e^2)^8 - 36884480*a^8*b^22*c^5*e^22*f^8*(a^2*c*f^2 - b^2
*c*e^2)^8 - 278604800*a^10*b^20*c^5*e^20*f^10*(a^2*c*f^2 - b^2*c*e^2)^8 + 2
774483200*a^12*b^18*c^5*e^18*f^12*(a^2*c*f^2 - b^2*c*e^2)^8 - 10869657600*a
^14*b^16*c^5*e^16*f^14*(a^2*c*f^2 - b^2*c*e^2)^8 + 25237416960*a^16*b^14*c^
5*e^14*f^16*(a^2*c*f^2 - b^2*c*e^2)^8 - 38348909568*a^18*b^12*c^5*e^12*f^18
*(a^2*c*f^2 - b^2*c*e^2)^8 + 39084659712*a^20*b^10*c^5*e^10*f^20*(a^2*c*f^2
 - b^2*c*e^2)^8 - 26118635520*a^22*b^8*c^5*e^8*f^22*(a^2*c*f^2 - b^2*c*e^2)
^8 + 10414620672*a^24*b^6*c^5*e^6*f^24*(a^2*c*f^2 - b^2*c*e^2)^8 - 17086545
92*a^26*b^4*c^5*e^4*f^26*(a^2*c*f^2 - b^2*c*e^2)^8 - 276561920*a^28*b^2*c^5
*e^2*f^28*(a^2*c*f^2 - b^2*c*e^2)^8 - 9704448*a^4*b^28*c^6*e^28*f^4*(a^2*c*
f^2 - b^2*c*e^2)^7 + 260614656*a^6*b^26*c^6*e^26*f^6*(a^2*c*f^2 - b^2*c*e^2
)^7 - 2166022464*a^8*b^24*c^6*e^24*f^8*(a^2*c*f^2 - b^2*c*e^2)^7 + 86261478
40*a^10*b^22*c^6*e^22*f^10*(a^2*c*f^2 - b^2*c*e^2)^7 - 16771503616*a^12*b^2
0*c^6*e^20*f^12*(a^2*c*f^2 - b^2*c*e^2)^7 + 3301800960*a^14*b^18*c^6*e^18*f
^14*(a^2*c*f^2 - b^2*c*e^2)^7 + 67337715968*a^16*b^16*c^6*e^16*f^16*(a^2*c*
f^2 - b^2*c*e^2)^7 - 189857873920*a^18*b^14*c^6*e^14*f^18*(a^2*c*f^2 - b^2*
c*e^2)^7 + 286100259840*a^20*b^12*c^6*e^12*f^20*(a^2*c*f^2 - b^2*c*e^2)^7 -
 275789894656*a^22*b^10*c^6*e^10*f^22*(a^2*c*f^2 - b^2*c*e^2)^7 + 173716537
344*a^24*b^8*c^6*e^8*f^24*(a^2*c*f^2 - b^2*c*e^2)^7 - 67416424448*a^26*b^6*
c^6*e^6*f^26*(a^2*c*f^2 - b^2*c*e^2)^7 + 12831686656*a^28*b^4*c^6*e^4*f^28*
(a^2*c*f^2 - b^2*c*e^2)^7 + 222560256*a^30*b^2*c^6*e^2*f^30*(a^2*c*f^2 - b^
2*c*e^2)^7 + 2099520*a^2*b^32*c^7*e^32*f^2*(a^2*c*f^2 - b^2*c*e^2)^6 - 1070
14608*a^4*b^30*c^7*e^30*f^4*(a^2*c*f^2 - b^2*c*e^2)^6 + 1848335616*a^6*b^28
*c^7*e^28*f^6*(a^2*c*f^2 - b^2*c*e^2)^6 - 15200005312*a^8*b^26*c^7*e^26*f^8
*(a^2*c*f^2 - b^2*c*e^2)^6 + 72612273792*a^10*b^24*c^7*e^24*f^10*(a^2*c*f^2
 - b^2*c*e^2)^6 - 221855779968*a^12*b^22*c^7*e^22*f^12*(a^2*c*f^2 - b^2*c*e
^2)^6 + 450717857536*a^14*b^20*c^7*e^20*f^14*(a^2*c*f^2 - b^2*c*e^2)^6 - 60
0578910208*a^16*b^18*c^7*e^18*f^16*(a^2*c*f^2 - b^2*c*e^2)^6 + 459464530688
*a^18*b^16*c^7*e^16*f^18*(a^2*c*f^2 - b^2*c*e^2)^6 - 33638947840*a^20*b^14*
c^7*e^14*f^20*(a^2*c*f^2 - b^2*c*e^2)^6 - 376299926528*a^22*b^12*c^7*e^12*f
^22*(a^2*c*f^2 - b^2*c*e^2)^6 + 488874068992*a^24*b^10*c^7*e^10*f^24*(a^2*c
*f^2 - b^2*c*e^2)^6 - 333407809536*a^26*b^8*c^7*e^8*f^26*(a^2*c*f^2 - b^2*c
*e^2)^6 + 134140313600*a^28*b^6*c^7*e^6*f^28*(a^2*c*f^2 - b^2*c*e^2)^6 - 28
220915712*a^30*b^4*c^7*e^4*f^30*(a^2*c*f^2 - b^2*c*e^2)^6 + 1230503936*a^32
*b^2*c^7*e^2*f^32*(a^2*c*f^2 - b^2*c*e^2)^6 + 3335904*a^2*b^34*c^8*e^34*f^2
*(a^2*c*f^2 - b^2*c*e^2)^5 - 290521728*a^4*b^32*c^8*e^32*f^4*(a^2*c*f^2 - b
^2*c*e^2)^5 + 4865684544*a^6*b^30*c^8*e^30*f^6*(a^2*c*f^2 - b^2*c*e^2)^5 -
40437394528*a^8*b^28*c^8*e^28*f^8*(a^2*c*f^2 - b^2*c*e^2)^5 + 205602254656*
a^10*b^26*c^8*e^26*f^10*(a^2*c*f^2 - b^2*c*e^2)^5 - 703885344192*a^12*b^24*
c^8*e^24*f^12*(a^2*c*f^2 - b^2*c*e^2)^5 + 1709253482624*a^14*b^22*c^8*e^22*
f^14*(a^2*c*f^2 - b^2*c*e^2)^5 - 3029282695168*a^16*b^20*c^8*e^20*f^16*(a^2
*c*f^2 - b^2*c*e^2)^5 + 3966230827520*a^18*b^18*c^8*e^18*f^18*(a^2*c*f^2 -
b^2*c*e^2)^5 - 3822339813632*a^20*b^16*c^8*e^16*f^20*(a^2*c*f^2 - b^2*c*e^2
)^5 + 2640438056960*a^22*b^14*c^8*e^14*f^22*(a^2*c*f^2 - b^2*c*e^2)^5 - 120
```

8501415936*a^24*b^12*c^8*e^12*f^24*(a^2*c*f^2 - b^2*c*e^2)^5 + 269338092544
*a^26*b^10*c^8*e^10*f^26*(a^2*c*f^2 - b^2*c*e^2)^5 + 53783212032*a^28*b^8*c
^8*e^8*f^28*(a^2*c*f^2 - b^2*c*e^2)^5 - 60985360384*a^30*b^6*c^8*e^6*f^30*(
a^2*c*f^2 - b^2*c*e^2)^5 + 17917083648*a^32*b^4*c^8*e^4*f^32*(a^2*c*f^2 - b
^2*c*e^2)^5 - 1558708224*a^34*b^2*c^8*e^2*f^34*(a^2*c*f^2 - b^2*c*e^2)^5 -
11917692*a^2*b^36*c^9*e^36*f^2*(a^2*c*f^2 - b^2*c*e^2)^4 - 224907516*a^4*b^
34*c^9*e^34*f^4*(a^2*c*f^2 - b^2*c*e^2)^4 + 5303932560*a^6*b^32*c^9*e^32*f^
6*(a^2*c*f^2 - b^2*c*e^2)^4 - 48206418480*a^8*b^30*c^9*e^30*f^8*(a^2*c*f^2
 - b^2*c*e^2)^4 + 261450609120*a^10*b^28*c^9*e^28*f^10*(a^2*c*f^2 - b^2*c*e^
2)^4 - 962361040256*a^12*b^26*c^9*e^26*f^12*(a^2*c*f^2 - b^2*c*e^2)^4 + 255
8559358080*a^14*b^24*c^9*e^24*f^14*(a^2*c*f^2 - b^2*c*e^2)^4 - 509180415065
6*a^16*b^22*c^9*e^22*f^16*(a^2*c*f^2 - b^2*c*e^2)^4 + 7750806514944*a^18*b^
20*c^9*e^20*f^18*(a^2*c*f^2 - b^2*c*e^2)^4 - 9137207485952*a^20*b^18*c^9*e^
18*f^20*(a^2*c*f^2 - b^2*c*e^2)^4 + 8384563280128*a^22*b^16*c^9*e^16*f^22*(
a^2*c*f^2 - b^2*c*e^2)^4 - 5975281259520*a^24*b^14*c^9*e^14*f^24*(a^2*c*f^2
 - b^2*c*e^2)^4 + 3269297268736*a^26*b^12*c^9*e^12*f^26*(a^2*c*f^2 - b^2*c*
e^2)^4 - 1339171540992*a^28*b^10*c^9*e^10*f^28*(a^2*c*f^2 - b^2*c*e^2)^4 +
391250194432*a^30*b^8*c^9*e^8*f^30*(a^2*c*f^2 - b^2*c*e^2)^4 - 74114154496*
a^32*b^6*c^9*e^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^4 + 7299203072*a^34*b^4*c^9*e
^4*f^34*(a^2*c*f^2 - b^2*c*e^2)^4 - 148635648*a^36*b^2*c^9*e^2*f^36*(a^2*c*
f^2 - b^2*c*e^2)^4 - 38704068*a^2*b^38*c^10*e^38*f^2*(a^2*c*f^2 - b^2*c*e^2
)^3 + 188845992*a^4*b^36*c^10*e^36*f^4*(a^2*c*f^2 - b^2*c*e^2)^3 + 11571242
04*a^6*b^34*c^10*e^34*f^6*(a^2*c*f^2 - b^2*c*e^2)^3 - 20586361424*a^8*b^32*
c^10*e^32*f^8*(a^2*c*f^2 - b^2*c*e^2)^3 + 135395499200*a^10*b^30*c^10*e^30*
f^10*(a^2*c*f^2 - b^2*c*e^2)^3 - 555513858464*a^12*b^28*c^10*e^28*f^12*(a^2
*c*f^2 - b^2*c*e^2)^3 + 1608776388864*a^14*b^26*c^10*e^26*f^14*(a^2*c*f^2 -
 b^2*c*e^2)^3 - 3473989271488*a^16*b^24*c^10*e^24*f^16*(a^2*c*f^2 - b^2*c*e
^2)^3 + 5766181411456*a^18*b^22*c^10*e^22*f^18*(a^2*c*f^2 - b^2*c*e^2)^3 -
7493983209472*a^20*b^20*c^10*e^20*f^20*(a^2*c*f^2 - b^2*c*e^2)^3 + 77139170
84672*a^22*b^18*c^10*e^18*f^22*(a^2*c*f^2 - b^2*c*e^2)^3 - 6328467293184*a^
24*b^16*c^10*e^16*f^24*(a^2*c*f^2 - b^2*c*e^2)^3 + 4142950034432*a^26*b^14*
c^10*e^14*f^26*(a^2*c*f^2 - b^2*c*e^2)^3 - 2152681536512*a^28*b^12*c^10*e^1
2*f^28*(a^2*c*f^2 - b^2*c*e^2)^3 + 874199511040*a^30*b^10*c^10*e^10*f^30*(a
^2*c*f^2 - b^2*c*e^2)^3 - 268759150592*a^32*b^8*c^10*e^8*f^32*(a^2*c*f^2 -
b^2*c*e^2)^3 + 58872545280*a^34*b^6*c^10*e^6*f^34*(a^2*c*f^2 - b^2*c*e^2)^3
 - 8151957504*a^36*b^4*c^10*e^4*f^36*(a^2*c*f^2 - b^2*c*e^2)^3 + 530841600*
a^38*b^2*c^10*e^2*f^38*(a^2*c*f^2 - b^2*c*e^2)^3 - 42743457*a^2*b^40*c^11*e
^40*f^2*(a^2*c*f^2 - b^2*c*e^2)^2 + 411055884*a^4*b^38*c^11*e^38*f^4*(a^2*c
*f^2 - b^2*c*e^2)^2 - 2180887236*a^6*b^36*c^11*e^36*f^6*(a^2*c*f^2 - b^2*c*
e^2)^2 + 6404946508*a^8*b^34*c^11*e^34*f^8*(a^2*c*f^2 - b^2*c*e^2)^2 - 5434
005264*a^10*b^32*c^11*e^32*f^10*(a^2*c*f^2 - b^2*c*e^2)^2 - 38868373520*a^1
2*b^30*c^11*e^30*f^12*(a^2*c*f^2 - b^2*c*e^2)^2 + 208447613600*a^14*b^28*c^
11*e^28*f^14*(a^2*c*f^2 - b^2*c*e^2)^2 - 579674999104*a^16*b^26*c^11*e^26*f
^16*(a^2*c*f^2 - b^2*c*e^2)^2 + 1104967566592*a^18*b^24*c^11*e^24*f^18*(a^2
*c*f^2 - b^2*c*e^2)^2 - 1554566531328*a^20*b^22*c^11*e^22*f^20*(a^2*c*f^2 -
 b^2*c*e^2)^2 + 1659734381312*a^22*b^20*c^11*e^20*f^22*(a^2*c*f^2 - b^2*c*e
^2)^2 - 1356361512192*a^24*b^18*c^11*e^18*f^24*(a^2*c*f^2 - b^2*c*e^2)^2 +
845331359744*a^26*b^16*c^11*e^16*f^26*(a^2*c*f^2 - b^2*c*e^2)^2 - 395676895
232*a^28*b^14*c^11*e^14*f^28*(a^2*c*f^2 - b^2*c*e^2)^2 + 134902689792*a^30*
b^12*c^11*e^12*f^30*(a^2*c*f^2 - b^2*c*e^2)^2 - 31670587392*a^32*b^10*c^11*
e^10*f^32*(a^2*c*f^2 - b^2*c*e^2)^2 + 4584669184*a^34*b^8*c^11*e^8*f^34*(a^
2*c*f^2 - b^2*c*e^2)^2 - 309657600*a^36*b^6*c^11*e^6*f^36*(a^2*c*f^2 - b^2*
c*e^2)^2)) + (2*a^(3/2)*b^5*c*e^5*f^3*((4096*C^3*e^3*(2*a^2*f^2 - b^2*e^2)^
3*(24*C*a^(21/2)*b^2*c^4*e*f^15*(a*c)^(5/2) - 30*C*a^(3/2)*b^12*c^5*e^11*f^
5*(a*c)^(3/2) + 24*C*a^(5/2)*b^10*c^4*e^9*f^7*(a*c)^(5/2) + 126*C*a^(7/2)*b
^10*c^5*e^9*f^7*(a*c)^(3/2) - 96*C*a^(9/2)*b^8*c^4*e^7*f^9*(a*c)^(5/2) - 19
8*C*a^(11/2)*b^8*c^5*e^7*f^9*(a*c)^(3/2) + 144*C*a^(13/2)*b^6*c^4*e^5*f^11*
(a*c)^(5/2) + 138*C*a^(15/2)*b^6*c^5*e^5*f^11*(a*c)^(3/2) - 96*C*a^(17/2)*b
^4*c^4*e^3*f^13*(a*c)^(5/2) - 36*C*a^(19/2)*b^4*c^5*e^3*f^13*(a*c)^(3/2)))/

```
(f^6*(a*f + b*e)^3*(a*f - b*e)^3*(b^2*c*e^2 - a^2*c*f^2)^(3/2)*(b^16*e^14*f
^4 - 4*a^2*b^14*e^12*f^6 + 6*a^4*b^12*e^10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*
b^8*e^6*f^12)) + (4096*C*e*(2*a^2*f^2 - b^2*e^2)*(64*C^3*a^(21/2)*c^3*e*f^1
1*(a*c)^(5/2) + 32*C^3*a^(5/2)*b^8*c^3*e^9*f^3*(a*c)^(5/2) - 160*C^3*a^(7/2
)*b^8*c^4*e^9*f^3*(a*c)^(3/2) - 160*C^3*a^(9/2)*b^6*c^3*e^7*f^5*(a*c)^(5/2)
 + 384*C^3*a^(11/2)*b^6*c^4*e^7*f^5*(a*c)^(3/2) + 288*C^3*a^(13/2)*b^4*c^3*
e^5*f^7*(a*c)^(5/2) - 392*C^3*a^(15/2)*b^4*c^4*e^5*f^7*(a*c)^(3/2) - 224*C^
3*a^(17/2)*b^2*c^3*e^3*f^9*(a*c)^(5/2) + 144*C^3*a^(19/2)*b^2*c^4*e^3*f^9*(
a*c)^(3/2) + 24*C^3*a^(3/2)*b^10*c^4*e^11*f*(a*c)^(3/2)))/(f^2*(a*f + b*e)*
(a*f - b*e)*(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^16*e^14*f^4 - 4*a^2*b^14*e^12*
f^6 + 6*a^4*b^12*e^10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12)))*(a*c)
^(3/2)*(4*a^2*c*f^2 - b^2*c*e^2)*(4*a^2*c*f^2 - 3*b^2*c*e^2)*(4*a^6*c*f^6 -
 3*b^6*c*e^6 + 8*a^2*b^4*c*e^4*f^2 - 8*a^4*b^2*c*e^2*f^4)^4)/(164025*b^46*c
^13*e^46 + 885735*b^44*c^12*e^44*(a^2*c*f^2 - b^2*c*e^2) + 117440512*a^30*c
^5*f^30*(a^2*c*f^2 - b^2*c*e^2)^8 - 385875968*a^32*c^6*f^32*(a^2*c*f^2 - b^
2*c*e^2)^7 + 419430400*a^34*c^7*f^34*(a^2*c*f^2 - b^2*c*e^2)^6 - 150994944*
a^36*c^8*f^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 236196*b^36*c^8*e^36*(a^2*c*f^2 -
 b^2*c*e^2)^5 + 1102248*b^38*c^9*e^38*(a^2*c*f^2 - b^2*c*e^2)^4 + 2053593*b
^40*c^10*e^40*(a^2*c*f^2 - b^2*c*e^2)^3 + 1909251*b^42*c^11*e^42*(a^2*c*f^2
 - b^2*c*e^2)^2 - 3937329*a^2*b^44*c^13*e^44*f^2 + 43893819*a^4*b^42*c^13*e
^42*f^4 - 301507155*a^6*b^40*c^13*e^40*f^6 + 1427514656*a^8*b^38*c^13*e^38*
f^8 - 4936911112*a^10*b^36*c^13*e^36*f^10 + 12893273616*a^12*b^34*c^13*e^34
*f^12 - 25921630432*a^14*b^32*c^13*e^32*f^14 + 40519286096*a^16*b^30*c^13*e
^30*f^16 - 49376608256*a^18*b^28*c^13*e^28*f^18 + 46721401856*a^20*b^26*c^1
3*e^26*f^20 - 33946324736*a^22*b^24*c^13*e^24*f^22 + 18556579328*a^24*b^22*
c^13*e^22*f^24 - 7375276032*a^26*b^20*c^13*e^20*f^26 + 2009817088*a^28*b^18
*c^13*e^18*f^28 - 335642624*a^30*b^16*c^13*e^16*f^30 + 25907200*a^32*b^14*c
^13*e^14*f^32 - 21130794*a^2*b^42*c^12*e^42*f^2*(a^2*c*f^2 - b^2*c*e^2) + 2
34399015*a^4*b^40*c^12*e^40*f^4*(a^2*c*f^2 - b^2*c*e^2) - 1604168280*a^6*b^
38*c^12*e^38*f^6*(a^2*c*f^2 - b^2*c*e^2) + 7579098492*a^8*b^36*c^12*e^36*f^
8*(a^2*c*f^2 - b^2*c*e^2) - 26212380172*a^10*b^34*c^12*e^34*f^10*(a^2*c*f^2
 - b^2*c*e^2) + 68672994096*a^12*b^32*c^12*e^32*f^12*(a^2*c*f^2 - b^2*c*e^2
) - 139160589504*a^14*b^30*c^12*e^30*f^14*(a^2*c*f^2 - b^2*c*e^2) + 2208591
91808*a^16*b^28*c^12*e^28*f^16*(a^2*c*f^2 - b^2*c*e^2) - 276344315328*a^18*
b^26*c^12*e^26*f^18*(a^2*c*f^2 - b^2*c*e^2) + 273130561984*a^20*b^24*c^12*e
^24*f^20*(a^2*c*f^2 - b^2*c*e^2) - 212730002688*a^22*b^22*c^12*e^22*f^22*(a
^2*c*f^2 - b^2*c*e^2) + 129574234368*a^24*b^20*c^12*e^20*f^24*(a^2*c*f^2 -
b^2*c*e^2) - 60770569216*a^26*b^18*c^12*e^18*f^26*(a^2*c*f^2 - b^2*c*e^2) +
 21304706048*a^28*b^16*c^12*e^16*f^28*(a^2*c*f^2 - b^2*c*e^2) - 5272965120*
a^30*b^14*c^12*e^14*f^30*(a^2*c*f^2 - b^2*c*e^2) + 819441664*a^32*b^12*c^12
*e^12*f^32*(a^2*c*f^2 - b^2*c*e^2) - 59392000*a^34*b^10*c^12*e^10*f^34*(a^2
*c*f^2 - b^2*c*e^2) + 9289728*a^6*b^24*c^5*e^24*f^6*(a^2*c*f^2 - b^2*c*e^2)
^8 - 36884480*a^8*b^22*c^5*e^22*f^8*(a^2*c*f^2 - b^2*c*e^2)^8 - 278604800*a
^10*b^20*c^5*e^20*f^10*(a^2*c*f^2 - b^2*c*e^2)^8 + 2774483200*a^12*b^18*c^5
*e^18*f^12*(a^2*c*f^2 - b^2*c*e^2)^8 - 10869657600*a^14*b^16*c^5*e^16*f^14*
(a^2*c*f^2 - b^2*c*e^2)^8 + 25237416960*a^16*b^14*c^5*e^14*f^16*(a^2*c*f^2
- b^2*c*e^2)^8 - 38348909568*a^18*b^12*c^5*e^12*f^18*(a^2*c*f^2 - b^2*c*e^2
)^8 + 39084659712*a^20*b^10*c^5*e^10*f^20*(a^2*c*f^2 - b^2*c*e^2)^8 - 26118
635520*a^22*b^8*c^5*e^8*f^22*(a^2*c*f^2 - b^2*c*e^2)^8 + 10414620672*a^24*b
^6*c^5*e^6*f^24*(a^2*c*f^2 - b^2*c*e^2)^8 - 1708654592*a^26*b^4*c^5*e^4*f^2
6*(a^2*c*f^2 - b^2*c*e^2)^8 - 276561920*a^28*b^2*c^5*e^2*f^28*(a^2*c*f^2 -
b^2*c*e^2)^8 - 9704448*a^4*b^28*c^6*e^28*f^4*(a^2*c*f^2 - b^2*c*e^2)^7 + 26
0614656*a^6*b^26*c^6*e^26*f^6*(a^2*c*f^2 - b^2*c*e^2)^7 - 2166022464*a^8*b^
24*c^6*e^24*f^8*(a^2*c*f^2 - b^2*c*e^2)^7 + 8626147840*a^10*b^22*c^6*e^22*f
^10*(a^2*c*f^2 - b^2*c*e^2)^7 - 16771503616*a^12*b^20*c^6*e^20*f^12*(a^2*c*
f^2 - b^2*c*e^2)^7 + 3301800960*a^14*b^18*c^6*e^18*f^14*(a^2*c*f^2 - b^2*c*
e^2)^7 + 67337715968*a^16*b^16*c^6*e^16*f^16*(a^2*c*f^2 - b^2*c*e^2)^7 - 18
9857873920*a^18*b^14*c^6*e^14*f^18*(a^2*c*f^2 - b^2*c*e^2)^7 + 286100259840
*a^20*b^12*c^6*e^12*f^20*(a^2*c*f^2 - b^2*c*e^2)^7 - 275789894656*a^22*b^10
```

```
*c^6*e^10*f^22*(a^2*c*f^2 - b^2*c*e^2)^7 + 173716537344*a^24*b^8*c^6*e^8*f^
24*(a^2*c*f^2 - b^2*c*e^2)^7 - 67416424448*a^26*b^6*c^6*e^6*f^26*(a^2*c*f^2
 - b^2*c*e^2)^7 + 12831686656*a^28*b^4*c^6*e^4*f^28*(a^2*c*f^2 - b^2*c*e^2)
^7 + 222560256*a^30*b^2*c^6*e^2*f^30*(a^2*c*f^2 - b^2*c*e^2)^7 + 2099520*a^
2*b^32*c^7*e^32*f^2*(a^2*c*f^2 - b^2*c*e^2)^6 - 107014608*a^4*b^30*c^7*e^30
*f^4*(a^2*c*f^2 - b^2*c*e^2)^6 + 1848335616*a^6*b^28*c^7*e^28*f^6*(a^2*c*f^
2 - b^2*c*e^2)^6 - 15200005312*a^8*b^26*c^7*e^26*f^8*(a^2*c*f^2 - b^2*c*e^2
)^6 + 72612273792*a^10*b^24*c^7*e^24*f^10*(a^2*c*f^2 - b^2*c*e^2)^6 - 22185
5779968*a^12*b^22*c^7*e^22*f^12*(a^2*c*f^2 - b^2*c*e^2)^6 + 450717857536*a^
14*b^20*c^7*e^20*f^14*(a^2*c*f^2 - b^2*c*e^2)^6 - 600578910208*a^16*b^18*c^
7*e^18*f^16*(a^2*c*f^2 - b^2*c*e^2)^6 + 459464530688*a^18*b^16*c^7*e^16*f^1
8*(a^2*c*f^2 - b^2*c*e^2)^6 - 33638947840*a^20*b^14*c^7*e^14*f^20*(a^2*c*f^
2 - b^2*c*e^2)^6 - 376299926528*a^22*b^12*c^7*e^12*f^22*(a^2*c*f^2 - b^2*c*
e^2)^6 + 488874068992*a^24*b^10*c^7*e^10*f^24*(a^2*c*f^2 - b^2*c*e^2)^6 - 3
33407809536*a^26*b^8*c^7*e^8*f^26*(a^2*c*f^2 - b^2*c*e^2)^6 + 134140313600*
a^28*b^6*c^7*e^6*f^28*(a^2*c*f^2 - b^2*c*e^2)^6 - 28220915712*a^30*b^4*c^7*
e^4*f^30*(a^2*c*f^2 - b^2*c*e^2)^6 + 1230503936*a^32*b^2*c^7*e^2*f^32*(a^2*
c*f^2 - b^2*c*e^2)^6 + 3335904*a^2*b^34*c^8*e^34*f^2*(a^2*c*f^2 - b^2*c*e^2
)^5 - 290521728*a^4*b^32*c^8*e^32*f^4*(a^2*c*f^2 - b^2*c*e^2)^5 + 486568454
4*a^6*b^30*c^8*e^30*f^6*(a^2*c*f^2 - b^2*c*e^2)^5 - 40437394528*a^8*b^28*c^
8*e^28*f^8*(a^2*c*f^2 - b^2*c*e^2)^5 + 205602254656*a^10*b^26*c^8*e^26*f^10
*(a^2*c*f^2 - b^2*c*e^2)^5 - 703885344192*a^12*b^24*c^8*e^24*f^12*(a^2*c*f^
2 - b^2*c*e^2)^5 + 1709253482624*a^14*b^22*c^8*e^22*f^14*(a^2*c*f^2 - b^2*c
*e^2)^5 - 3029282695168*a^16*b^20*c^8*e^20*f^16*(a^2*c*f^2 - b^2*c*e^2)^5 +
 3966230827520*a^18*b^18*c^8*e^18*f^18*(a^2*c*f^2 - b^2*c*e^2)^5 - 38223398
13632*a^20*b^16*c^8*e^16*f^20*(a^2*c*f^2 - b^2*c*e^2)^5 + 2640438056960*a^2
2*b^14*c^8*e^14*f^22*(a^2*c*f^2 - b^2*c*e^2)^5 - 1208501415936*a^24*b^12*c^
8*e^12*f^24*(a^2*c*f^2 - b^2*c*e^2)^5 + 269338092544*a^26*b^10*c^8*e^10*f^2
6*(a^2*c*f^2 - b^2*c*e^2)^5 + 53783212032*a^28*b^8*c^8*e^8*f^28*(a^2*c*f^2
 - b^2*c*e^2)^5 - 60985360384*a^30*b^6*c^8*e^6*f^30*(a^2*c*f^2 - b^2*c*e^2)^
5 + 17917083648*a^32*b^4*c^8*e^4*f^32*(a^2*c*f^2 - b^2*c*e^2)^5 - 155870822
4*a^34*b^2*c^8*e^2*f^34*(a^2*c*f^2 - b^2*c*e^2)^5 - 11917692*a^2*b^36*c^9*e
^36*f^2*(a^2*c*f^2 - b^2*c*e^2)^4 - 224907516*a^4*b^34*c^9*e^34*f^4*(a^2*c*
f^2 - b^2*c*e^2)^4 + 5303932560*a^6*b^32*c^9*e^32*f^6*(a^2*c*f^2 - b^2*c*e^
2)^4 - 48206418480*a^8*b^30*c^9*e^30*f^8*(a^2*c*f^2 - b^2*c*e^2)^4 + 261450
609120*a^10*b^28*c^9*e^28*f^10*(a^2*c*f^2 - b^2*c*e^2)^4 - 962361040256*a^1
2*b^26*c^9*e^26*f^12*(a^2*c*f^2 - b^2*c*e^2)^4 + 2558559358080*a^14*b^24*c^
9*e^24*f^14*(a^2*c*f^2 - b^2*c*e^2)^4 - 5091804150656*a^16*b^22*c^9*e^22*f^
16*(a^2*c*f^2 - b^2*c*e^2)^4 + 7750806514944*a^18*b^20*c^9*e^20*f^18*(a^2*c
*f^2 - b^2*c*e^2)^4 - 9137207485952*a^20*b^18*c^9*e^18*f^20*(a^2*c*f^2 - b^
2*c*e^2)^4 + 8384563280128*a^22*b^16*c^9*e^16*f^22*(a^2*c*f^2 - b^2*c*e^2)^
4 - 5975281259520*a^24*b^14*c^9*e^14*f^24*(a^2*c*f^2 - b^2*c*e^2)^4 + 32692
97268736*a^26*b^12*c^9*e^12*f^26*(a^2*c*f^2 - b^2*c*e^2)^4 - 1339171540992*
a^28*b^10*c^9*e^10*f^28*(a^2*c*f^2 - b^2*c*e^2)^4 + 391250194432*a^30*b^8*c
^9*e^8*f^30*(a^2*c*f^2 - b^2*c*e^2)^4 - 74114154496*a^32*b^6*c^9*e^6*f^32*(
a^2*c*f^2 - b^2*c*e^2)^4 + 7299203072*a^34*b^4*c^9*e^4*f^34*(a^2*c*f^2 - b^
2*c*e^2)^4 - 148635648*a^36*b^2*c^9*e^2*f^36*(a^2*c*f^2 - b^2*c*e^2)^4 - 38
704068*a^2*b^38*c^10*e^38*f^2*(a^2*c*f^2 - b^2*c*e^2)^3 + 188845992*a^4*b^3
6*c^10*e^36*f^4*(a^2*c*f^2 - b^2*c*e^2)^3 + 1157124204*a^6*b^34*c^10*e^34*f
^6*(a^2*c*f^2 - b^2*c*e^2)^3 - 20586361424*a^8*b^32*c^10*e^32*f^8*(a^2*c*f^
2 - b^2*c*e^2)^3 + 135395499200*a^10*b^30*c^10*e^30*f^10*(a^2*c*f^2 - b^2*c
*e^2)^3 - 555513858464*a^12*b^28*c^10*e^28*f^12*(a^2*c*f^2 - b^2*c*e^2)^3 +
 1608776388864*a^14*b^26*c^10*e^26*f^14*(a^2*c*f^2 - b^2*c*e^2)^3 - 3473989
271488*a^16*b^24*c^10*e^24*f^16*(a^2*c*f^2 - b^2*c*e^2)^3 + 5766181411456*a
^18*b^22*c^10*e^22*f^18*(a^2*c*f^2 - b^2*c*e^2)^3 - 7493983209472*a^20*b^20
*c^10*e^20*f^20*(a^2*c*f^2 - b^2*c*e^2)^3 + 7713917084672*a^22*b^18*c^10*e^
18*f^22*(a^2*c*f^2 - b^2*c*e^2)^3 - 6328467293184*a^24*b^16*c^10*e^16*f^24*
(a^2*c*f^2 - b^2*c*e^2)^3 + 4142950034432*a^26*b^14*c^10*e^14*f^26*(a^2*c*f
^2 - b^2*c*e^2)^3 - 2152681536512*a^28*b^12*c^10*e^12*f^28*(a^2*c*f^2 - b^2
```

*c*e^2)^3 + 874199511040*a^30*b^10*c^10*e^10*f^30*(a^2*c*f^2 - b^2*c*e^2)^3
 - 268759150592*a^32*b^8*c^10*e^8*f^32*(a^2*c*f^2 - b^2*c*e^2)^3 + 58872545
280*a^34*b^6*c^10*e^6*f^34*(a^2*c*f^2 - b^2*c*e^2)^3 - 8151957504*a^36*b^4*
c^10*e^4*f^36*(a^2*c*f^2 - b^2*c*e^2)^3 + 530841600*a^38*b^2*c^10*e^2*f^38*
(a^2*c*f^2 - b^2*c*e^2)^3 - 42743457*a^2*b^40*c^11*e^40*f^2*(a^2*c*f^2 - b^
2*c*e^2)^2 + 411055884*a^4*b^38*c^11*e^38*f^4*(a^2*c*f^2 - b^2*c*e^2)^2 - 2
180887236*a^6*b^36*c^11*e^36*f^6*(a^2*c*f^2 - b^2*c*e^2)^2 + 6404946508*a^8
*b^34*c^11*e^34*f^8*(a^2*c*f^2 - b^2*c*e^2)^2 - 5434005264*a^10*b^32*c^11*e
^32*f^10*(a^2*c*f^2 - b^2*c*e^2)^2 - 38868373520*a^12*b^30*c^11*e^30*f^12*(
a^2*c*f^2 - b^2*c*e^2)^2 + 208447613600*a^14*b^28*c^11*e^28*f^14*(a^2*c*f^2
 - b^2*c*e^2)^2 - 579674999104*a^16*b^26*c^11*e^26*f^16*(a^2*c*f^2 - b^2*c*
e^2)^2 + 1104967566592*a^18*b^24*c^11*e^24*f^18*(a^2*c*f^2 - b^2*c*e^2)^2 -
 1554566531328*a^20*b^22*c^11*e^22*f^20*(a^2*c*f^2 - b^2*c*e^2)^2 + 1659734
381312*a^22*b^20*c^11*e^20*f^22*(a^2*c*f^2 - b^2*c*e^2)^2 - 1356361512192*a
^24*b^18*c^11*e^18*f^24*(a^2*c*f^2 - b^2*c*e^2)^2 + 845331359744*a^26*b^16*
c^11*e^16*f^26*(a^2*c*f^2 - b^2*c*e^2)^2 - 395676895232*a^28*b^14*c^11*e^14
f^28(a^2*c*f^2 - b^2*c*e^2)^2 + 134902689792*a^30*b^12*c^11*e^12*f^30*(a^
2*c*f^2 - b^2*c*e^2)^2 - 31670587392*a^32*b^10*c^11*e^10*f^32*(a^2*c*f^2 -
b^2*c*e^2)^2 + 4584669184*a^34*b^8*c^11*e^8*f^34*(a^2*c*f^2 - b^2*c*e^2)^2
- 309657600*a^36*b^6*c^11*e^6*f^36*(a^2*c*f^2 - b^2*c*e^2)^2) + (4*a^(3/2)*
b^6*c^2*e^6*f^3*(a*c)^(3/2)*(2*a^2*c*f^2 - b^2*c*e^2)*(4*a^2*c*f^2 - 3*b^2*
c*e^2)*((16384*(12*C^4*a^(7/2)*b^4*c^3*e^7*(a*c)^(3/2) + 48*C^4*a^(15/2)*c^
3*e^3*f^4*(a*c)^(3/2) - 48*C^4*a^(11/2)*b^2*c^3*e^5*f^2*(a*c)^(3/2)))/(b^13
*e^12*f^3 - 3*a^2*b^11*e^10*f^5 + 3*a^4*b^9*e^8*f^7 - a^6*b^7*e^6*f^9) + (1
6384*C^4*e^4*(2*a^2*f^2 - b^2*e^2)^4*(5*a^(17/2)*b^2*c^4*e*f^14*(a*c)^(5/2)
 + 6*a^(3/2)*b^10*c^5*e^9*f^6*(a*c)^(3/2) - 5*a^(5/2)*b^8*c^4*e^7*f^8*(a*c)
^(5/2) - 18*a^(7/2)*b^8*c^5*e^7*f^8*(a*c)^(3/2) + 15*a^(9/2)*b^6*c^4*e^5*f^
10*(a*c)^(5/2) + 18*a^(11/2)*b^6*c^5*e^5*f^10*(a*c)^(3/2) - 15*a^(13/2)*b^4
*c^4*e^3*f^12*(a*c)^(5/2) - 6*a^(15/2)*b^4*c^5*e^3*f^12*(a*c)^(3/2)))/(f^8*
(a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)^2*(b^13*e^12*f^3 - 3*a^
2*b^11*e^10*f^5 + 3*a^4*b^9*e^8*f^7 - a^6*b^7*e^6*f^9)) - (16384*C^2*e^2*(2
*a^2*f^2 - b^2*e^2)^2*(20*C^2*a^(17/2)*c^3*e*f^10*(a*c)^(5/2) - 3*C^2*a^(3/
2)*b^8*c^4*e^9*f^2*(a*c)^(3/2) - 8*C^2*a^(5/2)*b^6*c^3*e^7*f^4*(a*c)^(5/2)
+ 11*C^2*a^(7/2)*b^6*c^4*e^7*f^4*(a*c)^(3/2) + 36*C^2*a^(9/2)*b^4*c^3*e^5*f
^6*(a*c)^(5/2) - 20*C^2*a^(11/2)*b^4*c^4*e^5*f^6*(a*c)^(3/2) - 48*C^2*a^(13
/2)*b^2*c^3*e^3*f^8*(a*c)^(5/2) + 12*C^2*a^(15/2)*b^2*c^4*e^3*f^8*(a*c)^(3/
2)))/(f^4*(a*f + b*e)^2*(a*f - b*e)^2*(a^2*c*f^2 - b^2*c*e^2)*(b^13*e^12*f^
3 - 3*a^2*b^11*e^10*f^5 + 3*a^4*b^9*e^8*f^7 - a^6*b^7*e^6*f^9)))*(4*a^6*c*f
^6 - 3*b^6*c*e^6 + 8*a^2*b^4*c*e^4*f^2 - 8*a^4*b^2*c*e^2*f^4)^4)/((b^2*c*e^
2 - a^2*c*f^2)^(1/2)*(164025*b^46*c^13*e^46 + 885735*b^44*c^12*e^44*(a^2*c*
f^2 - b^2*c*e^2) + 117440512*a^30*c^5*f^30*(a^2*c*f^2 - b^2*c*e^2)^8 - 3858
75968*a^32*c^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^7 + 419430400*a^34*c^7*f^34*(a^
2*c*f^2 - b^2*c*e^2)^6 - 150994944*a^36*c^8*f^36*(a^2*c*f^2 - b^2*c*e^2)^5
+ 236196*b^36*c^8*e^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 1102248*b^38*c^9*e^38*(a
^2*c*f^2 - b^2*c*e^2)^4 + 2053593*b^40*c^10*e^40*(a^2*c*f^2 - b^2*c*e^2)^3
+ 1909251*b^42*c^11*e^42*(a^2*c*f^2 - b^2*c*e^2)^2 - 3937329*a^2*b^44*c^13*
e^44*f^2 + 43893819*a^4*b^42*c^13*e^42*f^4 - 301507155*a^6*b^40*c^13*e^40*f
^6 + 1427514656*a^8*b^38*c^13*e^38*f^8 - 4936911112*a^10*b^36*c^13*e^36*f^1
0 + 12893273616*a^12*b^34*c^13*e^34*f^12 - 25921630432*a^14*b^32*c^13*e^32*
f^14 + 40519286096*a^16*b^30*c^13*e^30*f^16 - 49376608256*a^18*b^28*c^13*e^
28*f^18 + 46721401856*a^20*b^26*c^13*e^26*f^20 - 33946324736*a^22*b^24*c^13
*e^24*f^22 + 18556579328*a^24*b^22*c^13*e^22*f^24 - 7375276032*a^26*b^20*c^
13*e^20*f^26 + 2009817088*a^28*b^18*c^13*e^18*f^28 - 335642624*a^30*b^16*c^
13*e^16*f^30 + 25907200*a^32*b^14*c^13*e^14*f^32 - 21130794*a^2*b^42*c^12*e
^42*f^2*(a^2*c*f^2 - b^2*c*e^2) + 234399015*a^4*b^40*c^12*e^40*f^4*(a^2*c*f
^2 - b^2*c*e^2) - 1604168280*a^6*b^38*c^12*e^38*f^6*(a^2*c*f^2 - b^2*c*e^2)
 + 7579098492*a^8*b^36*c^12*e^36*f^8*(a^2*c*f^2 - b^2*c*e^2) - 26212380172*
a^10*b^34*c^12*e^34*f^10*(a^2*c*f^2 - b^2*c*e^2) + 68672994096*a^12*b^32*c^
12*e^32*f^12*(a^2*c*f^2 - b^2*c*e^2) - 139160589504*a^14*b^30*c^12*e^30*f^1

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4*(a^2*c*f^2 - b^2*c*e^2) + 220859191808*a^16*b^28*c^12*e^28*f^16*(a^2*c*f^
2 - b^2*c*e^2) - 276344315328*a^18*b^26*c^12*e^26*f^18*(a^2*c*f^2 - b^2*c*e
^2) + 273130561984*a^20*b^24*c^12*e^24*f^20*(a^2*c*f^2 - b^2*c*e^2) - 21273
0002688*a^22*b^22*c^12*e^22*f^22*(a^2*c*f^2 - b^2*c*e^2) + 129574234368*a^2
4*b^20*c^12*e^20*f^24*(a^2*c*f^2 - b^2*c*e^2) - 60770569216*a^26*b^18*c^12*
e^18*f^26*(a^2*c*f^2 - b^2*c*e^2) + 21304706048*a^28*b^16*c^12*e^16*f^28*(a
^2*c*f^2 - b^2*c*e^2) - 5272965120*a^30*b^14*c^12*e^14*f^30*(a^2*c*f^2 - b^
2*c*e^2) + 819441664*a^32*b^12*c^12*e^12*f^32*(a^2*c*f^2 - b^2*c*e^2) - 593
92000*a^34*b^10*c^12*e^10*f^34*(a^2*c*f^2 - b^2*c*e^2) + 9289728*a^6*b^24*c
^5*e^24*f^6*(a^2*c*f^2 - b^2*c*e^2)^8 - 36884480*a^8*b^22*c^5*e^22*f^8*(a^2
*c*f^2 - b^2*c*e^2)^8 - 278604800*a^10*b^20*c^5*e^20*f^10*(a^2*c*f^2 - b^2*
c*e^2)^8 + 2774483200*a^12*b^18*c^5*e^18*f^12*(a^2*c*f^2 - b^2*c*e^2)^8 - 1
0869657600*a^14*b^16*c^5*e^16*f^14*(a^2*c*f^2 - b^2*c*e^2)^8 + 25237416960*
a^16*b^14*c^5*e^14*f^16*(a^2*c*f^2 - b^2*c*e^2)^8 - 38348909568*a^18*b^12*c
^5*e^12*f^18*(a^2*c*f^2 - b^2*c*e^2)^8 + 39084659712*a^20*b^10*c^5*e^10*f^2
0*(a^2*c*f^2 - b^2*c*e^2)^8 - 26118635520*a^22*b^8*c^5*e^8*f^22*(a^2*c*f^2
 - b^2*c*e^2)^8 + 10414620672*a^24*b^6*c^5*e^6*f^24*(a^2*c*f^2 - b^2*c*e^2)^
8 - 1708654592*a^26*b^4*c^5*e^4*f^26*(a^2*c*f^2 - b^2*c*e^2)^8 - 276561920*
a^28*b^2*c^5*e^2*f^28*(a^2*c*f^2 - b^2*c*e^2)^8 - 9704448*a^4*b^28*c^6*e^28
*f^4*(a^2*c*f^2 - b^2*c*e^2)^7 + 260614656*a^6*b^26*c^6*e^26*f^6*(a^2*c*f^2
 - b^2*c*e^2)^7 - 2166022464*a^8*b^24*c^6*e^24*f^8*(a^2*c*f^2 - b^2*c*e^2)^
7 + 8626147840*a^10*b^22*c^6*e^22*f^10*(a^2*c*f^2 - b^2*c*e^2)^7 - 16771503
616*a^12*b^20*c^6*e^20*f^12*(a^2*c*f^2 - b^2*c*e^2)^7 + 3301800960*a^14*b^1
8*c^6*e^18*f^14*(a^2*c*f^2 - b^2*c*e^2)^7 + 67337715968*a^16*b^16*c^6*e^16*
f^16*(a^2*c*f^2 - b^2*c*e^2)^7 - 189857873920*a^18*b^14*c^6*e^14*f^18*(a^2*
c*f^2 - b^2*c*e^2)^7 + 286100259840*a^20*b^12*c^6*e^12*f^20*(a^2*c*f^2 - b^
2*c*e^2)^7 - 275789894656*a^22*b^10*c^6*e^10*f^22*(a^2*c*f^2 - b^2*c*e^2)^7
 + 173716537344*a^24*b^8*c^6*e^8*f^24*(a^2*c*f^2 - b^2*c*e^2)^7 - 674164244
48*a^26*b^6*c^6*e^6*f^26*(a^2*c*f^2 - b^2*c*e^2)^7 + 12831686656*a^28*b^4*c
^6*e^4*f^28*(a^2*c*f^2 - b^2*c*e^2)^7 + 222560256*a^30*b^2*c^6*e^2*f^30*(a^
2*c*f^2 - b^2*c*e^2)^7 + 2099520*a^2*b^32*c^7*e^32*f^2*(a^2*c*f^2 - b^2*c*e
^2)^6 - 107014608*a^4*b^30*c^7*e^30*f^4*(a^2*c*f^2 - b^2*c*e^2)^6 + 1848335
616*a^6*b^28*c^7*e^28*f^6*(a^2*c*f^2 - b^2*c*e^2)^6 - 15200005312*a^8*b^26*
c^7*e^26*f^8*(a^2*c*f^2 - b^2*c*e^2)^6 + 72612273792*a^10*b^24*c^7*e^24*f^1
0*(a^2*c*f^2 - b^2*c*e^2)^6 - 221855779968*a^12*b^22*c^7*e^22*f^12*(a^2*c*f
^2 - b^2*c*e^2)^6 + 450717857536*a^14*b^20*c^7*e^20*f^14*(a^2*c*f^2 - b^2*c
*e^2)^6 - 600578910208*a^16*b^18*c^7*e^18*f^16*(a^2*c*f^2 - b^2*c*e^2)^6 +
459464530688*a^18*b^16*c^7*e^16*f^18*(a^2*c*f^2 - b^2*c*e^2)^6 - 3363894784
0*a^20*b^14*c^7*e^14*f^20*(a^2*c*f^2 - b^2*c*e^2)^6 - 376299926528*a^22*b^1
2*c^7*e^12*f^22*(a^2*c*f^2 - b^2*c*e^2)^6 + 488874068992*a^24*b^10*c^7*e^10
*f^24*(a^2*c*f^2 - b^2*c*e^2)^6 - 333407809536*a^26*b^8*c^7*e^8*f^26*(a^2*c
*f^2 - b^2*c*e^2)^6 + 134140313600*a^28*b^6*c^7*e^6*f^28*(a^2*c*f^2 - b^2*c
*e^2)^6 - 28220915712*a^30*b^4*c^7*e^4*f^30*(a^2*c*f^2 - b^2*c*e^2)^6 + 123
0503936*a^32*b^2*c^7*e^2*f^32*(a^2*c*f^2 - b^2*c*e^2)^6 + 3335904*a^2*b^34*
c^8*e^34*f^2*(a^2*c*f^2 - b^2*c*e^2)^5 - 290521728*a^4*b^32*c^8*e^32*f^4*(a
^2*c*f^2 - b^2*c*e^2)^5 + 4865684544*a^6*b^30*c^8*e^30*f^6*(a^2*c*f^2 - b^2
*c*e^2)^5 - 40437394528*a^8*b^28*c^8*e^28*f^8*(a^2*c*f^2 - b^2*c*e^2)^5 + 2
05602254656*a^10*b^26*c^8*e^26*f^10*(a^2*c*f^2 - b^2*c*e^2)^5 - 70388534419
2*a^12*b^24*c^8*e^24*f^12*(a^2*c*f^2 - b^2*c*e^2)^5 + 1709253482624*a^14*b^
22*c^8*e^22*f^14*(a^2*c*f^2 - b^2*c*e^2)^5 - 3029282695168*a^16*b^20*c^8*e^
20*f^16*(a^2*c*f^2 - b^2*c*e^2)^5 + 3966230827520*a^18*b^18*c^8*e^18*f^18*(
a^2*c*f^2 - b^2*c*e^2)^5 - 3822339813632*a^20*b^16*c^8*e^16*f^20*(a^2*c*f^2
 - b^2*c*e^2)^5 + 2640438056960*a^22*b^14*c^8*e^14*f^22*(a^2*c*f^2 - b^2*c*
e^2)^5 - 1208501415936*a^24*b^12*c^8*e^12*f^24*(a^2*c*f^2 - b^2*c*e^2)^5 +
269338092544*a^26*b^10*c^8*e^10*f^26*(a^2*c*f^2 - b^2*c*e^2)^5 + 5378321203
2*a^28*b^8*c^8*e^8*f^28*(a^2*c*f^2 - b^2*c*e^2)^5 - 60985360384*a^30*b^6*c^
8*e^6*f^30*(a^2*c*f^2 - b^2*c*e^2)^5 + 17917083648*a^32*b^4*c^8*e^4*f^32*(a
^2*c*f^2 - b^2*c*e^2)^5 - 1558708224*a^34*b^2*c^8*e^2*f^34*(a^2*c*f^2 - b^2
*c*e^2)^5 - 11917692*a^2*b^36*c^9*e^36*f^2*(a^2*c*f^2 - b^2*c*e^2)^4 - 2249
```

07516*a^4*b^34*c^9*e^34*f^4*(a^2*c*f^2 - b^2*c*e^2)^4 + 5303932560*a^6*b^32
*c^9*e^32*f^6*(a^2*c*f^2 - b^2*c*e^2)^4 - 48206418480*a^8*b^30*c^9*e^30*f^8
*(a^2*c*f^2 - b^2*c*e^2)^4 + 261450609120*a^10*b^28*c^9*e^28*f^10*(a^2*c*f^
2 - b^2*c*e^2)^4 - 962361040256*a^12*b^26*c^9*e^26*f^12*(a^2*c*f^2 - b^2*c*
e^2)^4 + 2558559358080*a^14*b^24*c^9*e^24*f^14*(a^2*c*f^2 - b^2*c*e^2)^4 -
5091804150656*a^16*b^22*c^9*e^22*f^16*(a^2*c*f^2 - b^2*c*e^2)^4 + 775080651
4944*a^18*b^20*c^9*e^20*f^18*(a^2*c*f^2 - b^2*c*e^2)^4 - 9137207485952*a^20
*b^18*c^9*e^18*f^20*(a^2*c*f^2 - b^2*c*e^2)^4 + 8384563280128*a^22*b^16*c^9
*e^16*f^22*(a^2*c*f^2 - b^2*c*e^2)^4 - 5975281259520*a^24*b^14*c^9*e^14*f^2
4*(a^2*c*f^2 - b^2*c*e^2)^4 + 3269297268736*a^26*b^12*c^9*e^12*f^26*(a^2*c*
f^2 - b^2*c*e^2)^4 - 1339171540992*a^28*b^10*c^9*e^10*f^28*(a^2*c*f^2 - b^2
*c*e^2)^4 + 391250194432*a^30*b^8*c^9*e^8*f^30*(a^2*c*f^2 - b^2*c*e^2)^4 -
74114154496*a^32*b^6*c^9*e^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^4 + 7299203072*a^
34*b^4*c^9*e^4*f^34*(a^2*c*f^2 - b^2*c*e^2)^4 - 148635648*a^36*b^2*c^9*e^2*
f^36*(a^2*c*f^2 - b^2*c*e^2)^4 - 38704068*a^2*b^38*c^10*e^38*f^2*(a^2*c*f^2
 - b^2*c*e^2)^3 + 188845992*a^4*b^36*c^10*e^36*f^4*(a^2*c*f^2 - b^2*c*e^2)^
3 + 1157124204*a^6*b^34*c^10*e^34*f^6*(a^2*c*f^2 - b^2*c*e^2)^3 - 205863614
24*a^8*b^32*c^10*e^32*f^8*(a^2*c*f^2 - b^2*c*e^2)^3 + 135395499200*a^10*b^3
0*c^10*e^30*f^10*(a^2*c*f^2 - b^2*c*e^2)^3 - 555513858464*a^12*b^28*c^10*e^
28*f^12*(a^2*c*f^2 - b^2*c*e^2)^3 + 1608776388864*a^14*b^26*c^10*e^26*f^14*
(a^2*c*f^2 - b^2*c*e^2)^3 - 3473989271488*a^16*b^24*c^10*e^24*f^16*(a^2*c*f
^2 - b^2*c*e^2)^3 + 5766181411456*a^18*b^22*c^10*e^22*f^18*(a^2*c*f^2 - b^2
*c*e^2)^3 - 7493983209472*a^20*b^20*c^10*e^20*f^20*(a^2*c*f^2 - b^2*c*e^2)^
3 + 7713917084672*a^22*b^18*c^10*e^18*f^22*(a^2*c*f^2 - b^2*c*e^2)^3 - 6328
467293184*a^24*b^16*c^10*e^16*f^24*(a^2*c*f^2 - b^2*c*e^2)^3 + 414295003443
2*a^26*b^14*c^10*e^14*f^26*(a^2*c*f^2 - b^2*c*e^2)^3 - 2152681536512*a^28*b
^12*c^10*e^12*f^28*(a^2*c*f^2 - b^2*c*e^2)^3 + 874199511040*a^30*b^10*c^10*
e^10*f^30*(a^2*c*f^2 - b^2*c*e^2)^3 - 268759150592*a^32*b^8*c^10*e^8*f^32*(
a^2*c*f^2 - b^2*c*e^2)^3 + 58872545280*a^34*b^6*c^10*e^6*f^34*(a^2*c*f^2 -
b^2*c*e^2)^3 - 8151957504*a^36*b^4*c^10*e^4*f^36*(a^2*c*f^2 - b^2*c*e^2)^3
+ 530841600*a^38*b^2*c^10*e^2*f^38*(a^2*c*f^2 - b^2*c*e^2)^3 - 42743457*a^2
*b^40*c^11*e^40*f^2*(a^2*c*f^2 - b^2*c*e^2)^2 + 411055884*a^4*b^38*c^11*e^3
8*f^4*(a^2*c*f^2 - b^2*c*e^2)^2 - 2180887236*a^6*b^36*c^11*e^36*f^6*(a^2*c*
f^2 - b^2*c*e^2)^2 + 6404946508*a^8*b^34*c^11*e^34*f^8*(a^2*c*f^2 - b^2*c*e
^2)^2 - 5434005264*a^10*b^32*c^11*e^32*f^10*(a^2*c*f^2 - b^2*c*e^2)^2 - 388
68373520*a^12*b^30*c^11*e^30*f^12*(a^2*c*f^2 - b^2*c*e^2)^2 + 208447613600*
a^14*b^28*c^11*e^28*f^14*(a^2*c*f^2 - b^2*c*e^2)^2 - 579674999104*a^16*b^26
*c^11*e^26*f^16*(a^2*c*f^2 - b^2*c*e^2)^2 + 1104967566592*a^18*b^24*c^11*e^
24*f^18*(a^2*c*f^2 - b^2*c*e^2)^2 - 1554566531328*a^20*b^22*c^11*e^22*f^20*
(a^2*c*f^2 - b^2*c*e^2)^2 + 1659734381312*a^22*b^20*c^11*e^20*f^22*(a^2*c*f
^2 - b^2*c*e^2)^2 - 1356361512192*a^24*b^18*c^11*e^18*f^24*(a^2*c*f^2 - b^2
*c*e^2)^2 + 845331359744*a^26*b^16*c^11*e^16*f^26*(a^2*c*f^2 - b^2*c*e^2)^2
 - 395676895232*a^28*b^14*c^11*e^14*f^28*(a^2*c*f^2 - b^2*c*e^2)^2 + 134902
689792*a^30*b^12*c^11*e^12*f^30*(a^2*c*f^2 - b^2*c*e^2)^2 - 31670587392*a^3
2*b^10*c^11*e^10*f^32*(a^2*c*f^2 - b^2*c*e^2)^2 + 4584669184*a^34*b^8*c^11*
e^8*f^34*(a^2*c*f^2 - b^2*c*e^2)^2 - 309657600*a^36*b^6*c^11*e^6*f^36*(a^2*
c*f^2 - b^2*c*e^2)^2)))*(b^16*e^12*f^6*(a^2*c*f^2 - b^2*c*e^2)^2 - 4*a^2*b^
14*e^10*f^8*(a^2*c*f^2 - b^2*c*e^2)^2 + 6*a^4*b^12*e^8*f^10*(a^2*c*f^2 - b^
2*c*e^2)^2 - 4*a^6*b^10*e^6*f^12*(a^2*c*f^2 - b^2*c*e^2)^2 + a^8*b^8*e^4*f^
14*(a^2*c*f^2 - b^2*c*e^2)^2))/(((a + b*x)^(1/2) - a^(1/2))*(16384*C^4*a^6*
c^3*f^4 + 4096*C^4*a^2*b^4*c^3*e^4 - 16384*C^4*a^4*b^2*c^3*e^2*f^2)) + (8*a
^4*b^6*c^4*e^6*f^4*((4096*C^3*e^3*(2*a^2*f^2 - b^2*e^2)^3*(24*C*a^(21/2)*b^
2*c^4*e*f^15*(a*c)^(5/2) - 30*C*a^(3/2)*b^12*c^5*e^11*f^5*(a*c)^(3/2) + 24*
C*a^(5/2)*b^10*c^4*e^9*f^7*(a*c)^(5/2) + 126*C*a^(7/2)*b^10*c^5*e^9*f^7*(a*
c)^(3/2) - 96*C*a^(9/2)*b^8*c^4*e^7*f^9*(a*c)^(5/2) - 198*C*a^(11/2)*b^8*c^
5*e^7*f^9*(a*c)^(3/2) + 144*C*a^(13/2)*b^6*c^4*e^5*f^11*(a*c)^(5/2) + 138*C
*a^(15/2)*b^6*c^5*e^5*f^11*(a*c)^(3/2) - 96*C*a^(17/2)*b^4*c^4*e^3*f^13*(a*
c)^(5/2) - 36*C*a^(19/2)*b^4*c^5*e^3*f^13*(a*c)^(3/2)))/(f^6*(a*f + b*e)^3*
(a*f - b*e)^3*(b^2*c*e^2 - a^2*c*f^2)^(3/2)*(b^16*e^14*f^4 - 4*a^2*b^14*e^1

```
2*f^6 + 6*a^4*b^12*e^10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12)) + (4
096*C*e*(2*a^2*f^2 - b^2*e^2)*(64*C^3*a^(21/2)*c^3*e*f^11*(a*c)^(5/2) + 32*
C^3*a^(5/2)*b^8*c^3*e^9*f^3*(a*c)^(5/2) - 160*C^3*a^(7/2)*b^8*c^4*e^9*f^3*(
a*c)^(3/2) - 160*C^3*a^(9/2)*b^6*c^3*e^7*f^5*(a*c)^(5/2) + 384*C^3*a^(11/2)
*b^6*c^4*e^7*f^5*(a*c)^(3/2) + 288*C^3*a^(13/2)*b^4*c^3*e^5*f^7*(a*c)^(5/2)
 - 392*C^3*a^(15/2)*b^4*c^4*e^5*f^7*(a*c)^(3/2) - 224*C^3*a^(17/2)*b^2*c^3*
e^3*f^9*(a*c)^(5/2) + 144*C^3*a^(19/2)*b^2*c^4*e^3*f^9*(a*c)^(3/2) + 24*C^3
*a^(3/2)*b^10*c^4*e^11*f*(a*c)^(3/2)))/(f^2*(a*f + b*e)*(a*f - b*e)*(b^2*c*
e^2 - a^2*c*f^2)^(1/2)*(b^16*e^14*f^4 - 4*a^2*b^14*e^12*f^6 + 6*a^4*b^12*e^
10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12)))*(4*a^2*c*f^2 - 3*b^2*c*e
^2)*(4*a^6*c*f^6 - 3*b^6*c*e^6 + 8*a^2*b^4*c*e^4*f^2 - 8*a^4*b^2*c*e^2*f^4)
^4*(b^16*e^12*f^6*(a^2*c*f^2 - b^2*c*e^2)^2 - 4*a^2*b^14*e^10*f^8*(a^2*c*f^
2 - b^2*c*e^2)^2 + 6*a^4*b^12*e^8*f^10*(a^2*c*f^2 - b^2*c*e^2)^2 - 4*a^6*b^
10*e^6*f^12*(a^2*c*f^2 - b^2*c*e^2)^2 + a^8*b^8*e^4*f^14*(a^2*c*f^2 - b^2*c
*e^2)^2))/((16384*C^4*a^6*c^3*f^4 + 4096*C^4*a^2*b^4*c^3*e^4 - 16384*C^4*a^
4*b^2*c^3*e^2*f^2)*(164025*b^46*c^13*e^46 + 885735*b^44*c^12*e^44*(a^2*c*f^
2 - b^2*c*e^2) + 117440512*a^30*c^5*f^30*(a^2*c*f^2 - b^2*c*e^2)^8 - 385875
968*a^32*c^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^7 + 419430400*a^34*c^7*f^34*(a^2*
c*f^2 - b^2*c*e^2)^6 - 150994944*a^36*c^8*f^36*(a^2*c*f^2 - b^2*c*e^2)^5 +
236196*b^36*c^8*e^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 1102248*b^38*c^9*e^38*(a^2
*c*f^2 - b^2*c*e^2)^4 + 2053593*b^40*c^10*e^40*(a^2*c*f^2 - b^2*c*e^2)^3 +
1909251*b^42*c^11*e^42*(a^2*c*f^2 - b^2*c*e^2)^2 - 3937329*a^2*b^44*c^13*e^
44*f^2 + 43893819*a^4*b^42*c^13*e^42*f^4 - 301507155*a^6*b^40*c^13*e^40*f^6
 + 1427514656*a^8*b^38*c^13*e^38*f^8 - 4936911112*a^10*b^36*c^13*e^36*f^10
+ 12893273616*a^12*b^34*c^13*e^34*f^12 - 25921630432*a^14*b^32*c^13*e^32*f^
14 + 40519286096*a^16*b^30*c^13*e^30*f^16 - 49376608256*a^18*b^28*c^13*e^28
*f^18 + 46721401856*a^20*b^26*c^13*e^26*f^20 - 33946324736*a^22*b^24*c^13*e
^24*f^22 + 18556579328*a^24*b^22*c^13*e^22*f^24 - 7375276032*a^26*b^20*c^13
*e^20*f^26 + 2009817088*a^28*b^18*c^13*e^18*f^28 - 335642624*a^30*b^16*c^13
*e^16*f^30 + 25907200*a^32*b^14*c^13*e^14*f^32 - 21130794*a^2*b^42*c^12*e^4
2*f^2*(a^2*c*f^2 - b^2*c*e^2) + 234399015*a^4*b^40*c^12*e^40*f^4*(a^2*c*f^2
 - b^2*c*e^2) - 1604168280*a^6*b^38*c^12*e^38*f^6*(a^2*c*f^2 - b^2*c*e^2) +
 7579098492*a^8*b^36*c^12*e^36*f^8*(a^2*c*f^2 - b^2*c*e^2) - 26212380172*a^
10*b^34*c^12*e^34*f^10*(a^2*c*f^2 - b^2*c*e^2) + 68672994096*a^12*b^32*c^12
*e^32*f^12*(a^2*c*f^2 - b^2*c*e^2) - 139160589504*a^14*b^30*c^12*e^30*f^14*
(a^2*c*f^2 - b^2*c*e^2) + 220859191808*a^16*b^28*c^12*e^28*f^16*(a^2*c*f^2
- b^2*c*e^2) - 276344315328*a^18*b^26*c^12*e^26*f^18*(a^2*c*f^2 - b^2*c*e^2
) + 273130561984*a^20*b^24*c^12*e^24*f^20*(a^2*c*f^2 - b^2*c*e^2) - 2127300
02688*a^22*b^22*c^12*e^22*f^22*(a^2*c*f^2 - b^2*c*e^2) + 129574234368*a^24*
b^20*c^12*e^20*f^24*(a^2*c*f^2 - b^2*c*e^2) - 60770569216*a^26*b^18*c^12*e^
18*f^26*(a^2*c*f^2 - b^2*c*e^2) + 21304706048*a^28*b^16*c^12*e^16*f^28*(a^2
*c*f^2 - b^2*c*e^2) - 5272965120*a^30*b^14*c^12*e^14*f^30*(a^2*c*f^2 - b^2*
c*e^2) + 819441664*a^32*b^12*c^12*e^12*f^32*(a^2*c*f^2 - b^2*c*e^2) - 59392
000*a^34*b^10*c^12*e^10*f^34*(a^2*c*f^2 - b^2*c*e^2) + 9289728*a^6*b^24*c^5
*e^24*f^6*(a^2*c*f^2 - b^2*c*e^2)^8 - 36884480*a^8*b^22*c^5*e^22*f^8*(a^2*c
*f^2 - b^2*c*e^2)^8 - 278604800*a^10*b^20*c^5*e^20*f^10*(a^2*c*f^2 - b^2*c*
e^2)^8 + 2774483200*a^12*b^18*c^5*e^18*f^12*(a^2*c*f^2 - b^2*c*e^2)^8 - 108
69657600*a^14*b^16*c^5*e^16*f^14*(a^2*c*f^2 - b^2*c*e^2)^8 + 25237416960*a^
16*b^14*c^5*e^14*f^16*(a^2*c*f^2 - b^2*c*e^2)^8 - 38348909568*a^18*b^12*c^5
*e^12*f^18*(a^2*c*f^2 - b^2*c*e^2)^8 + 39084659712*a^20*b^10*c^5*e^10*f^20*
(a^2*c*f^2 - b^2*c*e^2)^8 - 26118635520*a^22*b^8*c^5*e^8*f^22*(a^2*c*f^2 -
b^2*c*e^2)^8 + 10414620672*a^24*b^6*c^5*e^6*f^24*(a^2*c*f^2 - b^2*c*e^2)^8
- 1708654592*a^26*b^4*c^5*e^4*f^26*(a^2*c*f^2 - b^2*c*e^2)^8 - 276561920*a^
28*b^2*c^5*e^2*f^28*(a^2*c*f^2 - b^2*c*e^2)^8 - 9704448*a^4*b^28*c^6*e^28*f
^4*(a^2*c*f^2 - b^2*c*e^2)^7 + 260614656*a^6*b^26*c^6*e^26*f^6*(a^2*c*f^2 -
 b^2*c*e^2)^7 - 2166022464*a^8*b^24*c^6*e^24*f^8*(a^2*c*f^2 - b^2*c*e^2)^7
+ 8626147840*a^10*b^22*c^6*e^22*f^10*(a^2*c*f^2 - b^2*c*e^2)^7 - 1677150361
6*a^12*b^20*c^6*e^20*f^12*(a^2*c*f^2 - b^2*c*e^2)^7 + 3301800960*a^14*b^18*
c^6*e^18*f^14*(a^2*c*f^2 - b^2*c*e^2)^7 + 67337715968*a^16*b^16*c^6*e^16*f^
```

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16*(a^2*c*f^2 - b^2*c*e^2)^7 - 189857873920*a^18*b^14*c^6*e^14*f^18*(a^2*c*
f^2 - b^2*c*e^2)^7 + 286100259840*a^20*b^12*c^6*e^12*f^20*(a^2*c*f^2 - b^2*
c*e^2)^7 - 275789894656*a^22*b^10*c^6*e^10*f^22*(a^2*c*f^2 - b^2*c*e^2)^7 +
 173716537344*a^24*b^8*c^6*e^8*f^24*(a^2*c*f^2 - b^2*c*e^2)^7 - 67416424448
*a^26*b^6*c^6*e^6*f^26*(a^2*c*f^2 - b^2*c*e^2)^7 + 12831686656*a^28*b^4*c^6
*e^4*f^28*(a^2*c*f^2 - b^2*c*e^2)^7 + 222560256*a^30*b^2*c^6*e^2*f^30*(a^2*
c*f^2 - b^2*c*e^2)^7 + 2099520*a^2*b^32*c^7*e^32*f^2*(a^2*c*f^2 - b^2*c*e^2
)^6 - 107014608*a^4*b^30*c^7*e^30*f^4*(a^2*c*f^2 - b^2*c*e^2)^6 + 184833561
6*a^6*b^28*c^7*e^28*f^6*(a^2*c*f^2 - b^2*c*e^2)^6 - 15200005312*a^8*b^26*c^
7*e^26*f^8*(a^2*c*f^2 - b^2*c*e^2)^6 + 72612273792*a^10*b^24*c^7*e^24*f^10*
(a^2*c*f^2 - b^2*c*e^2)^6 - 221855779968*a^12*b^22*c^7*e^22*f^12*(a^2*c*f^2
 - b^2*c*e^2)^6 + 450717857536*a^14*b^20*c^7*e^20*f^14*(a^2*c*f^2 - b^2*c*e
^2)^6 - 600578910208*a^16*b^18*c^7*e^18*f^16*(a^2*c*f^2 - b^2*c*e^2)^6 + 45
9464530688*a^18*b^16*c^7*e^16*f^18*(a^2*c*f^2 - b^2*c*e^2)^6 - 33638947840*
a^20*b^14*c^7*e^14*f^20*(a^2*c*f^2 - b^2*c*e^2)^6 - 376299926528*a^22*b^12*
c^7*e^12*f^22*(a^2*c*f^2 - b^2*c*e^2)^6 + 488874068992*a^24*b^10*c^7*e^10*f
^24*(a^2*c*f^2 - b^2*c*e^2)^6 - 333407809536*a^26*b^8*c^7*e^8*f^26*(a^2*c*f
^2 - b^2*c*e^2)^6 + 134140313600*a^28*b^6*c^7*e^6*f^28*(a^2*c*f^2 - b^2*c*e
^2)^6 - 28220915712*a^30*b^4*c^7*e^4*f^30*(a^2*c*f^2 - b^2*c*e^2)^6 + 12305
03936*a^32*b^2*c^7*e^2*f^32*(a^2*c*f^2 - b^2*c*e^2)^6 + 3335904*a^2*b^34*c^
8*e^34*f^2*(a^2*c*f^2 - b^2*c*e^2)^5 - 290521728*a^4*b^32*c^8*e^32*f^4*(a^2
*c*f^2 - b^2*c*e^2)^5 + 4865684544*a^6*b^30*c^8*e^30*f^6*(a^2*c*f^2 - b^2*c
*e^2)^5 - 40437394528*a^8*b^28*c^8*e^28*f^8*(a^2*c*f^2 - b^2*c*e^2)^5 + 205
602254656*a^10*b^26*c^8*e^26*f^10*(a^2*c*f^2 - b^2*c*e^2)^5 - 703885344192*
a^12*b^24*c^8*e^24*f^12*(a^2*c*f^2 - b^2*c*e^2)^5 + 1709253482624*a^14*b^22
*c^8*e^22*f^14*(a^2*c*f^2 - b^2*c*e^2)^5 - 3029282695168*a^16*b^20*c^8*e^20
*f^16*(a^2*c*f^2 - b^2*c*e^2)^5 + 3966230827520*a^18*b^18*c^8*e^18*f^18*(a^
2*c*f^2 - b^2*c*e^2)^5 - 3822339813632*a^20*b^16*c^8*e^16*f^20*(a^2*c*f^2 -
 b^2*c*e^2)^5 + 2640438056960*a^22*b^14*c^8*e^14*f^22*(a^2*c*f^2 - b^2*c*e^
2)^5 - 1208501415936*a^24*b^12*c^8*e^12*f^24*(a^2*c*f^2 - b^2*c*e^2)^5 + 26
9338092544*a^26*b^10*c^8*e^10*f^26*(a^2*c*f^2 - b^2*c*e^2)^5 + 53783212032*
a^28*b^8*c^8*e^8*f^28*(a^2*c*f^2 - b^2*c*e^2)^5 - 60985360384*a^30*b^6*c^8*
e^6*f^30*(a^2*c*f^2 - b^2*c*e^2)^5 + 17917083648*a^32*b^4*c^8*e^4*f^32*(a^2
*c*f^2 - b^2*c*e^2)^5 - 1558708224*a^34*b^2*c^8*e^2*f^34*(a^2*c*f^2 - b^2*c
*e^2)^5 - 11917692*a^2*b^36*c^9*e^36*f^2*(a^2*c*f^2 - b^2*c*e^2)^4 - 224907
516*a^4*b^34*c^9*e^34*f^4*(a^2*c*f^2 - b^2*c*e^2)^4 + 5303932560*a^6*b^32*c
^9*e^32*f^6*(a^2*c*f^2 - b^2*c*e^2)^4 - 48206418480*a^8*b^30*c^9*e^30*f^8*(
a^2*c*f^2 - b^2*c*e^2)^4 + 261450609120*a^10*b^28*c^9*e^28*f^10*(a^2*c*f^2
 - b^2*c*e^2)^4 - 962361040256*a^12*b^26*c^9*e^26*f^12*(a^2*c*f^2 - b^2*c*e^
2)^4 + 2558559358080*a^14*b^24*c^9*e^24*f^14*(a^2*c*f^2 - b^2*c*e^2)^4 - 50
91804150656*a^16*b^22*c^9*e^22*f^16*(a^2*c*f^2 - b^2*c*e^2)^4 + 77508065149
44*a^18*b^20*c^9*e^20*f^18*(a^2*c*f^2 - b^2*c*e^2)^4 - 9137207485952*a^20*b
^18*c^9*e^18*f^20*(a^2*c*f^2 - b^2*c*e^2)^4 + 8384563280128*a^22*b^16*c^9*e
^16*f^22*(a^2*c*f^2 - b^2*c*e^2)^4 - 5975281259520*a^24*b^14*c^9*e^14*f^24*
(a^2*c*f^2 - b^2*c*e^2)^4 + 3269297268736*a^26*b^12*c^9*e^12*f^26*(a^2*c*f^
2 - b^2*c*e^2)^4 - 1339171540992*a^28*b^10*c^9*e^10*f^28*(a^2*c*f^2 - b^2*c
*e^2)^4 + 391250194432*a^30*b^8*c^9*e^8*f^30*(a^2*c*f^2 - b^2*c*e^2)^4 - 74
114154496*a^32*b^6*c^9*e^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^4 + 7299203072*a^34
*b^4*c^9*e^4*f^34*(a^2*c*f^2 - b^2*c*e^2)^4 - 148635648*a^36*b^2*c^9*e^2*f^
36*(a^2*c*f^2 - b^2*c*e^2)^4 - 38704068*a^2*b^38*c^10*e^38*f^2*(a^2*c*f^2 -
 b^2*c*e^2)^3 + 188845992*a^4*b^36*c^10*e^36*f^4*(a^2*c*f^2 - b^2*c*e^2)^3
+ 1157124204*a^6*b^34*c^10*e^34*f^6*(a^2*c*f^2 - b^2*c*e^2)^3 - 20586361424
*a^8*b^32*c^10*e^32*f^8*(a^2*c*f^2 - b^2*c*e^2)^3 + 135395499200*a^10*b^30*
c^10*e^30*f^10*(a^2*c*f^2 - b^2*c*e^2)^3 - 555513858464*a^12*b^28*c^10*e^28
*f^12*(a^2*c*f^2 - b^2*c*e^2)^3 + 1608776388864*a^14*b^26*c^10*e^26*f^14*(a
^2*c*f^2 - b^2*c*e^2)^3 - 3473989271488*a^16*b^24*c^10*e^24*f^16*(a^2*c*f^2
 - b^2*c*e^2)^3 + 5766181411456*a^18*b^22*c^10*e^22*f^18*(a^2*c*f^2 - b^2*c
*e^2)^3 - 7493983209472*a^20*b^20*c^10*e^20*f^20*(a^2*c*f^2 - b^2*c*e^2)^3
+ 7713917084672*a^22*b^18*c^10*e^18*f^22*(a^2*c*f^2 - b^2*c*e^2)^3 - 632846
```

7293184*a^24*b^16*c^10*e^16*f^24*(a^2*c*f^2 - b^2*c*e^2)^3 + 4142950034432*
a^26*b^14*c^10*e^14*f^26*(a^2*c*f^2 - b^2*c*e^2)^3 - 2152681536512*a^28*b^1
2*c^10*e^12*f^28*(a^2*c*f^2 - b^2*c*e^2)^3 + 874199511040*a^30*b^10*c^10*e^
10*f^30*(a^2*c*f^2 - b^2*c*e^2)^3 - 268759150592*a^32*b^8*c^10*e^8*f^32*(a^
2*c*f^2 - b^2*c*e^2)^3 + 58872545280*a^34*b^6*c^10*e^6*f^34*(a^2*c*f^2 - b^
2*c*e^2)^3 - 8151957504*a^36*b^4*c^10*e^4*f^36*(a^2*c*f^2 - b^2*c*e^2)^3 +
530841600*a^38*b^2*c^10*e^2*f^38*(a^2*c*f^2 - b^2*c*e^2)^3 - 42743457*a^2*b
^40*c^11*e^40*f^2*(a^2*c*f^2 - b^2*c*e^2)^2 + 411055884*a^4*b^38*c^11*e^38*
f^4*(a^2*c*f^2 - b^2*c*e^2)^2 - 2180887236*a^6*b^36*c^11*e^36*f^6*(a^2*c*f^
2 - b^2*c*e^2)^2 + 6404946508*a^8*b^34*c^11*e^34*f^8*(a^2*c*f^2 - b^2*c*e^2
)^2 - 5434005264*a^10*b^32*c^11*e^32*f^10*(a^2*c*f^2 - b^2*c*e^2)^2 - 38868
373520*a^12*b^30*c^11*e^30*f^12*(a^2*c*f^2 - b^2*c*e^2)^2 + 208447613600*a^
14*b^28*c^11*e^28*f^14*(a^2*c*f^2 - b^2*c*e^2)^2 - 579674999104*a^16*b^26*c
^11*e^26*f^16*(a^2*c*f^2 - b^2*c*e^2)^2 + 1104967566592*a^18*b^24*c^11*e^24
f^18(a^2*c*f^2 - b^2*c*e^2)^2 - 1554566531328*a^20*b^22*c^11*e^22*f^20*(a
^2*c*f^2 - b^2*c*e^2)^2 + 1659734381312*a^22*b^20*c^11*e^20*f^22*(a^2*c*f^2
 - b^2*c*e^2)^2 - 1356361512192*a^24*b^18*c^11*e^18*f^24*(a^2*c*f^2 - b^2*c
*e^2)^2 + 845331359744*a^26*b^16*c^11*e^16*f^26*(a^2*c*f^2 - b^2*c*e^2)^2 -
 395676895232*a^28*b^14*c^11*e^14*f^28*(a^2*c*f^2 - b^2*c*e^2)^2 + 13490268
9792*a^30*b^12*c^11*e^12*f^30*(a^2*c*f^2 - b^2*c*e^2)^2 - 31670587392*a^32*
b^10*c^11*e^10*f^32*(a^2*c*f^2 - b^2*c*e^2)^2 + 4584669184*a^34*b^8*c^11*e^
8*f^34*(a^2*c*f^2 - b^2*c*e^2)^2 - 309657600*a^36*b^6*c^11*e^6*f^36*(a^2*c*
f^2 - b^2*c*e^2)^2)) - (4*a^(3/2)*b^6*c^2*e^6*f^3*(a*c)^(3/2)*(2*a^2*c*f^2
- b^2*c*e^2)*(4*a^2*c*f^2 - 3*b^2*c*e^2)*((4096*(16*C^4*a^4*b^8*c^5*e^10 +
64*C^4*a^12*c^5*e^2*f^8 - 92*C^4*a^6*b^6*c^5*e^8*f^2 + 192*C^4*a^8*b^4*c^5*
e^6*f^4 - 176*C^4*a^10*b^2*c^5*e^4*f^6))/(b^16*e^14*f^4 - 4*a^2*b^14*e^12*f
^6 + 6*a^4*b^12*e^10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12) + (4096*
C^4*e^4*(2*a^2*f^2 - b^2*e^2)^4*(9*a^2*b^14*c^7*e^12*f^6 - 43*a^4*b^12*c^7*
e^10*f^8 + 82*a^6*b^10*c^7*e^8*f^10 - 78*a^8*b^8*c^7*e^6*f^12 + 37*a^10*b^6
*c^7*e^4*f^14 - 7*a^12*b^4*c^7*e^2*f^16))/(f^8*(a*f + b*e)^4*(a*f - b*e)^4*
(a^2*c*f^2 - b^2*c*e^2)^2*(b^16*e^14*f^4 - 4*a^2*b^14*e^12*f^6 + 6*a^4*b^12
*e^10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^6*f^12)) + (4096*C^2*e^2*(2*a^2
*f^2 - b^2*e^2)^2*(16*C^2*a^14*c^6*f^14 + 9*C^2*a^2*b^12*c^6*e^12*f^2 - 54*
C^2*a^4*b^10*c^6*e^10*f^4 + 121*C^2*a^6*b^8*c^6*e^8*f^6 - 128*C^2*a^8*b^6*c
^6*e^6*f^8 + 80*C^2*a^10*b^4*c^6*e^4*f^10 - 44*C^2*a^12*b^2*c^6*e^2*f^12))/
(f^4*(a*f + b*e)^2*(a*f - b*e)^2*(a^2*c*f^2 - b^2*c*e^2)*(b^16*e^14*f^4 - 4
*a^2*b^14*e^12*f^6 + 6*a^4*b^12*e^10*f^8 - 4*a^6*b^10*e^8*f^10 + a^8*b^8*e^
6*f^12)))*(4*a^6*c*f^6 - 3*b^6*c*e^6 + 8*a^2*b^4*c*e^4*f^2 - 8*a^4*b^2*c*e^
2*f^4)^4*(b^16*e^12*f^6*(a^2*c*f^2 - b^2*c*e^2)^2 - 4*a^2*b^14*e^10*f^8*(a^
2*c*f^2 - b^2*c*e^2)^2 + 6*a^4*b^12*e^8*f^10*(a^2*c*f^2 - b^2*c*e^2)^2 - 4*
a^6*b^10*e^6*f^12*(a^2*c*f^2 - b^2*c*e^2)^2 + a^8*b^8*e^4*f^14*(a^2*c*f^2 -
 b^2*c*e^2)^2))/((b^2*c*e^2 - a^2*c*f^2)^(1/2)*(16384*C^4*a^6*c^3*f^4 + 409
6*C^4*a^2*b^4*c^3*e^4 - 16384*C^4*a^4*b^2*c^3*e^2*f^2)*(164025*b^46*c^13*e^
46 + 885735*b^44*c^12*e^44*(a^2*c*f^2 - b^2*c*e^2) + 117440512*a^30*c^5*f^3
0*(a^2*c*f^2 - b^2*c*e^2)^8 - 385875968*a^32*c^6*f^32*(a^2*c*f^2 - b^2*c*e^
2)^7 + 419430400*a^34*c^7*f^34*(a^2*c*f^2 - b^2*c*e^2)^6 - 150994944*a^36*c
^8*f^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 236196*b^36*c^8*e^36*(a^2*c*f^2 - b^2*c
*e^2)^5 + 1102248*b^38*c^9*e^38*(a^2*c*f^2 - b^2*c*e^2)^4 + 2053593*b^40*c^
10*e^40*(a^2*c*f^2 - b^2*c*e^2)^3 + 1909251*b^42*c^11*e^42*(a^2*c*f^2 - b^2
*c*e^2)^2 - 3937329*a^2*b^44*c^13*e^44*f^2 + 43893819*a^4*b^42*c^13*e^42*f^
4 - 301507155*a^6*b^40*c^13*e^40*f^6 + 1427514656*a^8*b^38*c^13*e^38*f^8 -
4936911112*a^10*b^36*c^13*e^36*f^10 + 12893273616*a^12*b^34*c^13*e^34*f^12
- 25921630432*a^14*b^32*c^13*e^32*f^14 + 40519286096*a^16*b^30*c^13*e^30*f^
16 - 49376608256*a^18*b^28*c^13*e^28*f^18 + 46721401856*a^20*b^26*c^13*e^26
*f^20 - 33946324736*a^22*b^24*c^13*e^24*f^22 + 18556579328*a^24*b^22*c^13*e
^22*f^24 - 7375276032*a^26*b^20*c^13*e^20*f^26 + 2009817088*a^28*b^18*c^13*
e^18*f^28 - 335642624*a^30*b^16*c^13*e^16*f^30 + 25907200*a^32*b^14*c^13*e^
14*f^32 - 21130794*a^2*b^42*c^12*e^42*f^2*(a^2*c*f^2 - b^2*c*e^2) + 2343990
15*a^4*b^40*c^12*e^40*f^4*(a^2*c*f^2 - b^2*c*e^2) - 1604168280*a^6*b^38*c^1

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2*e^38*f^6*(a^2*c*f^2 - b^2*c*e^2) + 7579098492*a^8*b^36*c^12*e^36*f^8*(a^2
*c*f^2 - b^2*c*e^2) - 26212380172*a^10*b^34*c^12*e^34*f^10*(a^2*c*f^2 - b^2
*c*e^2) + 68672994096*a^12*b^32*c^12*e^32*f^12*(a^2*c*f^2 - b^2*c*e^2) - 13
9160589504*a^14*b^30*c^12*e^30*f^14*(a^2*c*f^2 - b^2*c*e^2) + 220859191808*
a^16*b^28*c^12*e^28*f^16*(a^2*c*f^2 - b^2*c*e^2) - 276344315328*a^18*b^26*c
^12*e^26*f^18*(a^2*c*f^2 - b^2*c*e^2) + 273130561984*a^20*b^24*c^12*e^24*f^
20*(a^2*c*f^2 - b^2*c*e^2) - 212730002688*a^22*b^22*c^12*e^22*f^22*(a^2*c*f
^2 - b^2*c*e^2) + 129574234368*a^24*b^20*c^12*e^20*f^24*(a^2*c*f^2 - b^2*c*
e^2) - 60770569216*a^26*b^18*c^12*e^18*f^26*(a^2*c*f^2 - b^2*c*e^2) + 21304
706048*a^28*b^16*c^12*e^16*f^28*(a^2*c*f^2 - b^2*c*e^2) - 5272965120*a^30*b
^14*c^12*e^14*f^30*(a^2*c*f^2 - b^2*c*e^2) + 819441664*a^32*b^12*c^12*e^12*
f^32*(a^2*c*f^2 - b^2*c*e^2) - 59392000*a^34*b^10*c^12*e^10*f^34*(a^2*c*f^2
 - b^2*c*e^2) + 9289728*a^6*b^24*c^5*e^24*f^6*(a^2*c*f^2 - b^2*c*e^2)^8 - 3
6884480*a^8*b^22*c^5*e^22*f^8*(a^2*c*f^2 - b^2*c*e^2)^8 - 278604800*a^10*b^
20*c^5*e^20*f^10*(a^2*c*f^2 - b^2*c*e^2)^8 + 2774483200*a^12*b^18*c^5*e^18*
f^12*(a^2*c*f^2 - b^2*c*e^2)^8 - 10869657600*a^14*b^16*c^5*e^16*f^14*(a^2*c
*f^2 - b^2*c*e^2)^8 + 25237416960*a^16*b^14*c^5*e^14*f^16*(a^2*c*f^2 - b^2*
c*e^2)^8 - 38348909568*a^18*b^12*c^5*e^12*f^18*(a^2*c*f^2 - b^2*c*e^2)^8 +
39084659712*a^20*b^10*c^5*e^10*f^20*(a^2*c*f^2 - b^2*c*e^2)^8 - 26118635520
*a^22*b^8*c^5*e^8*f^22*(a^2*c*f^2 - b^2*c*e^2)^8 + 10414620672*a^24*b^6*c^5
*e^6*f^24*(a^2*c*f^2 - b^2*c*e^2)^8 - 1708654592*a^26*b^4*c^5*e^4*f^26*(a^2
*c*f^2 - b^2*c*e^2)^8 - 276561920*a^28*b^2*c^5*e^2*f^28*(a^2*c*f^2 - b^2*c*
e^2)^8 - 9704448*a^4*b^28*c^6*e^28*f^4*(a^2*c*f^2 - b^2*c*e^2)^7 + 26061465
6*a^6*b^26*c^6*e^26*f^6*(a^2*c*f^2 - b^2*c*e^2)^7 - 2166022464*a^8*b^24*c^6
*e^24*f^8*(a^2*c*f^2 - b^2*c*e^2)^7 + 8626147840*a^10*b^22*c^6*e^22*f^10*(a
^2*c*f^2 - b^2*c*e^2)^7 - 16771503616*a^12*b^20*c^6*e^20*f^12*(a^2*c*f^2 -
b^2*c*e^2)^7 + 3301800960*a^14*b^18*c^6*e^18*f^14*(a^2*c*f^2 - b^2*c*e^2)^7
 + 67337715968*a^16*b^16*c^6*e^16*f^16*(a^2*c*f^2 - b^2*c*e^2)^7 - 18985787
3920*a^18*b^14*c^6*e^14*f^18*(a^2*c*f^2 - b^2*c*e^2)^7 + 286100259840*a^20*
b^12*c^6*e^12*f^20*(a^2*c*f^2 - b^2*c*e^2)^7 - 275789894656*a^22*b^10*c^6*e
^10*f^22*(a^2*c*f^2 - b^2*c*e^2)^7 + 173716537344*a^24*b^8*c^6*e^8*f^24*(a^
2*c*f^2 - b^2*c*e^2)^7 - 67416424448*a^26*b^6*c^6*e^6*f^26*(a^2*c*f^2 - b^2
*c*e^2)^7 + 12831686656*a^28*b^4*c^6*e^4*f^28*(a^2*c*f^2 - b^2*c*e^2)^7 + 2
22560256*a^30*b^2*c^6*e^2*f^30*(a^2*c*f^2 - b^2*c*e^2)^7 + 2099520*a^2*b^32
*c^7*e^32*f^2*(a^2*c*f^2 - b^2*c*e^2)^6 - 107014608*a^4*b^30*c^7*e^30*f^4*(
a^2*c*f^2 - b^2*c*e^2)^6 + 1848335616*a^6*b^28*c^7*e^28*f^6*(a^2*c*f^2 - b^
2*c*e^2)^6 - 15200005312*a^8*b^26*c^7*e^26*f^8*(a^2*c*f^2 - b^2*c*e^2)^6 +
72612273792*a^10*b^24*c^7*e^24*f^10*(a^2*c*f^2 - b^2*c*e^2)^6 - 22185577996
8*a^12*b^22*c^7*e^22*f^12*(a^2*c*f^2 - b^2*c*e^2)^6 + 450717857536*a^14*b^2
0*c^7*e^20*f^14*(a^2*c*f^2 - b^2*c*e^2)^6 - 600578910208*a^16*b^18*c^7*e^18
*f^16*(a^2*c*f^2 - b^2*c*e^2)^6 + 459464530688*a^18*b^16*c^7*e^16*f^18*(a^2
*c*f^2 - b^2*c*e^2)^6 - 33638947840*a^20*b^14*c^7*e^14*f^20*(a^2*c*f^2 - b^
2*c*e^2)^6 - 376299926528*a^22*b^12*c^7*e^12*f^22*(a^2*c*f^2 - b^2*c*e^2)^6
 + 488874068992*a^24*b^10*c^7*e^10*f^24*(a^2*c*f^2 - b^2*c*e^2)^6 - 3334078
09536*a^26*b^8*c^7*e^8*f^26*(a^2*c*f^2 - b^2*c*e^2)^6 + 134140313600*a^28*b
^6*c^7*e^6*f^28*(a^2*c*f^2 - b^2*c*e^2)^6 - 28220915712*a^30*b^4*c^7*e^4*f^
30*(a^2*c*f^2 - b^2*c*e^2)^6 + 1230503936*a^32*b^2*c^7*e^2*f^32*(a^2*c*f^2
- b^2*c*e^2)^6 + 3335904*a^2*b^34*c^8*e^34*f^2*(a^2*c*f^2 - b^2*c*e^2)^5 -
290521728*a^4*b^32*c^8*e^32*f^4*(a^2*c*f^2 - b^2*c*e^2)^5 + 4865684544*a^6*
b^30*c^8*e^30*f^6*(a^2*c*f^2 - b^2*c*e^2)^5 - 40437394528*a^8*b^28*c^8*e^28
*f^8*(a^2*c*f^2 - b^2*c*e^2)^5 + 205602254656*a^10*b^26*c^8*e^26*f^10*(a^2*
c*f^2 - b^2*c*e^2)^5 - 703885344192*a^12*b^24*c^8*e^24*f^12*(a^2*c*f^2 - b^
2*c*e^2)^5 + 1709253482624*a^14*b^22*c^8*e^22*f^14*(a^2*c*f^2 - b^2*c*e^2)^
5 - 3029282695168*a^16*b^20*c^8*e^20*f^16*(a^2*c*f^2 - b^2*c*e^2)^5 + 39662
30827520*a^18*b^18*c^8*e^18*f^18*(a^2*c*f^2 - b^2*c*e^2)^5 - 3822339813632*
a^20*b^16*c^8*e^16*f^20*(a^2*c*f^2 - b^2*c*e^2)^5 + 2640438056960*a^22*b^14
*c^8*e^14*f^22*(a^2*c*f^2 - b^2*c*e^2)^5 - 1208501415936*a^24*b^12*c^8*e^12
*f^24*(a^2*c*f^2 - b^2*c*e^2)^5 + 269338092544*a^26*b^10*c^8*e^10*f^26*(a^2
*c*f^2 - b^2*c*e^2)^5 + 53783212032*a^28*b^8*c^8*e^8*f^28*(a^2*c*f^2 - b^2*
```

c*e^2)^5 - 60985360384*a^30*b^6*c^8*e^6*f^30*(a^2*c*f^2 - b^2*c*e^2)^5 + 17
917083648*a^32*b^4*c^8*e^4*f^32*(a^2*c*f^2 - b^2*c*e^2)^5 - 1558708224*a^34
*b^2*c^8*e^2*f^34*(a^2*c*f^2 - b^2*c*e^2)^5 - 11917692*a^2*b^36*c^9*e^36*f^
2*(a^2*c*f^2 - b^2*c*e^2)^4 - 224907516*a^4*b^34*c^9*e^34*f^4*(a^2*c*f^2 -
b^2*c*e^2)^4 + 5303932560*a^6*b^32*c^9*e^32*f^6*(a^2*c*f^2 - b^2*c*e^2)^4 -
48206418480*a^8*b^30*c^9*e^30*f^8*(a^2*c*f^2 - b^2*c*e^2)^4 + 261450609120
*a^10*b^28*c^9*e^28*f^10*(a^2*c*f^2 - b^2*c*e^2)^4 - 962361040256*a^12*b^26
*c^9*e^26*f^12*(a^2*c*f^2 - b^2*c*e^2)^4 + 2558559358080*a^14*b^24*c^9*e^24
f^14(a^2*c*f^2 - b^2*c*e^2)^4 - 5091804150656*a^16*b^22*c^9*e^22*f^16*(a^
2*c*f^2 - b^2*c*e^2)^4 + 7750806514944*a^18*b^20*c^9*e^20*f^18*(a^2*c*f^2 -
b^2*c*e^2)^4 - 9137207485952*a^20*b^18*c^9*e^18*f^20*(a^2*c*f^2 - b^2*c*e^
2)^4 + 8384563280128*a^22*b^16*c^9*e^16*f^22*(a^2*c*f^2 - b^2*c*e^2)^4 - 59
75281259520*a^24*b^14*c^9*e^14*f^24*(a^2*c*f^2 - b^2*c*e^2)^4 + 32692972687
36*a^26*b^12*c^9*e^12*f^26*(a^2*c*f^2 - b^2*c*e^2)^4 - 1339171540992*a^28*b
^10*c^9*e^10*f^28*(a^2*c*f^2 - b^2*c*e^2)^4 + 391250194432*a^30*b^8*c^9*e^8
f^30(a^2*c*f^2 - b^2*c*e^2)^4 - 74114154496*a^32*b^6*c^9*e^6*f^32*(a^2*c*
f^2 - b^2*c*e^2)^4 + 7299203072*a^34*b^4*c^9*e^4*f^34*(a^2*c*f^2 - b^2*c*e^
2)^4 - 148635648*a^36*b^2*c^9*e^2*f^36*(a^2*c*f^2 - b^2*c*e^2)^4 - 38704068
*a^2*b^38*c^10*e^38*f^2*(a^2*c*f^2 - b^2*c*e^2)^3 + 188845992*a^4*b^36*c^10
*e^36*f^4*(a^2*c*f^2 - b^2*c*e^2)^3 + 1157124204*a^6*b^34*c^10*e^34*f^6*(a^
2*c*f^2 - b^2*c*e^2)^3 - 20586361424*a^8*b^32*c^10*e^32*f^8*(a^2*c*f^2 - b^
2*c*e^2)^3 + 135395499200*a^10*b^30*c^10*e^30*f^10*(a^2*c*f^2 - b^2*c*e^2)^
3 - 555513858464*a^12*b^28*c^10*e^28*f^12*(a^2*c*f^2 - b^2*c*e^2)^3 + 16087
76388864*a^14*b^26*c^10*e^26*f^14*(a^2*c*f^2 - b^2*c*e^2)^3 - 3473989271488
*a^16*b^24*c^10*e^24*f^16*(a^2*c*f^2 - b^2*c*e^2)^3 + 5766181411456*a^18*b^
22*c^10*e^22*f^18*(a^2*c*f^2 - b^2*c*e^2)^3 - 7493983209472*a^20*b^20*c^10*
e^20*f^20*(a^2*c*f^2 - b^2*c*e^2)^3 + 7713917084672*a^22*b^18*c^10*e^18*f^2
2*(a^2*c*f^2 - b^2*c*e^2)^3 - 6328467293184*a^24*b^16*c^10*e^16*f^24*(a^2*c
*f^2 - b^2*c*e^2)^3 + 4142950034432*a^26*b^14*c^10*e^14*f^26*(a^2*c*f^2 - b
^2*c*e^2)^3 - 2152681536512*a^28*b^12*c^10*e^12*f^28*(a^2*c*f^2 - b^2*c*e^2
)^3 + 874199511040*a^30*b^10*c^10*e^10*f^30*(a^2*c*f^2 - b^2*c*e^2)^3 - 268
759150592*a^32*b^8*c^10*e^8*f^32*(a^2*c*f^2 - b^2*c*e^2)^3 + 58872545280*a^
34*b^6*c^10*e^6*f^34*(a^2*c*f^2 - b^2*c*e^2)^3 - 8151957504*a^36*b^4*c^10*e
^4*f^36*(a^2*c*f^2 - b^2*c*e^2)^3 + 530841600*a^38*b^2*c^10*e^2*f^38*(a^2*c
*f^2 - b^2*c*e^2)^3 - 42743457*a^2*b^40*c^11*e^40*f^2*(a^2*c*f^2 - b^2*c*e^
2)^2 + 411055884*a^4*b^38*c^11*e^38*f^4*(a^2*c*f^2 - b^2*c*e^2)^2 - 2180887
236*a^6*b^36*c^11*e^36*f^6*(a^2*c*f^2 - b^2*c*e^2)^2 + 6404946508*a^8*b^34*
c^11*e^34*f^8*(a^2*c*f^2 - b^2*c*e^2)^2 - 5434005264*a^10*b^32*c^11*e^32*f^
10*(a^2*c*f^2 - b^2*c*e^2)^2 - 38868373520*a^12*b^30*c^11*e^30*f^12*(a^2*c*
f^2 - b^2*c*e^2)^2 + 208447613600*a^14*b^28*c^11*e^28*f^14*(a^2*c*f^2 - b^2
*c*e^2)^2 - 579674999104*a^16*b^26*c^11*e^26*f^16*(a^2*c*f^2 - b^2*c*e^2)^2
+ 1104967566592*a^18*b^24*c^11*e^24*f^18*(a^2*c*f^2 - b^2*c*e^2)^2 - 15545
66531328*a^20*b^22*c^11*e^22*f^20*(a^2*c*f^2 - b^2*c*e^2)^2 + 1659734381312
*a^22*b^20*c^11*e^20*f^22*(a^2*c*f^2 - b^2*c*e^2)^2 - 1356361512192*a^24*b^
18*c^11*e^18*f^24*(a^2*c*f^2 - b^2*c*e^2)^2 + 845331359744*a^26*b^16*c^11*e
^16*f^26*(a^2*c*f^2 - b^2*c*e^2)^2 - 395676895232*a^28*b^14*c^11*e^14*f^28*
(a^2*c*f^2 - b^2*c*e^2)^2 + 134902689792*a^30*b^12*c^11*e^12*f^30*(a^2*c*f^
2 - b^2*c*e^2)^2 - 31670587392*a^32*b^10*c^11*e^10*f^32*(a^2*c*f^2 - b^2*c*
e^2)^2 + 4584669184*a^34*b^8*c^11*e^8*f^34*(a^2*c*f^2 - b^2*c*e^2)^2 - 3096
57600*a^36*b^6*c^11*e^6*f^36*(a^2*c*f^2 - b^2*c*e^2)^2))) - 2*atan((((((a^(
3/2)*f^3*(a*c)^(3/2)*(4*a^2*c*f^2 - b^2*c*e^2)^2*(4*a^2*c*f^2 - 3*b^2*c*e^2
)*(4*a^6*c*f^6 - 3*b^6*c*e^6 + 8*a^2*b^4*c*e^4*f^2 - 8*a^4*b^2*c*e^2*f^4)^4
)/(c^2*(164025*b^46*c^13*e^46 + 885735*b^44*c^12*e^44*(a^2*c*f^2 - b^2*c*e^
2) + 117440512*a^30*c^5*f^30*(a^2*c*f^2 - b^2*c*e^2)^8 - 385875968*a^32*c^6
f^32(a^2*c*f^2 - b^2*c*e^2)^7 + 419430400*a^34*c^7*f^34*(a^2*c*f^2 - b^2*
c*e^2)^6 - 150994944*a^36*c^8*f^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 236196*b^36*
c^8*e^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 1102248*b^38*c^9*e^38*(a^2*c*f^2 - b^2
*c*e^2)^4 + 2053593*b^40*c^10*e^40*(a^2*c*f^2 - b^2*c*e^2)^3 + 1909251*b^42
*c^11*e^42*(a^2*c*f^2 - b^2*c*e^2)^2 - 3937329*a^2*b^44*c^13*e^44*f^2 + 438

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93819*a^4*b^42*c^13*e^42*f^4 - 301507155*a^6*b^40*c^13*e^40*f^6 + 142751465
6*a^8*b^38*c^13*e^38*f^8 - 4936911112*a^10*b^36*c^13*e^36*f^10 + 1289327361
6*a^12*b^34*c^13*e^34*f^12 - 25921630432*a^14*b^32*c^13*e^32*f^14 + 4051928
6096*a^16*b^30*c^13*e^30*f^16 - 49376608256*a^18*b^28*c^13*e^28*f^18 + 4672
1401856*a^20*b^26*c^13*e^26*f^20 - 33946324736*a^22*b^24*c^13*e^24*f^22 + 1
8556579328*a^24*b^22*c^13*e^22*f^24 - 7375276032*a^26*b^20*c^13*e^20*f^26 +
 2009817088*a^28*b^18*c^13*e^18*f^28 - 335642624*a^30*b^16*c^13*e^16*f^30 +
 25907200*a^32*b^14*c^13*e^14*f^32 - 21130794*a^2*b^42*c^12*e^42*f^2*(a^2*c
*f^2 - b^2*c*e^2) + 234399015*a^4*b^40*c^12*e^40*f^4*(a^2*c*f^2 - b^2*c*e^2
) - 1604168280*a^6*b^38*c^12*e^38*f^6*(a^2*c*f^2 - b^2*c*e^2) + 7579098492*
a^8*b^36*c^12*e^36*f^8*(a^2*c*f^2 - b^2*c*e^2) - 26212380172*a^10*b^34*c^12
*e^34*f^10*(a^2*c*f^2 - b^2*c*e^2) + 68672994096*a^12*b^32*c^12*e^32*f^12*(
a^2*c*f^2 - b^2*c*e^2) - 139160589504*a^14*b^30*c^12*e^30*f^14*(a^2*c*f^2 -
 b^2*c*e^2) + 220859191808*a^16*b^28*c^12*e^28*f^16*(a^2*c*f^2 - b^2*c*e^2)
 - 276344315328*a^18*b^26*c^12*e^26*f^18*(a^2*c*f^2 - b^2*c*e^2) + 27313056
1984*a^20*b^24*c^12*e^24*f^20*(a^2*c*f^2 - b^2*c*e^2) - 212730002688*a^22*b
^22*c^12*e^22*f^22*(a^2*c*f^2 - b^2*c*e^2) + 129574234368*a^24*b^20*c^12*e^
20*f^24*(a^2*c*f^2 - b^2*c*e^2) - 60770569216*a^26*b^18*c^12*e^18*f^26*(a^2
*c*f^2 - b^2*c*e^2) + 21304706048*a^28*b^16*c^12*e^16*f^28*(a^2*c*f^2 - b^2
*c*e^2) - 5272965120*a^30*b^14*c^12*e^14*f^30*(a^2*c*f^2 - b^2*c*e^2) + 819
441664*a^32*b^12*c^12*e^12*f^32*(a^2*c*f^2 - b^2*c*e^2) - 59392000*a^34*b^1
0*c^12*e^10*f^34*(a^2*c*f^2 - b^2*c*e^2) + 9289728*a^6*b^24*c^5*e^24*f^6*(a
^2*c*f^2 - b^2*c*e^2)^8 - 36884480*a^8*b^22*c^5*e^22*f^8*(a^2*c*f^2 - b^2*c
*e^2)^8 - 278604800*a^10*b^20*c^5*e^20*f^10*(a^2*c*f^2 - b^2*c*e^2)^8 + 277
4483200*a^12*b^18*c^5*e^18*f^12*(a^2*c*f^2 - b^2*c*e^2)^8 - 10869657600*a^1
4*b^16*c^5*e^16*f^14*(a^2*c*f^2 - b^2*c*e^2)^8 + 25237416960*a^16*b^14*c^5*
e^14*f^16*(a^2*c*f^2 - b^2*c*e^2)^8 - 38348909568*a^18*b^12*c^5*e^12*f^18*(
a^2*c*f^2 - b^2*c*e^2)^8 + 39084659712*a^20*b^10*c^5*e^10*f^20*(a^2*c*f^2 -
 b^2*c*e^2)^8 - 26118635520*a^22*b^8*c^5*e^8*f^22*(a^2*c*f^2 - b^2*c*e^2)^8
 + 10414620672*a^24*b^6*c^5*e^6*f^24*(a^2*c*f^2 - b^2*c*e^2)^8 - 1708654592
*a^26*b^4*c^5*e^4*f^26*(a^2*c*f^2 - b^2*c*e^2)^8 - 276561920*a^28*b^2*c^5*e
^2*f^28*(a^2*c*f^2 - b^2*c*e^2)^8 - 9704448*a^4*b^28*c^6*e^28*f^4*(a^2*c*f^
2 - b^2*c*e^2)^7 + 260614656*a^6*b^26*c^6*e^26*f^6*(a^2*c*f^2 - b^2*c*e^2)^
7 - 2166022464*a^8*b^24*c^6*e^24*f^8*(a^2*c*f^2 - b^2*c*e^2)^7 + 8626147840
*a^10*b^22*c^6*e^22*f^10*(a^2*c*f^2 - b^2*c*e^2)^7 - 16771503616*a^12*b^20*
c^6*e^20*f^12*(a^2*c*f^2 - b^2*c*e^2)^7 + 3301800960*a^14*b^18*c^6*e^18*f^1
4*(a^2*c*f^2 - b^2*c*e^2)^7 + 67337715968*a^16*b^16*c^6*e^16*f^16*(a^2*c*f^
2 - b^2*c*e^2)^7 - 189857873920*a^18*b^14*c^6*e^14*f^18*(a^2*c*f^2 - b^2*c*
e^2)^7 + 286100259840*a^20*b^12*c^6*e^12*f^20*(a^2*c*f^2 - b^2*c*e^2)^7 - 2
75789894656*a^22*b^10*c^6*e^10*f^22*(a^2*c*f^2 - b^2*c*e^2)^7 + 17371653734
4*a^24*b^8*c^6*e^8*f^24*(a^2*c*f^2 - b^2*c*e^2)^7 - 67416424448*a^26*b^6*c^
6*e^6*f^26*(a^2*c*f^2 - b^2*c*e^2)^7 + 12831686656*a^28*b^4*c^6*e^4*f^28*(a
^2*c*f^2 - b^2*c*e^2)^7 + 222560256*a^30*b^2*c^6*e^2*f^30*(a^2*c*f^2 - b^2*
c*e^2)^7 + 2099520*a^2*b^32*c^7*e^32*f^2*(a^2*c*f^2 - b^2*c*e^2)^6 - 107014
608*a^4*b^30*c^7*e^30*f^4*(a^2*c*f^2 - b^2*c*e^2)^6 + 1848335616*a^6*b^28*c
^7*e^28*f^6*(a^2*c*f^2 - b^2*c*e^2)^6 - 15200005312*a^8*b^26*c^7*e^26*f^8*(
a^2*c*f^2 - b^2*c*e^2)^6 + 72612273792*a^10*b^24*c^7*e^24*f^10*(a^2*c*f^2 -
 b^2*c*e^2)^6 - 221855779968*a^12*b^22*c^7*e^22*f^12*(a^2*c*f^2 - b^2*c*e^2
)^6 + 450717857536*a^14*b^20*c^7*e^20*f^14*(a^2*c*f^2 - b^2*c*e^2)^6 - 6005
78910208*a^16*b^18*c^7*e^18*f^16*(a^2*c*f^2 - b^2*c*e^2)^6 + 459464530688*a
^18*b^16*c^7*e^16*f^18*(a^2*c*f^2 - b^2*c*e^2)^6 - 33638947840*a^20*b^14*c^
7*e^14*f^20*(a^2*c*f^2 - b^2*c*e^2)^6 - 376299926528*a^22*b^12*c^7*e^12*f^2
2*(a^2*c*f^2 - b^2*c*e^2)^6 + 488874068992*a^24*b^10*c^7*e^10*f^24*(a^2*c*f
^2 - b^2*c*e^2)^6 - 333407809536*a^26*b^8*c^7*e^8*f^26*(a^2*c*f^2 - b^2*c*e
^2)^6 + 134140313600*a^28*b^6*c^7*e^6*f^28*(a^2*c*f^2 - b^2*c*e^2)^6 - 2822
0915712*a^30*b^4*c^7*e^4*f^30*(a^2*c*f^2 - b^2*c*e^2)^6 + 1230503936*a^32*b
^2*c^7*e^2*f^32*(a^2*c*f^2 - b^2*c*e^2)^6 + 3335904*a^2*b^34*c^8*e^34*f^2*(
a^2*c*f^2 - b^2*c*e^2)^5 - 290521728*a^4*b^32*c^8*e^32*f^4*(a^2*c*f^2 - b^2
*c*e^2)^5 + 4865684544*a^6*b^30*c^8*e^30*f^6*(a^2*c*f^2 - b^2*c*e^2)^5 - 40
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437394528*a^8*b^28*c^8*e^28*f^8*(a^2*c*f^2 - b^2*c*e^2)^5 + 205602254656*a^
10*b^26*c^8*e^26*f^10*(a^2*c*f^2 - b^2*c*e^2)^5 - 703885344192*a^12*b^24*c^
8*e^24*f^12*(a^2*c*f^2 - b^2*c*e^2)^5 + 1709253482624*a^14*b^22*c^8*e^22*f^
14*(a^2*c*f^2 - b^2*c*e^2)^5 - 3029282695168*a^16*b^20*c^8*e^20*f^16*(a^2*c
*f^2 - b^2*c*e^2)^5 + 3966230827520*a^18*b^18*c^8*e^18*f^18*(a^2*c*f^2 - b^
2*c*e^2)^5 - 3822339813632*a^20*b^16*c^8*e^16*f^20*(a^2*c*f^2 - b^2*c*e^2)^
5 + 2640438056960*a^22*b^14*c^8*e^14*f^22*(a^2*c*f^2 - b^2*c*e^2)^5 - 12085
01415936*a^24*b^12*c^8*e^12*f^24*(a^2*c*f^2 - b^2*c*e^2)^5 + 269338092544*a
^26*b^10*c^8*e^10*f^26*(a^2*c*f^2 - b^2*c*e^2)^5 + 53783212032*a^28*b^8*c^8
*e^8*f^28*(a^2*c*f^2 - b^2*c*e^2)^5 - 60985360384*a^30*b^6*c^8*e^6*f^30*(a^
2*c*f^2 - b^2*c*e^2)^5 + 17917083648*a^32*b^4*c^8*e^4*f^32*(a^2*c*f^2 - b^2
*c*e^2)^5 - 1558708224*a^34*b^2*c^8*e^2*f^34*(a^2*c*f^2 - b^2*c*e^2)^5 - 11
917692*a^2*b^36*c^9*e^36*f^2*(a^2*c*f^2 - b^2*c*e^2)^4 - 224907516*a^4*b^34
*c^9*e^34*f^4*(a^2*c*f^2 - b^2*c*e^2)^4 + 5303932560*a^6*b^32*c^9*e^32*f^6*
(a^2*c*f^2 - b^2*c*e^2)^4 - 48206418480*a^8*b^30*c^9*e^30*f^8*(a^2*c*f^2 -
b^2*c*e^2)^4 + 261450609120*a^10*b^28*c^9*e^28*f^10*(a^2*c*f^2 - b^2*c*e^2)
^4 - 962361040256*a^12*b^26*c^9*e^26*f^12*(a^2*c*f^2 - b^2*c*e^2)^4 + 25585
59358080*a^14*b^24*c^9*e^24*f^14*(a^2*c*f^2 - b^2*c*e^2)^4 - 5091804150656*
a^16*b^22*c^9*e^22*f^16*(a^2*c*f^2 - b^2*c*e^2)^4 + 7750806514944*a^18*b^20
*c^9*e^20*f^18*(a^2*c*f^2 - b^2*c*e^2)^4 - 9137207485952*a^20*b^18*c^9*e^18
*f^20*(a^2*c*f^2 - b^2*c*e^2)^4 + 8384563280128*a^22*b^16*c^9*e^16*f^22*(a^
2*c*f^2 - b^2*c*e^2)^4 - 5975281259520*a^24*b^14*c^9*e^14*f^24*(a^2*c*f^2 -
 b^2*c*e^2)^4 + 3269297268736*a^26*b^12*c^9*e^12*f^26*(a^2*c*f^2 - b^2*c*e^
2)^4 - 1339171540992*a^28*b^10*c^9*e^10*f^28*(a^2*c*f^2 - b^2*c*e^2)^4 + 39
1250194432*a^30*b^8*c^9*e^8*f^30*(a^2*c*f^2 - b^2*c*e^2)^4 - 74114154496*a^
32*b^6*c^9*e^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^4 + 7299203072*a^34*b^4*c^9*e^4
*f^34*(a^2*c*f^2 - b^2*c*e^2)^4 - 148635648*a^36*b^2*c^9*e^2*f^36*(a^2*c*f^
2 - b^2*c*e^2)^4 - 38704068*a^2*b^38*c^10*e^38*f^2*(a^2*c*f^2 - b^2*c*e^2)^
3 + 188845992*a^4*b^36*c^10*e^36*f^4*(a^2*c*f^2 - b^2*c*e^2)^3 + 1157124204
*a^6*b^34*c^10*e^34*f^6*(a^2*c*f^2 - b^2*c*e^2)^3 - 20586361424*a^8*b^32*c^
10*e^32*f^8*(a^2*c*f^2 - b^2*c*e^2)^3 + 135395499200*a^10*b^30*c^10*e^30*f^
10*(a^2*c*f^2 - b^2*c*e^2)^3 - 555513858464*a^12*b^28*c^10*e^28*f^12*(a^2*c
*f^2 - b^2*c*e^2)^3 + 1608776388864*a^14*b^26*c^10*e^26*f^14*(a^2*c*f^2 - b
^2*c*e^2)^3 - 3473989271488*a^16*b^24*c^10*e^24*f^16*(a^2*c*f^2 - b^2*c*e^2
)^3 + 5766181411456*a^18*b^22*c^10*e^22*f^18*(a^2*c*f^2 - b^2*c*e^2)^3 - 74
93983209472*a^20*b^20*c^10*e^20*f^20*(a^2*c*f^2 - b^2*c*e^2)^3 + 7713917084
672*a^22*b^18*c^10*e^18*f^22*(a^2*c*f^2 - b^2*c*e^2)^3 - 6328467293184*a^24
*b^16*c^10*e^16*f^24*(a^2*c*f^2 - b^2*c*e^2)^3 + 4142950034432*a^26*b^14*c^
10*e^14*f^26*(a^2*c*f^2 - b^2*c*e^2)^3 - 2152681536512*a^28*b^12*c^10*e^12*
f^28*(a^2*c*f^2 - b^2*c*e^2)^3 + 874199511040*a^30*b^10*c^10*e^10*f^30*(a^2
*c*f^2 - b^2*c*e^2)^3 - 268759150592*a^32*b^8*c^10*e^8*f^32*(a^2*c*f^2 - b^
2*c*e^2)^3 + 58872545280*a^34*b^6*c^10*e^6*f^34*(a^2*c*f^2 - b^2*c*e^2)^3 -
 8151957504*a^36*b^4*c^10*e^4*f^36*(a^2*c*f^2 - b^2*c*e^2)^3 + 530841600*a^
38*b^2*c^10*e^2*f^38*(a^2*c*f^2 - b^2*c*e^2)^3 - 42743457*a^2*b^40*c^11*e^4
0*f^2*(a^2*c*f^2 - b^2*c*e^2)^2 + 411055884*a^4*b^38*c^11*e^38*f^4*(a^2*c*f
^2 - b^2*c*e^2)^2 - 2180887236*a^6*b^36*c^11*e^36*f^6*(a^2*c*f^2 - b^2*c*e^
2)^2 + 6404946508*a^8*b^34*c^11*e^34*f^8*(a^2*c*f^2 - b^2*c*e^2)^2 - 543400
5264*a^10*b^32*c^11*e^32*f^10*(a^2*c*f^2 - b^2*c*e^2)^2 - 38868373520*a^12*
b^30*c^11*e^30*f^12*(a^2*c*f^2 - b^2*c*e^2)^2 + 208447613600*a^14*b^28*c^11
*e^28*f^14*(a^2*c*f^2 - b^2*c*e^2)^2 - 579674999104*a^16*b^26*c^11*e^26*f^1
6*(a^2*c*f^2 - b^2*c*e^2)^2 + 1104967566592*a^18*b^24*c^11*e^24*f^18*(a^2*c
*f^2 - b^2*c*e^2)^2 - 1554566531328*a^20*b^22*c^11*e^22*f^20*(a^2*c*f^2 - b
^2*c*e^2)^2 + 1659734381312*a^22*b^20*c^11*e^20*f^22*(a^2*c*f^2 - b^2*c*e^2
)^2 - 1356361512192*a^24*b^18*c^11*e^18*f^24*(a^2*c*f^2 - b^2*c*e^2)^2 + 84
5331359744*a^26*b^16*c^11*e^16*f^26*(a^2*c*f^2 - b^2*c*e^2)^2 - 39567689523
2*a^28*b^14*c^11*e^14*f^28*(a^2*c*f^2 - b^2*c*e^2)^2 + 134902689792*a^30*b^
12*c^11*e^12*f^30*(a^2*c*f^2 - b^2*c*e^2)^2 - 31670587392*a^32*b^10*c^11*e^
10*f^32*(a^2*c*f^2 - b^2*c*e^2)^2 + 4584669184*a^34*b^8*c^11*e^8*f^34*(a^2*
c*f^2 - b^2*c*e^2)^2 - 309657600*a^36*b^6*c^11*e^6*f^36*(a^2*c*f^2 - b^2*c*
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e^2)^2)) - (a^(5/2)*f^5*(a*c)^(5/2)*(4*a^2*c*f^2 - 3*b^2*c*e^2)^3*(4*a^6*c*
f^6 - 3*b^6*c*e^6 + 8*a^2*b^4*c*e^4*f^2 - 8*a^4*b^2*c*e^2*f^4)^4)/(c^2*(a^2
*c*f^2 - b^2*c*e^2)*(164025*b^46*c^13*e^46 + 885735*b^44*c^12*e^44*(a^2*c*f
^2 - b^2*c*e^2) + 117440512*a^30*c^5*f^30*(a^2*c*f^2 - b^2*c*e^2)^8 - 38587
5968*a^32*c^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^7 + 419430400*a^34*c^7*f^34*(a^2
*c*f^2 - b^2*c*e^2)^6 - 150994944*a^36*c^8*f^36*(a^2*c*f^2 - b^2*c*e^2)^5 +
 236196*b^36*c^8*e^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 1102248*b^38*c^9*e^38*(a^
2*c*f^2 - b^2*c*e^2)^4 + 2053593*b^40*c^10*e^40*(a^2*c*f^2 - b^2*c*e^2)^3 +
 1909251*b^42*c^11*e^42*(a^2*c*f^2 - b^2*c*e^2)^2 - 3937329*a^2*b^44*c^13*e
^44*f^2 + 43893819*a^4*b^42*c^13*e^42*f^4 - 301507155*a^6*b^40*c^13*e^40*f^
6 + 1427514656*a^8*b^38*c^13*e^38*f^8 - 4936911112*a^10*b^36*c^13*e^36*f^10
 + 12893273616*a^12*b^34*c^13*e^34*f^12 - 25921630432*a^14*b^32*c^13*e^32*f
^14 + 40519286096*a^16*b^30*c^13*e^30*f^16 - 49376608256*a^18*b^28*c^13*e^2
8*f^18 + 46721401856*a^20*b^26*c^13*e^26*f^20 - 33946324736*a^22*b^24*c^13*
e^24*f^22 + 18556579328*a^24*b^22*c^13*e^22*f^24 - 7375276032*a^26*b^20*c^1
3*e^20*f^26 + 2009817088*a^28*b^18*c^13*e^18*f^28 - 335642624*a^30*b^16*c^1
3*e^16*f^30 + 25907200*a^32*b^14*c^13*e^14*f^32 - 21130794*a^2*b^42*c^12*e^
42*f^2*(a^2*c*f^2 - b^2*c*e^2) + 234399015*a^4*b^40*c^12*e^40*f^4*(a^2*c*f^
2 - b^2*c*e^2) - 1604168280*a^6*b^38*c^12*e^38*f^6*(a^2*c*f^2 - b^2*c*e^2)
+ 7579098492*a^8*b^36*c^12*e^36*f^8*(a^2*c*f^2 - b^2*c*e^2) - 26212380172*a
^10*b^34*c^12*e^34*f^10*(a^2*c*f^2 - b^2*c*e^2) + 68672994096*a^12*b^32*c^1
2*e^32*f^12*(a^2*c*f^2 - b^2*c*e^2) - 139160589504*a^14*b^30*c^12*e^30*f^14
*(a^2*c*f^2 - b^2*c*e^2) + 220859191808*a^16*b^28*c^12*e^28*f^16*(a^2*c*f^2
 - b^2*c*e^2) - 276344315328*a^18*b^26*c^12*e^26*f^18*(a^2*c*f^2 - b^2*c*e^
2) + 273130561984*a^20*b^24*c^12*e^24*f^20*(a^2*c*f^2 - b^2*c*e^2) - 212730
002688*a^22*b^22*c^12*e^22*f^22*(a^2*c*f^2 - b^2*c*e^2) + 129574234368*a^24
*b^20*c^12*e^20*f^24*(a^2*c*f^2 - b^2*c*e^2) - 60770569216*a^26*b^18*c^12*e
^18*f^26*(a^2*c*f^2 - b^2*c*e^2) + 21304706048*a^28*b^16*c^12*e^16*f^28*(a^
2*c*f^2 - b^2*c*e^2) - 5272965120*a^30*b^14*c^12*e^14*f^30*(a^2*c*f^2 - b^2
*c*e^2) + 819441664*a^32*b^12*c^12*e^12*f^32*(a^2*c*f^2 - b^2*c*e^2) - 5939
2000*a^34*b^10*c^12*e^10*f^34*(a^2*c*f^2 - b^2*c*e^2) + 9289728*a^6*b^24*c^
5*e^24*f^6*(a^2*c*f^2 - b^2*c*e^2)^8 - 36884480*a^8*b^22*c^5*e^22*f^8*(a^2*
c*f^2 - b^2*c*e^2)^8 - 278604800*a^10*b^20*c^5*e^20*f^10*(a^2*c*f^2 - b^2*c
*e^2)^8 + 2774483200*a^12*b^18*c^5*e^18*f^12*(a^2*c*f^2 - b^2*c*e^2)^8 - 10
869657600*a^14*b^16*c^5*e^16*f^14*(a^2*c*f^2 - b^2*c*e^2)^8 + 25237416960*a
^16*b^14*c^5*e^14*f^16*(a^2*c*f^2 - b^2*c*e^2)^8 - 38348909568*a^18*b^12*c^
5*e^12*f^18*(a^2*c*f^2 - b^2*c*e^2)^8 + 39084659712*a^20*b^10*c^5*e^10*f^20
*(a^2*c*f^2 - b^2*c*e^2)^8 - 26118635520*a^22*b^8*c^5*e^8*f^22*(a^2*c*f^2 -
 b^2*c*e^2)^8 + 10414620672*a^24*b^6*c^5*e^6*f^24*(a^2*c*f^2 - b^2*c*e^2)^8
 - 1708654592*a^26*b^4*c^5*e^4*f^26*(a^2*c*f^2 - b^2*c*e^2)^8 - 276561920*a
^28*b^2*c^5*e^2*f^28*(a^2*c*f^2 - b^2*c*e^2)^8 - 9704448*a^4*b^28*c^6*e^28*
f^4*(a^2*c*f^2 - b^2*c*e^2)^7 + 260614656*a^6*b^26*c^6*e^26*f^6*(a^2*c*f^2
- b^2*c*e^2)^7 - 2166022464*a^8*b^24*c^6*e^24*f^8*(a^2*c*f^2 - b^2*c*e^2)^7
 + 8626147840*a^10*b^22*c^6*e^22*f^10*(a^2*c*f^2 - b^2*c*e^2)^7 - 167715036
16*a^12*b^20*c^6*e^20*f^12*(a^2*c*f^2 - b^2*c*e^2)^7 + 3301800960*a^14*b^18
*c^6*e^18*f^14*(a^2*c*f^2 - b^2*c*e^2)^7 + 67337715968*a^16*b^16*c^6*e^16*f
^16*(a^2*c*f^2 - b^2*c*e^2)^7 - 189857873920*a^18*b^14*c^6*e^14*f^18*(a^2*c
*f^2 - b^2*c*e^2)^7 + 286100259840*a^20*b^12*c^6*e^12*f^20*(a^2*c*f^2 - b^2
*c*e^2)^7 - 275789894656*a^22*b^10*c^6*e^10*f^22*(a^2*c*f^2 - b^2*c*e^2)^7
+ 173716537344*a^24*b^8*c^6*e^8*f^24*(a^2*c*f^2 - b^2*c*e^2)^7 - 6741642444
8*a^26*b^6*c^6*e^6*f^26*(a^2*c*f^2 - b^2*c*e^2)^7 + 12831686656*a^28*b^4*c^
6*e^4*f^28*(a^2*c*f^2 - b^2*c*e^2)^7 + 222560256*a^30*b^2*c^6*e^2*f^30*(a^2
*c*f^2 - b^2*c*e^2)^7 + 2099520*a^2*b^32*c^7*e^32*f^2*(a^2*c*f^2 - b^2*c*e^
2)^6 - 107014608*a^4*b^30*c^7*e^30*f^4*(a^2*c*f^2 - b^2*c*e^2)^6 + 18483356
16*a^6*b^28*c^7*e^28*f^6*(a^2*c*f^2 - b^2*c*e^2)^6 - 15200005312*a^8*b^26*c
^7*e^26*f^8*(a^2*c*f^2 - b^2*c*e^2)^6 + 72612273792*a^10*b^24*c^7*e^24*f^10
*(a^2*c*f^2 - b^2*c*e^2)^6 - 221855779968*a^12*b^22*c^7*e^22*f^12*(a^2*c*f^
2 - b^2*c*e^2)^6 + 450717857536*a^14*b^20*c^7*e^20*f^14*(a^2*c*f^2 - b^2*c*
e^2)^6 - 600578910208*a^16*b^18*c^7*e^18*f^16*(a^2*c*f^2 - b^2*c*e^2)^6 + 4
```

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59464530688*a^18*b^16*c^7*e^16*f^18*(a^2*c*f^2 - b^2*c*e^2)^6 - 33638947840
*a^20*b^14*c^7*e^14*f^20*(a^2*c*f^2 - b^2*c*e^2)^6 - 376299926528*a^22*b^12
*c^7*e^12*f^22*(a^2*c*f^2 - b^2*c*e^2)^6 + 488874068992*a^24*b^10*c^7*e^10*
f^24*(a^2*c*f^2 - b^2*c*e^2)^6 - 333407809536*a^26*b^8*c^7*e^8*f^26*(a^2*c*
f^2 - b^2*c*e^2)^6 + 134140313600*a^28*b^6*c^7*e^6*f^28*(a^2*c*f^2 - b^2*c*
e^2)^6 - 28220915712*a^30*b^4*c^7*e^4*f^30*(a^2*c*f^2 - b^2*c*e^2)^6 + 1230
503936*a^32*b^2*c^7*e^2*f^32*(a^2*c*f^2 - b^2*c*e^2)^6 + 3335904*a^2*b^34*c
^8*e^34*f^2*(a^2*c*f^2 - b^2*c*e^2)^5 - 290521728*a^4*b^32*c^8*e^32*f^4*(a^
2*c*f^2 - b^2*c*e^2)^5 + 4865684544*a^6*b^30*c^8*e^30*f^6*(a^2*c*f^2 - b^2*
c*e^2)^5 - 40437394528*a^8*b^28*c^8*e^28*f^8*(a^2*c*f^2 - b^2*c*e^2)^5 + 20
5602254656*a^10*b^26*c^8*e^26*f^10*(a^2*c*f^2 - b^2*c*e^2)^5 - 703885344192
*a^12*b^24*c^8*e^24*f^12*(a^2*c*f^2 - b^2*c*e^2)^5 + 1709253482624*a^14*b^2
2*c^8*e^22*f^14*(a^2*c*f^2 - b^2*c*e^2)^5 - 3029282695168*a^16*b^20*c^8*e^2
0*f^16*(a^2*c*f^2 - b^2*c*e^2)^5 + 3966230827520*a^18*b^18*c^8*e^18*f^18*(a
^2*c*f^2 - b^2*c*e^2)^5 - 3822339813632*a^20*b^16*c^8*e^16*f^20*(a^2*c*f^2
- b^2*c*e^2)^5 + 2640438056960*a^22*b^14*c^8*e^14*f^22*(a^2*c*f^2 - b^2*c*e
^2)^5 - 1208501415936*a^24*b^12*c^8*e^12*f^24*(a^2*c*f^2 - b^2*c*e^2)^5 + 2
69338092544*a^26*b^10*c^8*e^10*f^26*(a^2*c*f^2 - b^2*c*e^2)^5 + 53783212032
*a^28*b^8*c^8*e^8*f^28*(a^2*c*f^2 - b^2*c*e^2)^5 - 60985360384*a^30*b^6*c^8
*e^6*f^30*(a^2*c*f^2 - b^2*c*e^2)^5 + 17917083648*a^32*b^4*c^8*e^4*f^32*(a^
2*c*f^2 - b^2*c*e^2)^5 - 1558708224*a^34*b^2*c^8*e^2*f^34*(a^2*c*f^2 - b^2*
c*e^2)^5 - 11917692*a^2*b^36*c^9*e^36*f^2*(a^2*c*f^2 - b^2*c*e^2)^4 - 22490
7516*a^4*b^34*c^9*e^34*f^4*(a^2*c*f^2 - b^2*c*e^2)^4 + 5303932560*a^6*b^32*
c^9*e^32*f^6*(a^2*c*f^2 - b^2*c*e^2)^4 - 48206418480*a^8*b^30*c^9*e^30*f^8*
(a^2*c*f^2 - b^2*c*e^2)^4 + 261450609120*a^10*b^28*c^9*e^28*f^10*(a^2*c*f^2
 - b^2*c*e^2)^4 - 962361040256*a^12*b^26*c^9*e^26*f^12*(a^2*c*f^2 - b^2*c*e
^2)^4 + 2558559358080*a^14*b^24*c^9*e^24*f^14*(a^2*c*f^2 - b^2*c*e^2)^4 - 5
091804150656*a^16*b^22*c^9*e^22*f^16*(a^2*c*f^2 - b^2*c*e^2)^4 + 7750806514
944*a^18*b^20*c^9*e^20*f^18*(a^2*c*f^2 - b^2*c*e^2)^4 - 9137207485952*a^20*
b^18*c^9*e^18*f^20*(a^2*c*f^2 - b^2*c*e^2)^4 + 8384563280128*a^22*b^16*c^9*
e^16*f^22*(a^2*c*f^2 - b^2*c*e^2)^4 - 5975281259520*a^24*b^14*c^9*e^14*f^24
*(a^2*c*f^2 - b^2*c*e^2)^4 + 3269297268736*a^26*b^12*c^9*e^12*f^26*(a^2*c*f
^2 - b^2*c*e^2)^4 - 1339171540992*a^28*b^10*c^9*e^10*f^28*(a^2*c*f^2 - b^2*
c*e^2)^4 + 391250194432*a^30*b^8*c^9*e^8*f^30*(a^2*c*f^2 - b^2*c*e^2)^4 - 7
4114154496*a^32*b^6*c^9*e^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^4 + 7299203072*a^3
4*b^4*c^9*e^4*f^34*(a^2*c*f^2 - b^2*c*e^2)^4 - 148635648*a^36*b^2*c^9*e^2*f
^36*(a^2*c*f^2 - b^2*c*e^2)^4 - 38704068*a^2*b^38*c^10*e^38*f^2*(a^2*c*f^2
- b^2*c*e^2)^3 + 188845992*a^4*b^36*c^10*e^36*f^4*(a^2*c*f^2 - b^2*c*e^2)^3
 + 1157124204*a^6*b^34*c^10*e^34*f^6*(a^2*c*f^2 - b^2*c*e^2)^3 - 2058636142
4*a^8*b^32*c^10*e^32*f^8*(a^2*c*f^2 - b^2*c*e^2)^3 + 135395499200*a^10*b^30
*c^10*e^30*f^10*(a^2*c*f^2 - b^2*c*e^2)^3 - 555513858464*a^12*b^28*c^10*e^2
8*f^12*(a^2*c*f^2 - b^2*c*e^2)^3 + 1608776388864*a^14*b^26*c^10*e^26*f^14*(
a^2*c*f^2 - b^2*c*e^2)^3 - 3473989271488*a^16*b^24*c^10*e^24*f^16*(a^2*c*f^
2 - b^2*c*e^2)^3 + 5766181411456*a^18*b^22*c^10*e^22*f^18*(a^2*c*f^2 - b^2*
c*e^2)^3 - 7493983209472*a^20*b^20*c^10*e^20*f^20*(a^2*c*f^2 - b^2*c*e^2)^3
 + 7713917084672*a^22*b^18*c^10*e^18*f^22*(a^2*c*f^2 - b^2*c*e^2)^3 - 63284
67293184*a^24*b^16*c^10*e^16*f^24*(a^2*c*f^2 - b^2*c*e^2)^3 + 4142950034432
*a^26*b^14*c^10*e^14*f^26*(a^2*c*f^2 - b^2*c*e^2)^3 - 2152681536512*a^28*b^
12*c^10*e^12*f^28*(a^2*c*f^2 - b^2*c*e^2)^3 + 874199511040*a^30*b^10*c^10*e
^10*f^30*(a^2*c*f^2 - b^2*c*e^2)^3 - 268759150592*a^32*b^8*c^10*e^8*f^32*(a
^2*c*f^2 - b^2*c*e^2)^3 + 58872545280*a^34*b^6*c^10*e^6*f^34*(a^2*c*f^2 - b
^2*c*e^2)^3 - 8151957504*a^36*b^4*c^10*e^4*f^36*(a^2*c*f^2 - b^2*c*e^2)^3 +
 530841600*a^38*b^2*c^10*e^2*f^38*(a^2*c*f^2 - b^2*c*e^2)^3 - 42743457*a^2*
b^40*c^11*e^40*f^2*(a^2*c*f^2 - b^2*c*e^2)^2 + 411055884*a^4*b^38*c^11*e^38
*f^4*(a^2*c*f^2 - b^2*c*e^2)^2 - 2180887236*a^6*b^36*c^11*e^36*f^6*(a^2*c*f
^2 - b^2*c*e^2)^2 + 6404946508*a^8*b^34*c^11*e^34*f^8*(a^2*c*f^2 - b^2*c*e^
2)^2 - 5434005264*a^10*b^32*c^11*e^32*f^10*(a^2*c*f^2 - b^2*c*e^2)^2 - 3886
8373520*a^12*b^30*c^11*e^30*f^12*(a^2*c*f^2 - b^2*c*e^2)^2 + 208447613600*a
^14*b^28*c^11*e^28*f^14*(a^2*c*f^2 - b^2*c*e^2)^2 - 579674999104*a^16*b^26*
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c^11*e^26*f^16*(a^2*c*f^2 - b^2*c*e^2)^2 + 1104967566592*a^18*b^24*c^11*e^2
4*f^18*(a^2*c*f^2 - b^2*c*e^2)^2 - 1554566531328*a^20*b^22*c^11*e^22*f^20*(
a^2*c*f^2 - b^2*c*e^2)^2 + 1659734381312*a^22*b^20*c^11*e^20*f^22*(a^2*c*f^
2 - b^2*c*e^2)^2 - 1356361512192*a^24*b^18*c^11*e^18*f^24*(a^2*c*f^2 - b^2*
c*e^2)^2 + 845331359744*a^26*b^16*c^11*e^16*f^26*(a^2*c*f^2 - b^2*c*e^2)^2
- 395676895232*a^28*b^14*c^11*e^14*f^28*(a^2*c*f^2 - b^2*c*e^2)^2 + 1349026
89792*a^30*b^12*c^11*e^12*f^30*(a^2*c*f^2 - b^2*c*e^2)^2 - 31670587392*a^32
*b^10*c^11*e^10*f^32*(a^2*c*f^2 - b^2*c*e^2)^2 + 4584669184*a^34*b^8*c^11*e
^8*f^34*(a^2*c*f^2 - b^2*c*e^2)^2 - 309657600*a^36*b^6*c^11*e^6*f^36*(a^2*c
*f^2 - b^2*c*e^2)^2)))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2
) - a^(1/2)) - (4*a^4*b*c*e*f^4*(4*a^2*c*f^2 - b^2*c*e^2)*(4*a^2*c*f^2 - 3*
b^2*c*e^2)*(4*a^6*c*f^6 - 3*b^6*c*e^6 + 8*a^2*b^4*c*e^4*f^2 - 8*a^4*b^2*c*e
^2*f^4)^4)/(164025*b^46*c^13*e^46 + 885735*b^44*c^12*e^44*(a^2*c*f^2 - b^2*
c*e^2) + 117440512*a^30*c^5*f^30*(a^2*c*f^2 - b^2*c*e^2)^8 - 385875968*a^32
*c^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^7 + 419430400*a^34*c^7*f^34*(a^2*c*f^2 -
b^2*c*e^2)^6 - 150994944*a^36*c^8*f^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 236196*b
^36*c^8*e^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 1102248*b^38*c^9*e^38*(a^2*c*f^2 -
 b^2*c*e^2)^4 + 2053593*b^40*c^10*e^40*(a^2*c*f^2 - b^2*c*e^2)^3 + 1909251*
b^42*c^11*e^42*(a^2*c*f^2 - b^2*c*e^2)^2 - 3937329*a^2*b^44*c^13*e^44*f^2 +
 43893819*a^4*b^42*c^13*e^42*f^4 - 301507155*a^6*b^40*c^13*e^40*f^6 + 14275
14656*a^8*b^38*c^13*e^38*f^8 - 4936911112*a^10*b^36*c^13*e^36*f^10 + 128932
73616*a^12*b^34*c^13*e^34*f^12 - 25921630432*a^14*b^32*c^13*e^32*f^14 + 405
19286096*a^16*b^30*c^13*e^30*f^16 - 49376608256*a^18*b^28*c^13*e^28*f^18 +
46721401856*a^20*b^26*c^13*e^26*f^20 - 33946324736*a^22*b^24*c^13*e^24*f^22
 + 18556579328*a^24*b^22*c^13*e^22*f^24 - 7375276032*a^26*b^20*c^13*e^20*f^
26 + 2009817088*a^28*b^18*c^13*e^18*f^28 - 335642624*a^30*b^16*c^13*e^16*f^
30 + 25907200*a^32*b^14*c^13*e^14*f^32 - 21130794*a^2*b^42*c^12*e^42*f^2*(a
^2*c*f^2 - b^2*c*e^2) + 234399015*a^4*b^40*c^12*e^40*f^4*(a^2*c*f^2 - b^2*c
*e^2) - 1604168280*a^6*b^38*c^12*e^38*f^6*(a^2*c*f^2 - b^2*c*e^2) + 7579098
492*a^8*b^36*c^12*e^36*f^8*(a^2*c*f^2 - b^2*c*e^2) - 26212380172*a^10*b^34*
c^12*e^34*f^10*(a^2*c*f^2 - b^2*c*e^2) + 68672994096*a^12*b^32*c^12*e^32*f^
12*(a^2*c*f^2 - b^2*c*e^2) - 139160589504*a^14*b^30*c^12*e^30*f^14*(a^2*c*f
^2 - b^2*c*e^2) + 220859191808*a^16*b^28*c^12*e^28*f^16*(a^2*c*f^2 - b^2*c*
e^2) - 276344315328*a^18*b^26*c^12*e^26*f^18*(a^2*c*f^2 - b^2*c*e^2) + 2731
30561984*a^20*b^24*c^12*e^24*f^20*(a^2*c*f^2 - b^2*c*e^2) - 212730002688*a^
22*b^22*c^12*e^22*f^22*(a^2*c*f^2 - b^2*c*e^2) + 129574234368*a^24*b^20*c^1
2*e^20*f^24*(a^2*c*f^2 - b^2*c*e^2) - 60770569216*a^26*b^18*c^12*e^18*f^26*
(a^2*c*f^2 - b^2*c*e^2) + 21304706048*a^28*b^16*c^12*e^16*f^28*(a^2*c*f^2 -
 b^2*c*e^2) - 5272965120*a^30*b^14*c^12*e^14*f^30*(a^2*c*f^2 - b^2*c*e^2) +
 819441664*a^32*b^12*c^12*e^12*f^32*(a^2*c*f^2 - b^2*c*e^2) - 59392000*a^34
*b^10*c^12*e^10*f^34*(a^2*c*f^2 - b^2*c*e^2) + 9289728*a^6*b^24*c^5*e^24*f^
6*(a^2*c*f^2 - b^2*c*e^2)^8 - 36884480*a^8*b^22*c^5*e^22*f^8*(a^2*c*f^2 - b
^2*c*e^2)^8 - 278604800*a^10*b^20*c^5*e^20*f^10*(a^2*c*f^2 - b^2*c*e^2)^8 +
 2774483200*a^12*b^18*c^5*e^18*f^12*(a^2*c*f^2 - b^2*c*e^2)^8 - 10869657600
*a^14*b^16*c^5*e^16*f^14*(a^2*c*f^2 - b^2*c*e^2)^8 + 25237416960*a^16*b^14*
c^5*e^14*f^16*(a^2*c*f^2 - b^2*c*e^2)^8 - 38348909568*a^18*b^12*c^5*e^12*f^
18*(a^2*c*f^2 - b^2*c*e^2)^8 + 39084659712*a^20*b^10*c^5*e^10*f^20*(a^2*c*f
^2 - b^2*c*e^2)^8 - 26118635520*a^22*b^8*c^5*e^8*f^22*(a^2*c*f^2 - b^2*c*e^
2)^8 + 10414620672*a^24*b^6*c^5*e^6*f^24*(a^2*c*f^2 - b^2*c*e^2)^8 - 170865
4592*a^26*b^4*c^5*e^4*f^26*(a^2*c*f^2 - b^2*c*e^2)^8 - 276561920*a^28*b^2*c
^5*e^2*f^28*(a^2*c*f^2 - b^2*c*e^2)^8 - 9704448*a^4*b^28*c^6*e^28*f^4*(a^2*
c*f^2 - b^2*c*e^2)^7 + 260614656*a^6*b^26*c^6*e^26*f^6*(a^2*c*f^2 - b^2*c*e
^2)^7 - 2166022464*a^8*b^24*c^6*e^24*f^8*(a^2*c*f^2 - b^2*c*e^2)^7 + 862614
7840*a^10*b^22*c^6*e^22*f^10*(a^2*c*f^2 - b^2*c*e^2)^7 - 16771503616*a^12*b
^20*c^6*e^20*f^12*(a^2*c*f^2 - b^2*c*e^2)^7 + 3301800960*a^14*b^18*c^6*e^18
*f^14*(a^2*c*f^2 - b^2*c*e^2)^7 + 67337715968*a^16*b^16*c^6*e^16*f^16*(a^2*
c*f^2 - b^2*c*e^2)^7 - 189857873920*a^18*b^14*c^6*e^14*f^18*(a^2*c*f^2 - b^
2*c*e^2)^7 + 286100259840*a^20*b^12*c^6*e^12*f^20*(a^2*c*f^2 - b^2*c*e^2)^7
 - 275789894656*a^22*b^10*c^6*e^10*f^22*(a^2*c*f^2 - b^2*c*e^2)^7 + 1737165
```

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37344*a^24*b^8*c^6*e^8*f^24*(a^2*c*f^2 - b^2*c*e^2)^7 - 67416424448*a^26*b^
6*c^6*e^6*f^26*(a^2*c*f^2 - b^2*c*e^2)^7 + 12831686656*a^28*b^4*c^6*e^4*f^2
8*(a^2*c*f^2 - b^2*c*e^2)^7 + 222560256*a^30*b^2*c^6*e^2*f^30*(a^2*c*f^2 -
b^2*c*e^2)^7 + 2099520*a^2*b^32*c^7*e^32*f^2*(a^2*c*f^2 - b^2*c*e^2)^6 - 10
7014608*a^4*b^30*c^7*e^30*f^4*(a^2*c*f^2 - b^2*c*e^2)^6 + 1848335616*a^6*b^
28*c^7*e^28*f^6*(a^2*c*f^2 - b^2*c*e^2)^6 - 15200005312*a^8*b^26*c^7*e^26*f
^8*(a^2*c*f^2 - b^2*c*e^2)^6 + 72612273792*a^10*b^24*c^7*e^24*f^10*(a^2*c*f
^2 - b^2*c*e^2)^6 - 221855779968*a^12*b^22*c^7*e^22*f^12*(a^2*c*f^2 - b^2*c
*e^2)^6 + 450717857536*a^14*b^20*c^7*e^20*f^14*(a^2*c*f^2 - b^2*c*e^2)^6 -
600578910208*a^16*b^18*c^7*e^18*f^16*(a^2*c*f^2 - b^2*c*e^2)^6 + 4594645306
88*a^18*b^16*c^7*e^16*f^18*(a^2*c*f^2 - b^2*c*e^2)^6 - 33638947840*a^20*b^1
4*c^7*e^14*f^20*(a^2*c*f^2 - b^2*c*e^2)^6 - 376299926528*a^22*b^12*c^7*e^12
*f^22*(a^2*c*f^2 - b^2*c*e^2)^6 + 488874068992*a^24*b^10*c^7*e^10*f^24*(a^2
*c*f^2 - b^2*c*e^2)^6 - 333407809536*a^26*b^8*c^7*e^8*f^26*(a^2*c*f^2 - b^2
*c*e^2)^6 + 134140313600*a^28*b^6*c^7*e^6*f^28*(a^2*c*f^2 - b^2*c*e^2)^6 -
28220915712*a^30*b^4*c^7*e^4*f^30*(a^2*c*f^2 - b^2*c*e^2)^6 + 1230503936*a^
32*b^2*c^7*e^2*f^32*(a^2*c*f^2 - b^2*c*e^2)^6 + 3335904*a^2*b^34*c^8*e^34*f
^2*(a^2*c*f^2 - b^2*c*e^2)^5 - 290521728*a^4*b^32*c^8*e^32*f^4*(a^2*c*f^2 -
 b^2*c*e^2)^5 + 4865684544*a^6*b^30*c^8*e^30*f^6*(a^2*c*f^2 - b^2*c*e^2)^5
- 40437394528*a^8*b^28*c^8*e^28*f^8*(a^2*c*f^2 - b^2*c*e^2)^5 + 20560225465
6*a^10*b^26*c^8*e^26*f^10*(a^2*c*f^2 - b^2*c*e^2)^5 - 703885344192*a^12*b^2
4*c^8*e^24*f^12*(a^2*c*f^2 - b^2*c*e^2)^5 + 1709253482624*a^14*b^22*c^8*e^2
2*f^14*(a^2*c*f^2 - b^2*c*e^2)^5 - 3029282695168*a^16*b^20*c^8*e^20*f^16*(a
^2*c*f^2 - b^2*c*e^2)^5 + 3966230827520*a^18*b^18*c^8*e^18*f^18*(a^2*c*f^2
- b^2*c*e^2)^5 - 3822339813632*a^20*b^16*c^8*e^16*f^20*(a^2*c*f^2 - b^2*c*e
^2)^5 + 2640438056960*a^22*b^14*c^8*e^14*f^22*(a^2*c*f^2 - b^2*c*e^2)^5 - 1
208501415936*a^24*b^12*c^8*e^12*f^24*(a^2*c*f^2 - b^2*c*e^2)^5 + 2693380925
44*a^26*b^10*c^8*e^10*f^26*(a^2*c*f^2 - b^2*c*e^2)^5 + 53783212032*a^28*b^8
*c^8*e^8*f^28*(a^2*c*f^2 - b^2*c*e^2)^5 - 60985360384*a^30*b^6*c^8*e^6*f^30
*(a^2*c*f^2 - b^2*c*e^2)^5 + 17917083648*a^32*b^4*c^8*e^4*f^32*(a^2*c*f^2 -
 b^2*c*e^2)^5 - 1558708224*a^34*b^2*c^8*e^2*f^34*(a^2*c*f^2 - b^2*c*e^2)^5
- 11917692*a^2*b^36*c^9*e^36*f^2*(a^2*c*f^2 - b^2*c*e^2)^4 - 224907516*a^4*
b^34*c^9*e^34*f^4*(a^2*c*f^2 - b^2*c*e^2)^4 + 5303932560*a^6*b^32*c^9*e^32*
f^6*(a^2*c*f^2 - b^2*c*e^2)^4 - 48206418480*a^8*b^30*c^9*e^30*f^8*(a^2*c*f^
2 - b^2*c*e^2)^4 + 261450609120*a^10*b^28*c^9*e^28*f^10*(a^2*c*f^2 - b^2*c*
e^2)^4 - 962361040256*a^12*b^26*c^9*e^26*f^12*(a^2*c*f^2 - b^2*c*e^2)^4 + 2
558559358080*a^14*b^24*c^9*e^24*f^14*(a^2*c*f^2 - b^2*c*e^2)^4 - 5091804150
656*a^16*b^22*c^9*e^22*f^16*(a^2*c*f^2 - b^2*c*e^2)^4 + 7750806514944*a^18*
b^20*c^9*e^20*f^18*(a^2*c*f^2 - b^2*c*e^2)^4 - 9137207485952*a^20*b^18*c^9*
e^18*f^20*(a^2*c*f^2 - b^2*c*e^2)^4 + 8384563280128*a^22*b^16*c^9*e^16*f^22
*(a^2*c*f^2 - b^2*c*e^2)^4 - 5975281259520*a^24*b^14*c^9*e^14*f^24*(a^2*c*f
^2 - b^2*c*e^2)^4 + 3269297268736*a^26*b^12*c^9*e^12*f^26*(a^2*c*f^2 - b^2*
c*e^2)^4 - 1339171540992*a^28*b^10*c^9*e^10*f^28*(a^2*c*f^2 - b^2*c*e^2)^4
+ 391250194432*a^30*b^8*c^9*e^8*f^30*(a^2*c*f^2 - b^2*c*e^2)^4 - 7411415449
6*a^32*b^6*c^9*e^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^4 + 7299203072*a^34*b^4*c^9
*e^4*f^34*(a^2*c*f^2 - b^2*c*e^2)^4 - 148635648*a^36*b^2*c^9*e^2*f^36*(a^2*
c*f^2 - b^2*c*e^2)^4 - 38704068*a^2*b^38*c^10*e^38*f^2*(a^2*c*f^2 - b^2*c*e
^2)^3 + 188845992*a^4*b^36*c^10*e^36*f^4*(a^2*c*f^2 - b^2*c*e^2)^3 + 115712
4204*a^6*b^34*c^10*e^34*f^6*(a^2*c*f^2 - b^2*c*e^2)^3 - 20586361424*a^8*b^3
2*c^10*e^32*f^8*(a^2*c*f^2 - b^2*c*e^2)^3 + 135395499200*a^10*b^30*c^10*e^3
0*f^10*(a^2*c*f^2 - b^2*c*e^2)^3 - 555513858464*a^12*b^28*c^10*e^28*f^12*(a
^2*c*f^2 - b^2*c*e^2)^3 + 1608776388864*a^14*b^26*c^10*e^26*f^14*(a^2*c*f^2
 - b^2*c*e^2)^3 - 3473989271488*a^16*b^24*c^10*e^24*f^16*(a^2*c*f^2 - b^2*c
*e^2)^3 + 5766181411456*a^18*b^22*c^10*e^22*f^18*(a^2*c*f^2 - b^2*c*e^2)^3
- 7493983209472*a^20*b^20*c^10*e^20*f^20*(a^2*c*f^2 - b^2*c*e^2)^3 + 771391
7084672*a^22*b^18*c^10*e^18*f^22*(a^2*c*f^2 - b^2*c*e^2)^3 - 6328467293184*
a^24*b^16*c^10*e^16*f^24*(a^2*c*f^2 - b^2*c*e^2)^3 + 4142950034432*a^26*b^1
4*c^10*e^14*f^26*(a^2*c*f^2 - b^2*c*e^2)^3 - 2152681536512*a^28*b^12*c^10*e
^12*f^28*(a^2*c*f^2 - b^2*c*e^2)^3 + 874199511040*a^30*b^10*c^10*e^10*f^30*
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(a^2*c*f^2 - b^2*c*e^2)^3 - 268759150592*a^32*b^8*c^10*e^8*f^32*(a^2*c*f^2
- b^2*c*e^2)^3 + 58872545280*a^34*b^6*c^10*e^6*f^34*(a^2*c*f^2 - b^2*c*e^2)
^3 - 8151957504*a^36*b^4*c^10*e^4*f^36*(a^2*c*f^2 - b^2*c*e^2)^3 + 53084160
0*a^38*b^2*c^10*e^2*f^38*(a^2*c*f^2 - b^2*c*e^2)^3 - 42743457*a^2*b^40*c^11
*e^40*f^2*(a^2*c*f^2 - b^2*c*e^2)^2 + 411055884*a^4*b^38*c^11*e^38*f^4*(a^2
*c*f^2 - b^2*c*e^2)^2 - 2180887236*a^6*b^36*c^11*e^36*f^6*(a^2*c*f^2 - b^2*
c*e^2)^2 + 6404946508*a^8*b^34*c^11*e^34*f^8*(a^2*c*f^2 - b^2*c*e^2)^2 - 54
34005264*a^10*b^32*c^11*e^32*f^10*(a^2*c*f^2 - b^2*c*e^2)^2 - 38868373520*a
^12*b^30*c^11*e^30*f^12*(a^2*c*f^2 - b^2*c*e^2)^2 + 208447613600*a^14*b^28*
c^11*e^28*f^14*(a^2*c*f^2 - b^2*c*e^2)^2 - 579674999104*a^16*b^26*c^11*e^26
*f^16*(a^2*c*f^2 - b^2*c*e^2)^2 + 1104967566592*a^18*b^24*c^11*e^24*f^18*(a
^2*c*f^2 - b^2*c*e^2)^2 - 1554566531328*a^20*b^22*c^11*e^22*f^20*(a^2*c*f^2
 - b^2*c*e^2)^2 + 1659734381312*a^22*b^20*c^11*e^20*f^22*(a^2*c*f^2 - b^2*c
*e^2)^2 - 1356361512192*a^24*b^18*c^11*e^18*f^24*(a^2*c*f^2 - b^2*c*e^2)^2
+ 845331359744*a^26*b^16*c^11*e^16*f^26*(a^2*c*f^2 - b^2*c*e^2)^2 - 3956768
95232*a^28*b^14*c^11*e^14*f^28*(a^2*c*f^2 - b^2*c*e^2)^2 + 134902689792*a^3
0*b^12*c^11*e^12*f^30*(a^2*c*f^2 - b^2*c*e^2)^2 - 31670587392*a^32*b^10*c^1
1*e^10*f^32*(a^2*c*f^2 - b^2*c*e^2)^2 + 4584669184*a^34*b^8*c^11*e^8*f^34*(
a^2*c*f^2 - b^2*c*e^2)^2 - 309657600*a^36*b^6*c^11*e^6*f^36*(a^2*c*f^2 - b^
2*c*e^2)^2) + (2*a^4*b*c*e*f^4*(2*a^2*c*f^2 - b^2*c*e^2)*(4*a^2*c*f^2 - 3*b
^2*c*e^2)^2*(4*a^6*c*f^6 - 3*b^6*c*e^6 + 8*a^2*b^4*c*e^4*f^2 - 8*a^4*b^2*c*
e^2*f^4)^4)/((a^2*c*f^2 - b^2*c*e^2)*(164025*b^46*c^13*e^46 + 885735*b^44*c
^12*e^44*(a^2*c*f^2 - b^2*c*e^2) + 117440512*a^30*c^5*f^30*(a^2*c*f^2 - b^2
*c*e^2)^8 - 385875968*a^32*c^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^7 + 419430400*a
^34*c^7*f^34*(a^2*c*f^2 - b^2*c*e^2)^6 - 150994944*a^36*c^8*f^36*(a^2*c*f^2
 - b^2*c*e^2)^5 + 236196*b^36*c^8*e^36*(a^2*c*f^2 - b^2*c*e^2)^5 + 1102248*
b^38*c^9*e^38*(a^2*c*f^2 - b^2*c*e^2)^4 + 2053593*b^40*c^10*e^40*(a^2*c*f^2
 - b^2*c*e^2)^3 + 1909251*b^42*c^11*e^42*(a^2*c*f^2 - b^2*c*e^2)^2 - 393732
9*a^2*b^44*c^13*e^44*f^2 + 43893819*a^4*b^42*c^13*e^42*f^4 - 301507155*a^6*
b^40*c^13*e^40*f^6 + 1427514656*a^8*b^38*c^13*e^38*f^8 - 4936911112*a^10*b^
36*c^13*e^36*f^10 + 12893273616*a^12*b^34*c^13*e^34*f^12 - 25921630432*a^14
*b^32*c^13*e^32*f^14 + 40519286096*a^16*b^30*c^13*e^30*f^16 - 49376608256*a
^18*b^28*c^13*e^28*f^18 + 46721401856*a^20*b^26*c^13*e^26*f^20 - 3394632473
6*a^22*b^24*c^13*e^24*f^22 + 18556579328*a^24*b^22*c^13*e^22*f^24 - 7375276
032*a^26*b^20*c^13*e^20*f^26 + 2009817088*a^28*b^18*c^13*e^18*f^28 - 335642
624*a^30*b^16*c^13*e^16*f^30 + 25907200*a^32*b^14*c^13*e^14*f^32 - 21130794
*a^2*b^42*c^12*e^42*f^2*(a^2*c*f^2 - b^2*c*e^2) + 234399015*a^4*b^40*c^12*e
^40*f^4*(a^2*c*f^2 - b^2*c*e^2) - 1604168280*a^6*b^38*c^12*e^38*f^6*(a^2*c*
f^2 - b^2*c*e^2) + 7579098492*a^8*b^36*c^12*e^36*f^8*(a^2*c*f^2 - b^2*c*e^2
) - 26212380172*a^10*b^34*c^12*e^34*f^10*(a^2*c*f^2 - b^2*c*e^2) + 68672994
096*a^12*b^32*c^12*e^32*f^12*(a^2*c*f^2 - b^2*c*e^2) - 139160589504*a^14*b^
30*c^12*e^30*f^14*(a^2*c*f^2 - b^2*c*e^2) + 220859191808*a^16*b^28*c^12*e^2
8*f^16*(a^2*c*f^2 - b^2*c*e^2) - 276344315328*a^18*b^26*c^12*e^26*f^18*(a^2
*c*f^2 - b^2*c*e^2) + 273130561984*a^20*b^24*c^12*e^24*f^20*(a^2*c*f^2 - b^
2*c*e^2) - 212730002688*a^22*b^22*c^12*e^22*f^22*(a^2*c*f^2 - b^2*c*e^2) +
129574234368*a^24*b^20*c^12*e^20*f^24*(a^2*c*f^2 - b^2*c*e^2) - 60770569216
*a^26*b^18*c^12*e^18*f^26*(a^2*c*f^2 - b^2*c*e^2) + 21304706048*a^28*b^16*c
^12*e^16*f^28*(a^2*c*f^2 - b^2*c*e^2) - 5272965120*a^30*b^14*c^12*e^14*f^30
*(a^2*c*f^2 - b^2*c*e^2) + 819441664*a^32*b^12*c^12*e^12*f^32*(a^2*c*f^2 -
b^2*c*e^2) - 59392000*a^34*b^10*c^12*e^10*f^34*(a^2*c*f^2 - b^2*c*e^2) + 92
89728*a^6*b^24*c^5*e^24*f^6*(a^2*c*f^2 - b^2*c*e^2)^8 - 36884480*a^8*b^22*c
^5*e^22*f^8*(a^2*c*f^2 - b^2*c*e^2)^8 - 278604800*a^10*b^20*c^5*e^20*f^10*(
a^2*c*f^2 - b^2*c*e^2)^8 + 2774483200*a^12*b^18*c^5*e^18*f^12*(a^2*c*f^2 -
b^2*c*e^2)^8 - 10869657600*a^14*b^16*c^5*e^16*f^14*(a^2*c*f^2 - b^2*c*e^2)^
8 + 25237416960*a^16*b^14*c^5*e^14*f^16*(a^2*c*f^2 - b^2*c*e^2)^8 - 3834890
9568*a^18*b^12*c^5*e^12*f^18*(a^2*c*f^2 - b^2*c*e^2)^8 + 39084659712*a^20*b
^10*c^5*e^10*f^20*(a^2*c*f^2 - b^2*c*e^2)^8 - 26118635520*a^22*b^8*c^5*e^8*
f^22*(a^2*c*f^2 - b^2*c*e^2)^8 + 10414620672*a^24*b^6*c^5*e^6*f^24*(a^2*c*f
^2 - b^2*c*e^2)^8 - 1708654592*a^26*b^4*c^5*e^4*f^26*(a^2*c*f^2 - b^2*c*e^2
```

)^8 - 276561920*a^28*b^2*c^5*e^2*f^28*(a^2*c*f^2 - b^2*c*e^2)^8 - 9704448*a
^4*b^28*c^6*e^28*f^4*(a^2*c*f^2 - b^2*c*e^2)^7 + 260614656*a^6*b^26*c^6*e^2
6*f^6*(a^2*c*f^2 - b^2*c*e^2)^7 - 2166022464*a^8*b^24*c^6*e^24*f^8*(a^2*c*f
^2 - b^2*c*e^2)^7 + 8626147840*a^10*b^22*c^6*e^22*f^10*(a^2*c*f^2 - b^2*c*e
^2)^7 - 16771503616*a^12*b^20*c^6*e^20*f^12*(a^2*c*f^2 - b^2*c*e^2)^7 + 330
1800960*a^14*b^18*c^6*e^18*f^14*(a^2*c*f^2 - b^2*c*e^2)^7 + 67337715968*a^1
6*b^16*c^6*e^16*f^16*(a^2*c*f^2 - b^2*c*e^2)^7 - 189857873920*a^18*b^14*c^6
*e^14*f^18*(a^2*c*f^2 - b^2*c*e^2)^7 + 286100259840*a^20*b^12*c^6*e^12*f^20
*(a^2*c*f^2 - b^2*c*e^2)^7 - 275789894656*a^22*b^10*c^6*e^10*f^22*(a^2*c*f^
2 - b^2*c*e^2)^7 + 173716537344*a^24*b^8*c^6*e^8*f^24*(a^2*c*f^2 - b^2*c*e^
2)^7 - 67416424448*a^26*b^6*c^6*e^6*f^26*(a^2*c*f^2 - b^2*c*e^2)^7 + 128316
86656*a^28*b^4*c^6*e^4*f^28*(a^2*c*f^2 - b^2*c*e^2)^7 + 222560256*a^30*b^2*
c^6*e^2*f^30*(a^2*c*f^2 - b^2*c*e^2)^7 + 2099520*a^2*b^32*c^7*e^32*f^2*(a^2
*c*f^2 - b^2*c*e^2)^6 - 107014608*a^4*b^30*c^7*e^30*f^4*(a^2*c*f^2 - b^2*c*
e^2)^6 + 1848335616*a^6*b^28*c^7*e^28*f^6*(a^2*c*f^2 - b^2*c*e^2)^6 - 15200
005312*a^8*b^26*c^7*e^26*f^8*(a^2*c*f^2 - b^2*c*e^2)^6 + 72612273792*a^10*b
^24*c^7*e^24*f^10*(a^2*c*f^2 - b^2*c*e^2)^6 - 221855779968*a^12*b^22*c^7*e^
22*f^12*(a^2*c*f^2 - b^2*c*e^2)^6 + 450717857536*a^14*b^20*c^7*e^20*f^14*(a
^2*c*f^2 - b^2*c*e^2)^6 - 600578910208*a^16*b^18*c^7*e^18*f^16*(a^2*c*f^2 -
b^2*c*e^2)^6 + 459464530688*a^18*b^16*c^7*e^16*f^18*(a^2*c*f^2 - b^2*c*e^2
)^6 - 33638947840*a^20*b^14*c^7*e^14*f^20*(a^2*c*f^2 - b^2*c*e^2)^6 - 37629
9926528*a^22*b^12*c^7*e^12*f^22*(a^2*c*f^2 - b^2*c*e^2)^6 + 488874068992*a^
24*b^10*c^7*e^10*f^24*(a^2*c*f^2 - b^2*c*e^2)^6 - 333407809536*a^26*b^8*c^7
*e^8*f^26*(a^2*c*f^2 - b^2*c*e^2)^6 + 134140313600*a^28*b^6*c^7*e^6*f^28*(a
^2*c*f^2 - b^2*c*e^2)^6 - 28220915712*a^30*b^4*c^7*e^4*f^30*(a^2*c*f^2 - b^
2*c*e^2)^6 + 1230503936*a^32*b^2*c^7*e^2*f^32*(a^2*c*f^2 - b^2*c*e^2)^6 + 3
335904*a^2*b^34*c^8*e^34*f^2*(a^2*c*f^2 - b^2*c*e^2)^5 - 290521728*a^4*b^32
*c^8*e^32*f^4*(a^2*c*f^2 - b^2*c*e^2)^5 + 4865684544*a^6*b^30*c^8*e^30*f^6*
(a^2*c*f^2 - b^2*c*e^2)^5 - 40437394528*a^8*b^28*c^8*e^28*f^8*(a^2*c*f^2 -
b^2*c*e^2)^5 + 205602254656*a^10*b^26*c^8*e^26*f^10*(a^2*c*f^2 - b^2*c*e^2)
^5 - 703885344192*a^12*b^24*c^8*e^24*f^12*(a^2*c*f^2 - b^2*c*e^2)^5 + 17092
53482624*a^14*b^22*c^8*e^22*f^14*(a^2*c*f^2 - b^2*c*e^2)^5 - 3029282695168*
a^16*b^20*c^8*e^20*f^16*(a^2*c*f^2 - b^2*c*e^2)^5 + 3966230827520*a^18*b^18
*c^8*e^18*f^18*(a^2*c*f^2 - b^2*c*e^2)^5 - 3822339813632*a^20*b^16*c^8*e^16
f^20(a^2*c*f^2 - b^2*c*e^2)^5 + 2640438056960*a^22*b^14*c^8*e^14*f^22*(a^
2*c*f^2 - b^2*c*e^2)^5 - 1208501415936*a^24*b^12*c^8*e^12*f^24*(a^2*c*f^2 -
b^2*c*e^2)^5 + 269338092544*a^26*b^10*c^8*e^10*f^26*(a^2*c*f^2 - b^2*c*e^2
)^5 + 53783212032*a^28*b^8*c^8*e^8*f^28*(a^2*c*f^2 - b^2*c*e^2)^5 - 6098536
0384*a^30*b^6*c^8*e^6*f^30*(a^2*c*f^2 - b^2*c*e^2)^5 + 17917083648*a^32*b^4
*c^8*e^4*f^32*(a^2*c*f^2 - b^2*c*e^2)^5 - 1558708224*a^34*b^2*c^8*e^2*f^34*
(a^2*c*f^2 - b^2*c*e^2)^5 - 11917692*a^2*b^36*c^9*e^36*f^2*(a^2*c*f^2 - b^2
*c*e^2)^4 - 224907516*a^4*b^34*c^9*e^34*f^4*(a^2*c*f^2 - b^2*c*e^2)^4 + 530
3932560*a^6*b^32*c^9*e^32*f^6*(a^2*c*f^2 - b^2*c*e^2)^4 - 48206418480*a^8*b
^30*c^9*e^30*f^8*(a^2*c*f^2 - b^2*c*e^2)^4 + 261450609120*a^10*b^28*c^9*e^2
8*f^10*(a^2*c*f^2 - b^2*c*e^2)^4 - 962361040256*a^12*b^26*c^9*e^26*f^12*(a^
2*c*f^2 - b^2*c*e^2)^4 + 2558559358080*a^14*b^24*c^9*e^24*f^14*(a^2*c*f^2 -
b^2*c*e^2)^4 - 5091804150656*a^16*b^22*c^9*e^22*f^16*(a^2*c*f^2 - b^2*c*e^
2)^4 + 7750806514944*a^18*b^20*c^9*e^20*f^18*(a^2*c*f^2 - b^2*c*e^2)^4 - 91
37207485952*a^20*b^18*c^9*e^18*f^20*(a^2*c*f^2 - b^2*c*e^2)^4 + 83845632801
28*a^22*b^16*c^9*e^16*f^22*(a^2*c*f^2 - b^2*c*e^2)^4 - 5975281259520*a^24*b
^14*c^9*e^14*f^24*(a^2*c*f^2 - b^2*c*e^2)^4 + 3269297268736*a^26*b^12*c^9*e
^12*f^26*(a^2*c*f^2 - b^2*c*e^2)^4 - 1339171540992*a^28*b^10*c^9*e^10*f^28*
(a^2*c*f^2 - b^2*c*e^2)^4 + 391250194432*a^30*b^8*c^9*e^8*f^30*(a^2*c*f^2 -
b^2*c*e^2)^4 - 74114154496*a^32*b^6*c^9*e^6*f^32*(a^2*c*f^2 - b^2*c*e^2)^4
+ 7299203072*a^34*b^4*c^9*e^4*f^34*(a^2*c*f^2 - b^2*c*e^2)^4 - 148635648*a
^36*b^2*c^9*e^2*f^36*(a^2*c*f^2 - b^2*c*e^2)^4 - 38704068*a^2*b^38*c^10*e^3
8*f^2*(a^2*c*f^2 - b^2*c*e^2)^3 + 188845992*a^4*b^36*c^10*e^36*f^4*(a^2*c*f
^2 - b^2*c*e^2)^3 + 1157124204*a^6*b^34*c^10*e^34*f^6*(a^2*c*f^2 - b^2*c*e^
2)^3 - 20586361424*a^8*b^32*c^10*e^32*f^8*(a^2*c*f^2 - b^2*c*e^2)^3 + 13539

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5499200*a^10*b^30*c^10*e^30*f^10*(a^2*c*f^2 - b^2*c*e^2)^3 - 555513858464*a
^12*b^28*c^10*e^28*f^12*(a^2*c*f^2 - b^2*c*e^2)^3 + 1608776388864*a^14*b^26
*c^10*e^26*f^14*(a^2*c*f^2 - b^2*c*e^2)^3 - 3473989271488*a^16*b^24*c^10*e^
24*f^16*(a^2*c*f^2 - b^2*c*e^2)^3 + 5766181411456*a^18*b^22*c^10*e^22*f^18*
(a^2*c*f^2 - b^2*c*e^2)^3 - 7493983209472*a^20*b^20*c^10*e^20*f^20*(a^2*c*f
^2 - b^2*c*e^2)^3 + 7713917084672*a^22*b^18*c^10*e^18*f^22*(a^2*c*f^2 - b^2
*c*e^2)^3 - 6328467293184*a^24*b^16*c^10*e^16*f^24*(a^2*c*f^2 - b^2*c*e^2)^
3 + 4142950034432*a^26*b^14*c^10*e^14*f^26*(a^2*c*f^2 - b^2*c*e^2)^3 - 2152
681536512*a^28*b^12*c^10*e^12*f^28*(a^2*c*f^2 - b^2*c*e^2)^3 + 874199511040
*a^30*b^10*c^10*e^10*f^30*(a^2*c*f^2 - b^2*c*e^2)^3 - 268759150592*a^32*b^8
*c^10*e^8*f^32*(a^2*c*f^2 - b^2*c*e^2)^3 + 58872545280*a^34*b^6*c^10*e^6*f^
34*(a^2*c*f^2 - b^2*c*e^2)^3 - 8151957504*a^36*b^4*c^10*e^4*f^36*(a^2*c*f^2
 - b^2*c*e^2)^3 + 530841600*a^38*b^2*c^10*e^2*f^38*(a^2*c*f^2 - b^2*c*e^2)^
3 - 42743457*a^2*b^40*c^11*e^40*f^2*(a^2*c*f^2 - b^2*c*e^2)^2 + 411055884*a
^4*b^38*c^11*e^38*f^4*(a^2*c*f^2 - b^2*c*e^2)^2 - 2180887236*a^6*b^36*c^11*
e^36*f^6*(a^2*c*f^2 - b^2*c*e^2)^2 + 6404946508*a^8*b^34*c^11*e^34*f^8*(a^2
*c*f^2 - b^2*c*e^2)^2 - 5434005264*a^10*b^32*c^11*e^32*f^10*(a^2*c*f^2 - b^
2*c*e^2)^2 - 38868373520*a^12*b^30*c^11*e^30*f^12*(a^2*c*f^2 - b^2*c*e^2)^2
 + 208447613600*a^14*b^28*c^11*e^28*f^14*(a^2*c*f^2 - b^2*c*e^2)^2 - 579674
999104*a^16*b^26*c^11*e^26*f^16*(a^2*c*f^2 - b^2*c*e^2)^2 + 1104967566592*a
^18*b^24*c^11*e^24*f^18*(a^2*c*f^2 - b^2*c*e^2)^2 - 1554566531328*a^20*b^22
*c^11*e^22*f^20*(a^2*c*f^2 - b^2*c*e^2)^2 + 1659734381312*a^22*b^20*c^11*e^
20*f^22*(a^2*c*f^2 - b^2*c*e^2)^2 - 1356361512192*a^24*b^18*c^11*e^18*f^24*
(a^2*c*f^2 - b^2*c*e^2)^2 + 845331359744*a^26*b^16*c^11*e^16*f^26*(a^2*c*f^
2 - b^2*c*e^2)^2 - 395676895232*a^28*b^14*c^11*e^14*f^28*(a^2*c*f^2 - b^2*c
*e^2)^2 + 134902689792*a^30*b^12*c^11*e^12*f^30*(a^2*c*f^2 - b^2*c*e^2)^2 -
 31670587392*a^32*b^10*c^11*e^10*f^32*(a^2*c*f^2 - b^2*c*e^2)^2 + 458466918
4*a^34*b^8*c^11*e^8*f^34*(a^2*c*f^2 - b^2*c*e^2)^2 - 309657600*a^36*b^6*c^1
1*e^6*f^36*(a^2*c*f^2 - b^2*c*e^2)^2)))*(236196*b^36*c^8*e^36*(b^2*c*e^2 -
a^2*c*f^2)^(11/2) - 385875968*a^32*c^6*f^32*(b^2*c*e^2 - a^2*c*f^2)^(15/2)
- 419430400*a^34*c^7*f^34*(b^2*c*e^2 - a^2*c*f^2)^(13/2) - 150994944*a^36*c
^8*f^36*(b^2*c*e^2 - a^2*c*f^2)^(11/2) - 117440512*a^30*c^5*f^30*(b^2*c*e^2
 - a^2*c*f^2)^(17/2) - 1102248*b^38*c^9*e^38*(b^2*c*e^2 - a^2*c*f^2)^(9/2)
+ 2053593*b^40*c^10*e^40*(b^2*c*e^2 - a^2*c*f^2)^(7/2) - 1909251*b^42*c^11*
e^42*(b^2*c*e^2 - a^2*c*f^2)^(5/2) + 885735*b^44*c^12*e^44*(b^2*c*e^2 - a^2
*c*f^2)^(3/2) - 164025*b^46*c^13*e^46*(b^2*c*e^2 - a^2*c*f^2)^(1/2) - 92897
28*a^6*b^24*c^5*e^24*f^6*(b^2*c*e^2 - a^2*c*f^2)^(17/2) + 36884480*a^8*b^22
*c^5*e^22*f^8*(b^2*c*e^2 - a^2*c*f^2)^(17/2) + 278604800*a^10*b^20*c^5*e^20
*f^10*(b^2*c*e^2 - a^2*c*f^2)^(17/2) - 2774483200*a^12*b^18*c^5*e^18*f^12*(
b^2*c*e^2 - a^2*c*f^2)^(17/2) + 10869657600*a^14*b^16*c^5*e^16*f^14*(b^2*c*
e^2 - a^2*c*f^2)^(17/2) - 25237416960*a^16*b^14*c^5*e^14*f^16*(b^2*c*e^2 -
a^2*c*f^2)^(17/2) + 38348909568*a^18*b^12*c^5*e^12*f^18*(b^2*c*e^2 - a^2*c*
f^2)^(17/2) - 39084659712*a^20*b^10*c^5*e^10*f^20*(b^2*c*e^2 - a^2*c*f^2)^(
17/2) + 26118635520*a^22*b^8*c^5*e^8*f^22*(b^2*c*e^2 - a^2*c*f^2)^(17/2) -
10414620672*a^24*b^6*c^5*e^6*f^24*(b^2*c*e^2 - a^2*c*f^2)^(17/2) + 17086545
92*a^26*b^4*c^5*e^4*f^26*(b^2*c*e^2 - a^2*c*f^2)^(17/2) + 276561920*a^28*b^
2*c^5*e^2*f^28*(b^2*c*e^2 - a^2*c*f^2)^(17/2) - 9704448*a^4*b^28*c^6*e^28*f
^4*(b^2*c*e^2 - a^2*c*f^2)^(15/2) + 260614656*a^6*b^26*c^6*e^26*f^6*(b^2*c*
e^2 - a^2*c*f^2)^(15/2) - 2166022464*a^8*b^24*c^6*e^24*f^8*(b^2*c*e^2 - a^2
*c*f^2)^(15/2) + 8626147840*a^10*b^22*c^6*e^22*f^10*(b^2*c*e^2 - a^2*c*f^2)
^(15/2) - 16771503616*a^12*b^20*c^6*e^20*f^12*(b^2*c*e^2 - a^2*c*f^2)^(15/2
) + 3301800960*a^14*b^18*c^6*e^18*f^14*(b^2*c*e^2 - a^2*c*f^2)^(15/2) + 673
37715968*a^16*b^16*c^6*e^16*f^16*(b^2*c*e^2 - a^2*c*f^2)^(15/2) - 189857873
920*a^18*b^14*c^6*e^14*f^18*(b^2*c*e^2 - a^2*c*f^2)^(15/2) + 286100259840*a
^20*b^12*c^6*e^12*f^20*(b^2*c*e^2 - a^2*c*f^2)^(15/2) - 275789894656*a^22*b
^10*c^6*e^10*f^22*(b^2*c*e^2 - a^2*c*f^2)^(15/2) + 173716537344*a^24*b^8*c^
6*e^8*f^24*(b^2*c*e^2 - a^2*c*f^2)^(15/2) - 67416424448*a^26*b^6*c^6*e^6*f^
26*(b^2*c*e^2 - a^2*c*f^2)^(15/2) + 12831686656*a^28*b^4*c^6*e^4*f^28*(b^2*
c*e^2 - a^2*c*f^2)^(15/2) + 222560256*a^30*b^2*c^6*e^2*f^30*(b^2*c*e^2 - a^
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2*c*f^2)^(15/2) - 2099520*a^2*b^32*c^7*e^32*f^2*(b^2*c*e^2 - a^2*c*f^2)^(13
/2) + 107014608*a^4*b^30*c^7*e^30*f^4*(b^2*c*e^2 - a^2*c*f^2)^(13/2) - 1848
335616*a^6*b^28*c^7*e^28*f^6*(b^2*c*e^2 - a^2*c*f^2)^(13/2) + 15200005312*a
^8*b^26*c^7*e^26*f^8*(b^2*c*e^2 - a^2*c*f^2)^(13/2) - 72612273792*a^10*b^24
*c^7*e^24*f^10*(b^2*c*e^2 - a^2*c*f^2)^(13/2) + 221855779968*a^12*b^22*c^7*
e^22*f^12*(b^2*c*e^2 - a^2*c*f^2)^(13/2) - 450717857536*a^14*b^20*c^7*e^20*
f^14*(b^2*c*e^2 - a^2*c*f^2)^(13/2) + 600578910208*a^16*b^18*c^7*e^18*f^16*
(b^2*c*e^2 - a^2*c*f^2)^(13/2) - 459464530688*a^18*b^16*c^7*e^16*f^18*(b^2*
c*e^2 - a^2*c*f^2)^(13/2) + 33638947840*a^20*b^14*c^7*e^14*f^20*(b^2*c*e^2
 - a^2*c*f^2)^(13/2) + 376299926528*a^22*b^12*c^7*e^12*f^22*(b^2*c*e^2 - a^2
*c*f^2)^(13/2) - 488874068992*a^24*b^10*c^7*e^10*f^24*(b^2*c*e^2 - a^2*c*f^
2)^(13/2) + 333407809536*a^26*b^8*c^7*e^8*f^26*(b^2*c*e^2 - a^2*c*f^2)^(13/
2) - 134140313600*a^28*b^6*c^7*e^6*f^28*(b^2*c*e^2 - a^2*c*f^2)^(13/2) + 28
220915712*a^30*b^4*c^7*e^4*f^30*(b^2*c*e^2 - a^2*c*f^2)^(13/2) - 1230503936
*a^32*b^2*c^7*e^2*f^32*(b^2*c*e^2 - a^2*c*f^2)^(13/2) + 3335904*a^2*b^34*c^
8*e^34*f^2*(b^2*c*e^2 - a^2*c*f^2)^(11/2) - 290521728*a^4*b^32*c^8*e^32*f^4
*(b^2*c*e^2 - a^2*c*f^2)^(11/2) + 4865684544*a^6*b^30*c^8*e^30*f^6*(b^2*c*e
^2 - a^2*c*f^2)^(11/2) - 40437394528*a^8*b^28*c^8*e^28*f^8*(b^2*c*e^2 - a^2
*c*f^2)^(11/2) + 205602254656*a^10*b^26*c^8*e^26*f^10*(b^2*c*e^2 - a^2*c*f^
2)^(11/2) - 703885344192*a^12*b^24*c^8*e^24*f^12*(b^2*c*e^2 - a^2*c*f^2)^(1
1/2) + 1709253482624*a^14*b^22*c^8*e^22*f^14*(b^2*c*e^2 - a^2*c*f^2)^(11/2)
 - 3029282695168*a^16*b^20*c^8*e^20*f^16*(b^2*c*e^2 - a^2*c*f^2)^(11/2) + 3
966230827520*a^18*b^18*c^8*e^18*f^18*(b^2*c*e^2 - a^2*c*f^2)^(11/2) - 38223
39813632*a^20*b^16*c^8*e^16*f^20*(b^2*c*e^2 - a^2*c*f^2)^(11/2) + 264043805
6960*a^22*b^14*c^8*e^14*f^22*(b^2*c*e^2 - a^2*c*f^2)^(11/2) - 1208501415936
*a^24*b^12*c^8*e^12*f^24*(b^2*c*e^2 - a^2*c*f^2)^(11/2) + 269338092544*a^26
*b^10*c^8*e^10*f^26*(b^2*c*e^2 - a^2*c*f^2)^(11/2) + 53783212032*a^28*b^8*c
^8*e^8*f^28*(b^2*c*e^2 - a^2*c*f^2)^(11/2) - 60985360384*a^30*b^6*c^8*e^6*f
^30*(b^2*c*e^2 - a^2*c*f^2)^(11/2) + 17917083648*a^32*b^4*c^8*e^4*f^32*(b^2
*c*e^2 - a^2*c*f^2)^(11/2) - 1558708224*a^34*b^2*c^8*e^2*f^34*(b^2*c*e^2 -
a^2*c*f^2)^(11/2) + 11917692*a^2*b^36*c^9*e^36*f^2*(b^2*c*e^2 - a^2*c*f^2)^
(9/2) + 224907516*a^4*b^34*c^9*e^34*f^4*(b^2*c*e^2 - a^2*c*f^2)^(9/2) - 530
3932560*a^6*b^32*c^9*e^32*f^6*(b^2*c*e^2 - a^2*c*f^2)^(9/2) + 48206418480*a
^8*b^30*c^9*e^30*f^8*(b^2*c*e^2 - a^2*c*f^2)^(9/2) - 261450609120*a^10*b^28
*c^9*e^28*f^10*(b^2*c*e^2 - a^2*c*f^2)^(9/2) + 962361040256*a^12*b^26*c^9*e
^26*f^12*(b^2*c*e^2 - a^2*c*f^2)^(9/2) - 2558559358080*a^14*b^24*c^9*e^24*f
^14*(b^2*c*e^2 - a^2*c*f^2)^(9/2) + 5091804150656*a^16*b^22*c^9*e^22*f^16*(
b^2*c*e^2 - a^2*c*f^2)^(9/2) - 7750806514944*a^18*b^20*c^9*e^20*f^18*(b^2*c
*e^2 - a^2*c*f^2)^(9/2) + 9137207485952*a^20*b^18*c^9*e^18*f^20*(b^2*c*e^2
 - a^2*c*f^2)^(9/2) - 8384563280128*a^22*b^16*c^9*e^16*f^22*(b^2*c*e^2 - a^2
*c*f^2)^(9/2) + 5975281259520*a^24*b^14*c^9*e^14*f^24*(b^2*c*e^2 - a^2*c*f^
2)^(9/2) - 3269297268736*a^26*b^12*c^9*e^12*f^26*(b^2*c*e^2 - a^2*c*f^2)^(9
/2) + 1339171540992*a^28*b^10*c^9*e^10*f^28*(b^2*c*e^2 - a^2*c*f^2)^(9/2) -
 391250194432*a^30*b^8*c^9*e^8*f^30*(b^2*c*e^2 - a^2*c*f^2)^(9/2) + 7411415
4496*a^32*b^6*c^9*e^6*f^32*(b^2*c*e^2 - a^2*c*f^2)^(9/2) - 7299203072*a^34*
b^4*c^9*e^4*f^34*(b^2*c*e^2 - a^2*c*f^2)^(9/2) + 148635648*a^36*b^2*c^9*e^2
*f^36*(b^2*c*e^2 - a^2*c*f^2)^(9/2) - 38704068*a^2*b^38*c^10*e^38*f^2*(b^2*
c*e^2 - a^2*c*f^2)^(7/2) + 188845992*a^4*b^36*c^10*e^36*f^4*(b^2*c*e^2 - a^
2*c*f^2)^(7/2) + 1157124204*a^6*b^34*c^10*e^34*f^6*(b^2*c*e^2 - a^2*c*f^2)^
(7/2) - 20586361424*a^8*b^32*c^10*e^32*f^8*(b^2*c*e^2 - a^2*c*f^2)^(7/2) +
135395499200*a^10*b^30*c^10*e^30*f^10*(b^2*c*e^2 - a^2*c*f^2)^(7/2) - 55551
3858464*a^12*b^28*c^10*e^28*f^12*(b^2*c*e^2 - a^2*c*f^2)^(7/2) + 1608776388
864*a^14*b^26*c^10*e^26*f^14*(b^2*c*e^2 - a^2*c*f^2)^(7/2) - 3473989271488*
a^16*b^24*c^10*e^24*f^16*(b^2*c*e^2 - a^2*c*f^2)^(7/2) + 5766181411456*a^18
*b^22*c^10*e^22*f^18*(b^2*c*e^2 - a^2*c*f^2)^(7/2) - 7493983209472*a^20*b^2
0*c^10*e^20*f^20*(b^2*c*e^2 - a^2*c*f^2)^(7/2) + 7713917084672*a^22*b^18*c^
10*e^18*f^22*(b^2*c*e^2 - a^2*c*f^2)^(7/2) - 6328467293184*a^24*b^16*c^10*e
^16*f^24*(b^2*c*e^2 - a^2*c*f^2)^(7/2) + 4142950034432*a^26*b^14*c^10*e^14*
f^26*(b^2*c*e^2 - a^2*c*f^2)^(7/2) - 2152681536512*a^28*b^12*c^10*e^12*f^28
```

```
*(b^2*c*e^2 - a^2*c*f^2)^(7/2) + 874199511040*a^30*b^10*c^10*e^10*f^30*(b^2
*c*e^2 - a^2*c*f^2)^(7/2) - 268759150592*a^32*b^8*c^10*e^8*f^32*(b^2*c*e^2
- a^2*c*f^2)^(7/2) + 58872545280*a^34*b^6*c^10*e^6*f^34*(b^2*c*e^2 - a^2*c*
f^2)^(7/2) - 8151957504*a^36*b^4*c^10*e^4*f^36*(b^2*c*e^2 - a^2*c*f^2)^(7/2
) + 530841600*a^38*b^2*c^10*e^2*f^38*(b^2*c*e^2 - a^2*c*f^2)^(7/2) + 427434
57*a^2*b^40*c^11*e^40*f^2*(b^2*c*e^2 - a^2*c*f^2)^(5/2) - 411055884*a^4*b^3
8*c^11*e^38*f^4*(b^2*c*e^2 - a^2*c*f^2)^(5/2) + 2180887236*a^6*b^36*c^11*e^
36*f^6*(b^2*c*e^2 - a^2*c*f^2)^(5/2) - 6404946508*a^8*b^34*c^11*e^34*f^8*(b
^2*c*e^2 - a^2*c*f^2)^(5/2) + 5434005264*a^10*b^32*c^11*e^32*f^10*(b^2*c*e^
2 - a^2*c*f^2)^(5/2) + 38868373520*a^12*b^30*c^11*e^30*f^12*(b^2*c*e^2 - a^
2*c*f^2)^(5/2) - 208447613600*a^14*b^28*c^11*e^28*f^14*(b^2*c*e^2 - a^2*c*f
^2)^(5/2) + 579674999104*a^16*b^26*c^11*e^26*f^16*(b^2*c*e^2 - a^2*c*f^2)^(
5/2) - 1104967566592*a^18*b^24*c^11*e^24*f^18*(b^2*c*e^2 - a^2*c*f^2)^(5/2)
 + 1554566531328*a^20*b^22*c^11*e^22*f^20*(b^2*c*e^2 - a^2*c*f^2)^(5/2) - 1
659734381312*a^22*b^20*c^11*e^20*f^22*(b^2*c*e^2 - a^2*c*f^2)^(5/2) + 13563
61512192*a^24*b^18*c^11*e^18*f^24*(b^2*c*e^2 - a^2*c*f^2)^(5/2) - 845331359
744*a^26*b^16*c^11*e^16*f^26*(b^2*c*e^2 - a^2*c*f^2)^(5/2) + 395676895232*a
^28*b^14*c^11*e^14*f^28*(b^2*c*e^2 - a^2*c*f^2)^(5/2) - 134902689792*a^30*b
^12*c^11*e^12*f^30*(b^2*c*e^2 - a^2*c*f^2)^(5/2) + 31670587392*a^32*b^10*c^
11*e^10*f^32*(b^2*c*e^2 - a^2*c*f^2)^(5/2) - 4584669184*a^34*b^8*c^11*e^8*f
^34*(b^2*c*e^2 - a^2*c*f^2)^(5/2) + 309657600*a^36*b^6*c^11*e^6*f^36*(b^2*c
*e^2 - a^2*c*f^2)^(5/2) - 21130794*a^2*b^42*c^12*e^42*f^2*(b^2*c*e^2 - a^2*
c*f^2)^(3/2) + 234399015*a^4*b^40*c^12*e^40*f^4*(b^2*c*e^2 - a^2*c*f^2)^(3/
2) - 1604168280*a^6*b^38*c^12*e^38*f^6*(b^2*c*e^2 - a^2*c*f^2)^(3/2) + 7579
098492*a^8*b^36*c^12*e^36*f^8*(b^2*c*e^2 - a^2*c*f^2)^(3/2) - 26212380172*a
^10*b^34*c^12*e^34*f^10*(b^2*c*e^2 - a^2*c*f^2)^(3/2) + 68672994096*a^12*b^
32*c^12*e^32*f^12*(b^2*c*e^2 - a^2*c*f^2)^(3/2) - 139160589504*a^14*b^30*c^
12*e^30*f^14*(b^2*c*e^2 - a^2*c*f^2)^(3/2) + 220859191808*a^16*b^28*c^12*e^
28*f^16*(b^2*c*e^2 - a^2*c*f^2)^(3/2) - 276344315328*a^18*b^26*c^12*e^26*f^
18*(b^2*c*e^2 - a^2*c*f^2)^(3/2) + 273130561984*a^20*b^24*c^12*e^24*f^20*(b
^2*c*e^2 - a^2*c*f^2)^(3/2) - 212730002688*a^22*b^22*c^12*e^22*f^22*(b^2*c*
e^2 - a^2*c*f^2)^(3/2) + 129574234368*a^24*b^20*c^12*e^20*f^24*(b^2*c*e^2 -
 a^2*c*f^2)^(3/2) - 60770569216*a^26*b^18*c^12*e^18*f^26*(b^2*c*e^2 - a^2*c
*f^2)^(3/2) + 21304706048*a^28*b^16*c^12*e^16*f^28*(b^2*c*e^2 - a^2*c*f^2)^
(3/2) - 5272965120*a^30*b^14*c^12*e^14*f^30*(b^2*c*e^2 - a^2*c*f^2)^(3/2) +
 819441664*a^32*b^12*c^12*e^12*f^32*(b^2*c*e^2 - a^2*c*f^2)^(3/2) - 5939200
0*a^34*b^10*c^12*e^10*f^34*(b^2*c*e^2 - a^2*c*f^2)^(3/2) + 3937329*a^2*b^44
*c^13*e^44*f^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2) - 43893819*a^4*b^42*c^13*e^42*
f^4*(b^2*c*e^2 - a^2*c*f^2)^(1/2) + 301507155*a^6*b^40*c^13*e^40*f^6*(b^2*c
*e^2 - a^2*c*f^2)^(1/2) - 1427514656*a^8*b^38*c^13*e^38*f^8*(b^2*c*e^2 - a^
2*c*f^2)^(1/2) + 4936911112*a^10*b^36*c^13*e^36*f^10*(b^2*c*e^2 - a^2*c*f^2
)^(1/2) - 12893273616*a^12*b^34*c^13*e^34*f^12*(b^2*c*e^2 - a^2*c*f^2)^(1/2
) + 25921630432*a^14*b^32*c^13*e^32*f^14*(b^2*c*e^2 - a^2*c*f^2)^(1/2) - 40
519286096*a^16*b^30*c^13*e^30*f^16*(b^2*c*e^2 - a^2*c*f^2)^(1/2) + 49376608
256*a^18*b^28*c^13*e^28*f^18*(b^2*c*e^2 - a^2*c*f^2)^(1/2) - 46721401856*a^
20*b^26*c^13*e^26*f^20*(b^2*c*e^2 - a^2*c*f^2)^(1/2) + 33946324736*a^22*b^2
4*c^13*e^24*f^22*(b^2*c*e^2 - a^2*c*f^2)^(1/2) - 18556579328*a^24*b^22*c^13
*e^22*f^24*(b^2*c*e^2 - a^2*c*f^2)^(1/2) + 7375276032*a^26*b^20*c^13*e^20*f
^26*(b^2*c*e^2 - a^2*c*f^2)^(1/2) - 2009817088*a^28*b^18*c^13*e^18*f^28*(b^
2*c*e^2 - a^2*c*f^2)^(1/2) + 335642624*a^30*b^16*c^13*e^16*f^30*(b^2*c*e^2
- a^2*c*f^2)^(1/2) - 25907200*a^32*b^14*c^13*e^14*f^32*(b^2*c*e^2 - a^2*c*f
^2)^(1/2)))/(16384*a^(17/2)*b^19*c*e^19*f^15*(a*c)^(13/2) - 2048*a^(13/2)*b
^21*c*e^21*f^13*(a*c)^(13/2) - 57344*a^(21/2)*b^17*c*e^17*f^17*(a*c)^(13/2)
 + 114688*a^(25/2)*b^15*c*e^15*f^19*(a*c)^(13/2) - 143360*a^(29/2)*b^13*c*e
^13*f^21*(a*c)^(13/2) + 114688*a^(33/2)*b^11*c*e^11*f^23*(a*c)^(13/2) - 573
44*a^(37/2)*b^9*c*e^9*f^25*(a*c)^(13/2) + 16384*a^(41/2)*b^7*c*e^7*f^27*(a*
c)^(13/2) - 2048*a^(45/2)*b^5*c*e^5*f^29*(a*c)^(13/2) + 486*a^(3/2)*b^31*c^
6*e^31*f^3*(a*c)^(3/2) - 3240*a^(5/2)*b^29*c^5*e^29*f^5*(a*c)^(5/2) + 8640*
a^(7/2)*b^27*c^4*e^27*f^7*(a*c)^(7/2) - 2592*a^(7/2)*b^29*c^6*e^29*f^5*(a*c
```

```
)^(3/2) - 11520*a^(9/2)*b^25*c^3*e^25*f^9*(a*c)^(9/2) + 19008*a^(9/2)*b^27*
c^5*e^27*f^7*(a*c)^(5/2) + 7680*a^(11/2)*b^23*c^2*e^23*f^11*(a*c)^(11/2) -
55296*a^(11/2)*b^25*c^4*e^25*f^9*(a*c)^(7/2) + 5184*a^(11/2)*b^27*c^6*e^27*
f^7*(a*c)^(3/2) + 79872*a^(13/2)*b^23*c^3*e^23*f^11*(a*c)^(9/2) - 44064*a^(
13/2)*b^25*c^5*e^25*f^9*(a*c)^(5/2) - 57344*a^(15/2)*b^21*c^2*e^21*f^13*(a*
c)^(11/2) + 145152*a^(15/2)*b^23*c^4*e^23*f^11*(a*c)^(7/2) - 4608*a^(15/2)*
b^25*c^6*e^25*f^9*(a*c)^(3/2) - 233472*a^(17/2)*b^21*c^3*e^21*f^13*(a*c)^(9
/2) + 50304*a^(17/2)*b^23*c^5*e^23*f^11*(a*c)^(5/2) + 184320*a^(19/2)*b^19*
c^2*e^19*f^15*(a*c)^(11/2) - 199424*a^(19/2)*b^21*c^4*e^21*f^13*(a*c)^(7/2)
 + 1536*a^(19/2)*b^23*c^6*e^23*f^11*(a*c)^(3/2) + 371712*a^(21/2)*b^19*c^3*
e^19*f^15*(a*c)^(9/2) - 28160*a^(21/2)*b^21*c^5*e^21*f^13*(a*c)^(5/2) - 331
776*a^(23/2)*b^17*c^2*e^17*f^17*(a*c)^(11/2) + 150592*a^(23/2)*b^19*c^4*e^1
9*f^15*(a*c)^(7/2) - 346368*a^(25/2)*b^17*c^3*e^17*f^17*(a*c)^(9/2) + 6144*
a^(25/2)*b^19*c^5*e^19*f^15*(a*c)^(5/2) + 363520*a^(27/2)*b^15*c^2*e^15*f^1
9*(a*c)^(11/2) - 58880*a^(27/2)*b^17*c^4*e^17*f^17*(a*c)^(7/2) + 187392*a^(
29/2)*b^15*c^3*e^15*f^19*(a*c)^(9/2) - 245760*a^(31/2)*b^13*c^2*e^13*f^21*(
a*c)^(11/2) + 9216*a^(31/2)*b^15*c^4*e^15*f^19*(a*c)^(7/2) - 53760*a^(33/2)
*b^13*c^3*e^13*f^21*(a*c)^(9/2) + 98304*a^(35/2)*b^11*c^2*e^11*f^23*(a*c)^(
11/2) + 6144*a^(37/2)*b^11*c^3*e^11*f^23*(a*c)^(9/2) - 20480*a^(39/2)*b^9*c
^2*e^9*f^25*(a*c)^(11/2) + 1536*a^(43/2)*b^7*c^2*e^7*f^27*(a*c)^(11/2)))))/
(f^2*(a*f + b*e)*(a*f - b*e)*(b^2*c*e^2 - a^2*c*f^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**2/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

3.33 $\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\sqrt{ac-bcx}(e+fx)^3}\,dx$

Optimal. Leaf size=363

$$\frac{\left(a^2-b^2x^2\right)\left(2a^2f^2(2Ce-Bf)-b^2e\left(f(Be-3Af)+Ce^2\right)\right)}{2f\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)^2}+\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2\sqrt{a+bx}(e+fx)^2\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2\cdots}}{\cdots}$$

Rubi [A] time = 0.59, antiderivative size = 361, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1610, 1651, 807, 725, 204}

$$\frac{\left(a^2-b^2x^2\right)\left(2a^2f^2(2Ce-Bf)-b^2\left(ef(Be-3Af)+Ce^3\right)\right)}{2f\sqrt{a+bx}(e+fx)\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)^2}+\frac{f\left(a^2-b^2x^2\right)\left(A+\frac{e(Ce-Bf)}{f^2}\right)}{2\sqrt{a+bx}(e+fx)^2\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)}+\frac{\sqrt{a^2c-b^2cx^2}\left(A\left(a^2b^2f^2+2b^4e^2\right)+a^2b^2e(Ce-3Bf)+2a^4Cf^2\right)\tan^{-1}\left(\frac{\sqrt{c}\left(a^2f+b^2ex\right)}{\sqrt{a^2c-b^2cx^2}\sqrt{b^2e^2-a^2f^2}}\right)}{2\sqrt{c}\sqrt{a+bx}\sqrt{ac-bcx}\left(b^2e^2-a^2f^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x]

[Out] (f*(A + (e*(C*e - B*f))/f^2)*(a^2 - b^2*x^2))/(2*(b^2*e^2 - a^2*f^2)*Sqrt[a
 + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^2) + ((2*a^2*f^2*(2*C*e - B*f) - b^2*(C
*e^3 + e*f*(B*e - 3*A*f)))*(a^2 - b^2*x^2))/(2*f*(b^2*e^2 - a^2*f^2)^2*Sqrt
[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)) + ((2*a^4*C*f^2 + a^2*b^2*e*(C*e - 3
*B*f) + A*(2*b^4*e^2 + a^2*b^2*f^2))*Sqrt[a^2*c - b^2*c*x^2]*ArcTan[(Sqrt[c
]*(a^2*f + b^2*e*x))/(Sqrt[b^2*e^2 - a^2*f^2]*Sqrt[a^2*c - b^2*c*x^2])])/(2
*Sqrt[c]*(b^2*e^2 - a^2*f^2)^(5/2)*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
```

```
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^3}\,dx = \frac{\sqrt{a^2c-b^2cx^2}\int \frac{A+Bx+Cx^2}{(e+fx)^3\sqrt{a^2c-b^2cx^2}}\,dx}{\sqrt{a+bx}\,\sqrt{ac-bcx}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2} + \frac{\sqrt{a^2c-b^2cx^2}\int \frac{2c\left(Ab^2e+a^2(Ce\right.}{}}{2c\left(b^2e^2-a^2f^2\right)}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2\left(Ce^3\right.\right.}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2\left(Ce^3\right.\right.}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a}}$$

$$= \frac{f\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(a^2-b^2x^2\right)}{2\left(b^2e^2-a^2f^2\right)\sqrt{a+bx}\,\sqrt{ac-bcx}\,(e+fx)^2} + \frac{\left(2a^2f^2(2Ce-Bf)-b^2\left(Ce^3\right.\right.}{2f\left(b^2e^2-a^2f^2\right)^2\sqrt{a}}$$

Mathematica [A] time = 1.31, size = 492, normalized size = 1.36

$$\frac{\frac{b^2\sqrt{a-bx}\left(f(Af-Be)+Ce^2\right)\left(2(e+fx)\left(a^2f^2+2b^2e^2\right)\tanh^{-1}\left(\frac{\sqrt{a-bx}\,\sqrt{be-af}}{\sqrt{a+bx}\,\sqrt{-af-be}}\right)+3ef\sqrt{a-bx}\,\sqrt{a+bx}\,\sqrt{-af-be}\,\sqrt{be-af}\right)}{(e+fx)(-af-be)^{5/2}(be-af)^{5/2}}+\frac{2f(bx-a)\sqrt{a+bx}\,(Bf-2Ce)}{(e+fx)\left(a^2f^2-b^2e^2\right)}+\frac{f(bx-a)\sqrt{a+bx}\left(f(Af-Be)+Ce^2\right)}{(e+fx)^2(af-be)(af+be)}+\frac{4b^2e\sqrt{a-bx}\,(2Ce-Bf)\tanh^{-1}\left(\frac{\sqrt{a-bx}\,\sqrt{be-af}}{\sqrt{a+bx}\,\sqrt{-af-be}}\right)}{(-af-be)^{3/2}(be-af)^{3/2}}+\frac{4C\sqrt{a-bx}\,\tanh^{-1}\left(\frac{\sqrt{a-bx}\,\sqrt{be-af}}{\sqrt{a+bx}\,\sqrt{-af-be}}\right)}{\sqrt{-af-be}\,\sqrt{be-af}}}{2f^2\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e + f*x)^3),x
]
```

```
[Out] ((f*(C*e^2 + f*(-(B*e) + A*f))*(-a + b*x)*Sqrt[a + b*x])/((-(b*e) + a*f)*(b
*e + a*f)*(e + f*x)^2) + (2*f*(-2*C*e + B*f)*(-a + b*x)*Sqrt[a + b*x])/((-(
b^2*e^2) + a^2*f^2)*(e + f*x)) + (4*C*Sqrt[a - b*x]*ArcTanh[(Sqrt[b*e - a*f
]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])]])/(Sqrt[-(b*e) - a*f]*S
qrt[b*e - a*f]) + (4*b^2*e*(2*C*e - B*f)*Sqrt[a - b*x]*ArcTanh[(Sqrt[b*e -
a*f]*Sqrt[a - b*x])/(Sqrt[-(b*e) - a*f]*Sqrt[a + b*x])]])/((-(b*e) - a*f)^(3
/2)*(b*e - a*f)^(3/2)) + (b^2*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[a - b*x]*(3*e
*f*Sqrt[-(b*e) - a*f]*Sqrt[b*e - a*f]*Sqrt[a - b*x]*Sqrt[a + b*x] + 2*(2*b^
2*e^2 + a^2*f^2)*(e + f*x)*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[a - b*x])/(Sqrt[-(
b*e) - a*f]*Sqrt[a + b*x])]]))/((-(b*e) - a*f)^(5/2)*(b*e - a*f)^(5/2)*(e +
f*x)))/(2*f^2*Sqrt[c*(a - b*x)])
```

IntegrateAlgebraic [A] time = 0.00, size = 610, normalized size = 1.68

$$\frac{\left(-2a^4Cf^2-a^2Ab^2f^2+3a^2b^2Bef-a^2b^2Ce^2-2Ab^4e^2\right)\tanh^{-1}\left(\frac{\sqrt{ac-bcx}\,\sqrt{af-be}}{\sqrt{c}\,\sqrt{a+bx}\,\sqrt{af+be}}\right)}{\sqrt{c}\,(be-af)^2\sqrt{af-be}\,(af+be)^{5/2}}-\frac{ab\sqrt{ac-bcx}\left(-\frac{2a^3Bf^3(ac-bcx)}{a+bx}+\frac{4a^3Cef^2(ac-bcx)}{a+bx}+2a^3Bcf^3-4a^3cCef^2+\frac{a^2Abf^3(ac-bcx)}{a+bx}+a^2Abcf^3+\frac{a^2bBef^2(ac-bcx)}{a+bx}+a^2bBcef^2-\frac{3a^2bCe^2f(ac-bcx)}{a+bx}-3a^2bcCe^2f-\frac{4Ab^3e^2f(ac-bcx)}{a+bx}+\frac{3aAb^2ef^2(ac-bcx)}{a+bx}-3aAb^2cef^2+\frac{2b^3Be^3(ac-bcx)}{a+bx}-\frac{ab^2Be^2f(ac-bcx)}{a+bx}+ab^2Bce^2f-\frac{ab^2Ce^3(ac-bcx)}{a+bx}+ab^2cCe^3-4Ab^3ce^2f+2b^3Bce^3\right)}{\sqrt{a+bx}\,(be-af)^2(af+be)^2\left(\frac{be(ac-bcx)}{a+bx}-\frac{af(ac-bcx)}{a+bx}+acf+bce\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]*(e +
f*x)^3),x]
```

```
[Out] -((a*b*Sqrt[a*c - b*c*x]*(2*b^3*B*c*e^3 + a*b^2*c*C*e^3 - 4*A*b^3*c*e^2*f +
 a*b^2*B*c*e^2*f - 3*a^2*b*c*C*e^2*f - 3*a*A*b^2*c*e*f^2 + a^2*b*B*c*e*f^2
- 4*a^3*c*C*e*f^2 + a^2*A*b*c*f^3 + 2*a^3*B*c*f^3 + (2*b^3*B*e^3*(a*c - b*c
*x))/(a + b*x) - (a*b^2*C*e^3*(a*c - b*c*x))/(a + b*x) - (4*A*b^3*e^2*f*(a*
c - b*c*x))/(a + b*x) - (a*b^2*B*e^2*f*(a*c - b*c*x))/(a + b*x) - (3*a^2*b*
C*e^2*f*(a*c - b*c*x))/(a + b*x) + (3*a*A*b^2*e*f^2*(a*c - b*c*x))/(a + b*x
) + (a^2*b*B*e*f^2*(a*c - b*c*x))/(a + b*x) + (4*a^3*C*e*f^2*(a*c - b*c*x))
/(a + b*x) + (a^2*A*b*f^3*(a*c - b*c*x))/(a + b*x) - (2*a^3*B*f^3*(a*c - b*
c*x))/(a + b*x)))/((b*e - a*f)^2*(b*e + a*f)^2*Sqrt[a + b*x]*(b*c*e + a*c*f
 + (b*e*(a*c - b*c*x))/(a + b*x) - (a*f*(a*c - b*c*x))/(a + b*x))^2)) + ((-
2*A*b^4*e^2 - a^2*b^2*C*e^2 + 3*a^2*b^2*B*e*f - a^2*A*b^2*f^2 - 2*a^4*C*f^2
)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[a*c - b*c*x])/(Sqrt[c]*Sqrt[b*e + a*f]*S
qrt[a + b*x])])/(Sqrt[c]*(b*e - a*f)^2*Sqrt[-(b*e) + a*f]*(b*e + a*f)^(5/2)
)
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

giac [**B**] time = 9.49, size = 1658, normalized size = 4.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="giac")
```

```
[Out] -(2*C*a^4*sqrt(-c)*c^2*f^2 + A*a^2*b^2*sqrt(-c)*c^2*f^2 - 3*B*a^2*b^2*sqrt(
-c)*c^2*f*e + C*a^2*b^2*sqrt(-c)*c^2*e^2 + 2*A*b^4*sqrt(-c)*c^2*e^2)*arctan
(1/2*(2*b*c^2*e + (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*
c)*c))^2*f)/(sqrt(a^2*f^2 - b^2*e^2)*c^2))/((a^4*f^4*abs(c) - 2*a^2*b^2*f^2
*abs(c)*e^2 + b^4*abs(c)*e^4)*sqrt(a^2*f^2 - b^2*e^2)*c^2) + 2*(16*B*a^6*b*
sqrt(-c)*c^8*f^5 - 32*C*a^6*b*sqrt(-c)*c^8*f^4*e - 24*A*a^4*b^3*sqrt(-c)*c^
8*f^4*e + 4*A*a^4*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x
- a*c)*c))^2*sqrt(-c)*c^6*f^5 + 8*B*a^4*b^3*sqrt(-c)*c^8*f^3*e^2 + 20*B*a^4
*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt
(-c)*c^6*f^4*e + 4*B*a^4*b*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b
*c*x - a*c)*c))^4*sqrt(-c)*c^4*f^5 + 8*C*a^4*b^3*sqrt(-c)*c^8*f^2*e^3 - 44*
C*a^4*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2
*sqrt(-c)*c^6*f^3*e^2 - 40*A*a^2*b^4*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*
a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)*c^6*f^3*e^2 - 8*C*a^4*b*(sqrt(-b*c*x +
 a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sqrt(-c)*c^4*f^4*e - 6*
A*a^2*b^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4
*sqrt(-c)*c^4*f^4*e - A*a^2*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2
 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f^5 + 16*B*a^2*b^4*(sqrt(-b*c*x + a*c)*
sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)*c^6*f^2*e^3 + 10*B*a
^2*b^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sq
rt(-c)*c^4*f^3*e^2 + 3*B*a^2*b^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^
2 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f^4*e + 8*C*a^2*b^4*(sqrt(-b*c*x + a*c
)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)*c^6*f*e^4 - 14*C*a
^2*b^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sq
rt(-c)*c^4*f^2*e^3 - 12*A*b^5*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 +
 (b*c*x - a*c)*c))^4*sqrt(-c)*c^4*f^2*e^3 - 5*C*a^2*b^2*(sqrt(-b*c*x + a*c)
*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f^3*e^2 - 2*A*b
^4*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*sqrt(-
```

```
c)*c^2*f^3*e^2 + 4*B*b^5*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c
*x - a*c)*c))^4*sqrt(-c)*c^4*f*e^4 + 4*C*b^5*(sqrt(-b*c*x + a*c)*sqrt(-c) -
 sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*sqrt(-c)*c^4*e^5 + 2*C*b^4*(sqrt(-b*c*x
 + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*sqrt(-c)*c^2*f*e^4)/(
(a^4*f^6*abs(c) - 2*a^2*b^2*f^4*abs(c)*e^2 + b^4*f^2*abs(c)*e^4)*(4*a^2*c^4
*f + 4*b*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*
c^2*e + (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*f
)^2)
```

maple [B] time = 0.00, size = 1848, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] -1/2*(A*a^2*b^2*c*f^4*x^2*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)
^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))+2*A*b^4*c*e^2*f^2*x^2*ln(2*(b^2
*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/
(f*x+e))-3*B*a^2*b^2*c*e*f^3*x^2*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)
*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))+2*C*a^4*c*f^4*x^2*ln(2*(
b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*
f)/(f*x+e))+C*a^2*b^2*c*e^2*f^2*x^2*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e
^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))+2*A*a^2*b^2*c*e*f^3*x
*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)
^(1/2)*f)/(f*x+e))+4*A*b^4*c*e^3*f*x*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*
e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))-6*B*a^2*b^2*c*e^2*f^
2*x*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)
*c)^(1/2)*f)/(f*x+e))+4*C*a^4*c*e*f^3*x*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b
^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))+2*C*a^2*b^2*c*e^3
*f*x*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2
)*c)^(1/2)*f)/(f*x+e))+A*a^2*b^2*c*e^2*f^2*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^
2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))+2*A*b^4*c*e^4*
ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^
(1/2)*f)/(f*x+e))-3*B*a^2*b^2*c*e^3*f*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2
*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))+2*C*a^4*c*e^2*f^2*l
n(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(
1/2)*f)/(f*x+e))+C*a^2*b^2*c*e^4*ln(2*(b^2*c*e*x+a^2*c*f+((a^2*f^2-b^2*e^2)
*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*f)/(f*x+e))-3*((a^2*f^2-b^2*e^2)*c/f
^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*A*b^2*e*f^3*x+2*((a^2*f^2-b^2*e^2)*c/f^2
)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)*B*a^2*f^4*x+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2
)*(-(b^2*x^2-a^2)*c)^(1/2)*B*b^2*e^2*f^2*x-4*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2
)*(-(b^2*x^2-a^2)*c)^(1/2)*C*a^2*e*f^3*x+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-
(b^2*x^2-a^2)*c)^(1/2)*C*b^2*e^3*f*x+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2
*x^2-a^2)*c)^(1/2)*A*a^2*f^4-4*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a
^2)*c)^(1/2)*A*b^2*e^2*f^2+((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*
c)^(1/2)*B*a^2*e*f^3+2*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(
1/2)*B*b^2*e^3*f-3*((a^2*f^2-b^2*e^2)*c/f^2)^(1/2)*(-(b^2*x^2-a^2)*c)^(1/2)
*C*a^2*e^2*f^2)*(-(b*x-a)*c)^(1/2)*(b*x+a)^(1/2)/(-(b^2*x^2-a^2)*c)^(1/2)/(
a*f-b*e)/(a*f+b*e)/(a^2*f^2-b^2*e^2)/(f*x+e)^2/((a^2*f^2-b^2*e^2)*c/f^2)^(1
/2)/c/f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(f*x+e)^3/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*f-b*e>0)', see `assume?` for more
 details)Is a*f-b*e positive, negative or zero?
```

mupad [B] time = 0.01, size = 9344, normalized size = 25.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^3*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] ((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(4*C*a^4*c^3*f^2 + 2*C*a^2*b^2*c^3*e^
2))/(((a + b*x)^(1/2) - a^(1/2))*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a^4*b*e*f^4
)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(68*C*a^4*c^2*f^2 - 14*C*a^2*b^
2*c^2*e^2))/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^5 - 2*a^2*b^3*e^3*f^2 + a
^4*b*e*f^4)) - ((68*C*a^4*c*f^2 - 14*C*a^2*b^2*c*e^2)*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^5 - 2*a^2*b^3*e^3*f
^2 + a^4*b*e*f^4)) - ((4*C*a^4*f^2 + 2*C*a^2*b^2*e^2)*((a*c - b*c*x)^(1/2)
- (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(1/2))^7*(b^5*e^5 - 2*a^2*b^3*e^3*f
^2 + a^4*b*e*f^4)) - (a^(1/2)*(a*c)^(1/2)*(48*C*a^4*c*f^3 - 24*C*a^2*b^2*c*
e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(((a + b*x)^(1/2) - a^(1/2))^
4*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)) + (a^(1/2)*(a*c)^(1/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(24*C*a^4*f^3 + 12*C*a^2*b^2*e^2*f))/(
((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^6 - 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^
4)) + (a^(1/2)*(a*c)^(1/2)*(24*C*a^4*c^2*f^3 + 12*C*a^2*b^2*c^2*e^2*f)*((a*
c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^6
- 2*a^2*b^4*e^4*f^2 + a^4*b^2*e^2*f^4)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))^8/((a + b*x)^(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^6*(16*a^2*c*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6
) + ((16*a^2*c^3*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2
)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^
2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/
2))^4) - (8*a^(1/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b
*e*((a + b*x)^(1/2) - a^(1/2))^7) + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*
c*x)^(1/2) - (a*c)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c
*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2)
 - a^(1/2))^5) + (8*a^(1/2)*c^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^
(1/2))^3)/(b*e*((a + b*x)^(1/2) - a^(1/2))^3)) + (((4*A*a^4*f^4 - 10*A*a^2*
b^2*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(
1/2))^7*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*f^2)) - ((4*A*a^4*c^3*f^4
- 10*A*a^2*b^2*c^3*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(((a + b*x
)^(1/2) - a^(1/2))*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*f^2)) - ((4*A*a
^4*c^2*f^4 - 58*A*a^2*b^2*c^2*e^2*f^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^
3)/(((a + b*x)^(1/2) - a^(1/2))^3*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a^2*b^3*e^5*
f^2)) + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5*(4*A*a^4*c*f^4 - 58*A*a^2*b^
2*c*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/2))^5*(b^5*e^7 + a^4*b*e^3*f^4 - 2*a
^2*b^3*e^5*f^2)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))
^6*(16*A*b^4*e^4*f - 8*A*a^4*f^5 + 28*A*a^2*b^2*e^2*f^3))/(((a + b*x)^(1/2)
 - a^(1/2))^6*(b^6*e^8 - 2*a^2*b^4*e^6*f^2 + a^4*b^2*e^4*f^4)) + (a^(1/2)*(
a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4*(16*A*a^4*c*f^5 + 32*A*b^4
*c*e^4*f - 72*A*a^2*b^2*c*e^2*f^3))/(((a + b*x)^(1/2) - a^(1/2))^4*(b^6*e^8
 - 2*a^2*b^4*e^6*f^2 + a^4*b^2*e^4*f^4)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c
*x)^(1/2) - (a*c)^(1/2))^2*(16*A*b^4*c^2*e^4*f - 8*A*a^4*c^2*f^5 + 28*A*a^2
*b^2*c^2*e^2*f^3))/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^8 - 2*a^2*b^4*e^6*
f^2 + a^4*b^2*e^4*f^4)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8/((a + b*x)^
(1/2) - a^(1/2))^8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(16*a^2*c
*f^2 + 4*b^2*c*e^2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6) + ((16*a^2*c^3
*f^2 + 4*b^2*c^3*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*e^2*((a +
 b*x)^(1/2) - a^(1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^2)*((a*c - b*c*x
```

```
)^(1/2) - (a*c)^(1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^4) - (8*a^(1
/2)*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b*e*((a + b*x)^(1
/2) - a^(1/2))^7) + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*
c)^(1/2)))/(b*e*((a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c*f*(a*c)^(1/2)*(
(a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2) - a^(1/2))^5) +
 (8*a^(1/2)*c^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*(
(a + b*x)^(1/2) - a^(1/2))^3)) - (((32*B*a^4*c^2*f^3 + 22*B*a^2*b^2*c^2*e^2
*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(((a + b*x)^(1/2) - a^(1/2))^3*(
b^5*e^6 + a^4*b*e^2*f^4 - 2*a^2*b^3*e^4*f^2)) - ((32*B*a^4*c*f^3 + 22*B*a^2
*b^2*c*e^2*f)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^5)/(((a + b*x)^(1/2) - a^
(1/2))^5*(b^5*e^6 + a^4*b*e^2*f^4 - 2*a^2*b^3*e^4*f^2)) + (a^(1/2)*(a*c)^(1
/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2*(8*B*a^4*c^2*f^4 + 8*B*b^4*c^2*e^
4 + 20*B*a^2*b^2*c^2*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/2))^2*(b^6*e^7 - 2*
a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^4)) + (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(
1/2) - (a*c)^(1/2))^6*(8*B*a^4*f^4 + 8*B*b^4*e^4 + 20*B*a^2*b^2*e^2*f^2))/(
((a + b*x)^(1/2) - a^(1/2))^6*(b^6*e^7 - 2*a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^
4)) - (a^(1/2)*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^4*(16*B*a^4*
c*f^4 - 16*B*b^4*c*e^4 + 24*B*a^2*b^2*c*e^2*f^2))/(((a + b*x)^(1/2) - a^(1/
2))^4*(b^6*e^7 - 2*a^2*b^4*e^5*f^2 + a^4*b^2*e^3*f^4)) - (6*B*a^2*b*f*((a*c
 - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(((a + b*x)^(1/2) - a^(1/2))^7*(a^4*f^4 +
 b^4*e^4 - 2*a^2*b^2*e^2*f^2)) + (6*B*a^2*b*c^3*f*((a*c - b*c*x)^(1/2) - (a
*c)^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))*(a^4*f^4 + b^4*e^4 - 2*a^2*b^2*e^2
*f^2)))/(((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^8/((a + b*x)^(1/2) - a^(1/2))^
8 + c^4 + (((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^6*(16*a^2*c*f^2 + 4*b^2*c*e^
2))/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^6) + ((16*a^2*c^3*f^2 + 4*b^2*c^3*
e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/(b^2*e^2*((a + b*x)^(1/2) - a^(
1/2))^2) - ((32*a^2*c^2*f^2 - 6*b^2*c^2*e^2)*((a*c - b*c*x)^(1/2) - (a*c)^(
1/2))^4)/(b^2*e^2*((a + b*x)^(1/2) - a^(1/2))^4) - (8*a^(1/2)*f*(a*c)^(1/2)
*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^7)/(b*e*((a + b*x)^(1/2) - a^(1/2))^7)
 + (8*a^(1/2)*c^3*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/(b*e*(
(a + b*x)^(1/2) - a^(1/2))) - (8*a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/
2) - (a*c)^(1/2))^5)/(b*e*((a + b*x)^(1/2) - a^(1/2))^5) + (8*a^(1/2)*c^2*f
*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/(b*e*((a + b*x)^(1/2) -
 a^(1/2))^3)) + (C*a^2*(2*a^2*f^2 + b^2*e^2)*(2*atan(((((a*c - b*c*x)^(1/2)
 - (a*c)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (a^2
*c*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + 2
*a^(1/2)*b*c*e*f*(a*c)^(1/2))/(2*b*c*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2))) + 2*
atan((((((4*(4*C^2*a^8*f^4 + C^2*a^4*b^4*e^4 + 4*C^2*a^6*b^2*e^2*f^2))/(b^
10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b
^2*e^2*f^8) - (C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2*(12*a^10*c*f^10 - 4*b^10*c*e
^10 + 28*a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f^4 + 88*a^6*b^4*c*e^4*f^6 -
52*a^8*b^2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)
*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a
^8*b^2*e^2*f^8)))/(4*b*c^2*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (C*a^(3/2)*(2
*a^2*f^2 + b^2*e^2)*(8*C*a^(17/2)*f^7*(a*c)^(1/2) - 12*C*a^(13/2)*b^2*e^2*f
^5*(a*c)^(1/2) + 4*C*a^(5/2)*b^6*e^6*f*(a*c)^(1/2)))/(2*b*c^2*e*f*(a*c)^(1/
2)*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 - 4
*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)
))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 + (
((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(((4*(4*C^2*a^8*c*f^4 + C^2*a^4*b^4*c*e
^4 + 4*C^2*a^6*b^2*c*e^2*f^2))/(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e
^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8) + (C^2*a^4*(2*a^2*f^2 + b^2*e
^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c^2*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^4*b
^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^4*f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f + b
*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2
+ 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)))/(4*b*c^2*e*(b^
2*c*e^2 - a^2*c*f^2)^(1/2)) + (8*C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2)/(b*e*(a*f
 + b*e)^4*(a*f - b*e)^4*(b^2*c*e^2 - a^2*c*f^2)^(3/2)) - (C*a^(3/2)*(2*a^2*
f^2 + b^2*e^2)*(8*C*a^(17/2)*c*f^7*(a*c)^(1/2) + 4*C*a^(5/2)*b^6*c*e^6*f*(a
```

```
*c)^(1/2) - 12*C*a^(13/2)*b^2*c*e^2*f^5*(a*c)^(1/2)))/(2*b*c^2*e*f*(a*c)^(1
/2)*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 -
4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8
))))/((a + b*x)^(1/2) - a^(1/2)) - ((((4*(4*C^2*a^8*f^4 + C^2*a^4*b^4*e^4 +
 4*C^2*a^6*b^2*e^2*f^2))/(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4
 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8) - (C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2*
(12*a^10*c*f^10 - 4*b^10*c*e^10 + 28*a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f
^4 + 88*a^6*b^4*c*e^4*f^6 - 52*a^8*b^2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*
e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6
*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)))/(2*a^(1/2)*c*f*(a*c)^(1/2)*(b
^2*c*e^2 - a^2*c*f^2)^(1/2)) + (4*C^2*a^(9/2)*f*(a*c)^(1/2)*(2*a^2*f^2 + b^
2*e^2)^2)/(b^2*c*e^2*(a*f + b*e)^4*(a*f - b*e)^4*(b^2*c*e^2 - a^2*c*f^2)^(3
/2)))*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2
- ((4*(4*C^2*a^8*c*f^4 + C^2*a^4*b^4*c*e^4 + 4*C^2*a^6*b^2*c*e^2*f^2))/(b^1
0*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^
2*e^2*f^8) + (C^2*a^4*(2*a^2*f^2 + b^2*e^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c^2
*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^4*b^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^4*
f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^
2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4
*f^6 + a^8*b^2*e^2*f^8)))/(2*a^(1/2)*c*f*(a*c)^(1/2)*(b^2*c*e^2 - a^2*c*f^2
)^(1/2)))*(b^10*e^10*(a^2*c*f^2 - b^2*c*e^2) - 4*a^2*b^8*e^8*f^2*(a^2*c*f^2
 - b^2*c*e^2) + 6*a^4*b^6*e^6*f^4*(a^2*c*f^2 - b^2*c*e^2) - 4*a^6*b^4*e^4*f
^6*(a^2*c*f^2 - b^2*c*e^2) + a^8*b^2*e^2*f^8*(a^2*c*f^2 - b^2*c*e^2)))/(16*
C^2*a^8*f^4 + 4*C^2*a^4*b^4*e^4 + 16*C^2*a^6*b^2*e^2*f^2))))/(2*(a*f + b*e)
^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (A*b^2*(a^2*f^2 + 2*b^2*e
^2)*(2*atan(((((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/
((a + b*x)^(1/2) - a^(1/2)) - (a^2*c*f^2*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)
))/((a + b*x)^(1/2) - a^(1/2)) + 2*a^(1/2)*b*c*e*f*(a*c)^(1/2))/(2*b*c*e*(b
^2*c*e^2 - a^2*c*f^2)^(1/2))) + 2*atan((((((a*c - b*c*x)^(1/2) - (a*c)^(1/2
))*(((4*(4*A^2*b^8*c*e^4 + A^2*a^4*b^4*c*f^4 + 4*A^2*a^2*b^6*c*e^2*f^2))/(b
^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*
b^2*e^2*f^8) + (A^2*b^4*(a^2*f^2 + 2*b^2*e^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c
^2*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^4*b^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^
4*f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 -
b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e
^4*f^6 + a^8*b^2*e^2*f^8)))/(4*b*c^2*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (8*
A^2*b^3*(a^2*f^2 + 2*b^2*e^2)^2)/(e*(a*f + b*e)^4*(a*f - b*e)^4*(b^2*c*e^2
- a^2*c*f^2)^(3/2)) - (A*b*(a^2*f^2 + 2*b^2*e^2)*(4*A*a^(13/2)*b^2*c*f^7*(a
*c)^(1/2) + 8*A*a^(1/2)*b^8*c*e^6*f*(a*c)^(1/2) - 12*A*a^(5/2)*b^6*c*e^4*f^
3*(a*c)^(1/2)))/(2*a^(1/2)*c^2*e*f*(a*c)^(1/2)*(a*f + b*e)^2*(a*f - b*e)^2*
(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^
6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8))))/((a + b*x)^(1/2) - a^(1/2))
 + ((((4*(4*A^2*b^8*e^4 + A^2*a^4*b^4*f^4 + 4*A^2*a^2*b^6*e^2*f^2))/(b^10*e
^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e
^2*f^8) - (A^2*b^4*(a^2*f^2 + 2*b^2*e^2)^2*(12*a^10*c*f^10 - 4*b^10*c*e^10
+ 28*a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f^4 + 88*a^6*b^4*c*e^4*f^6 - 52*a
^8*b^2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^
10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b
^2*e^2*f^8)))/(4*b*c^2*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (A*b*(a^2*f^2 + 2
*b^2*e^2)*(4*A*a^(13/2)*b^2*f^7*(a*c)^(1/2) - 12*A*a^(5/2)*b^6*e^4*f^3*(a*c
)^(1/2) + 8*A*a^(1/2)*b^8*e^6*f*(a*c)^(1/2)))/(2*a^(1/2)*c^2*e*f*(a*c)^(1/2
)*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2)*(b^10*e^10 - 4*
a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8))
)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 - ((
((4*(4*A^2*b^8*e^4 + A^2*a^4*b^4*f^4 + 4*A^2*a^2*b^6*e^2*f^2))/(b^10*e^10 -
 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^
8) - (A^2*b^4*(a^2*f^2 + 2*b^2*e^2)^2*(12*a^10*c*f^10 - 4*b^10*c*e^10 + 28*
a^2*b^8*c*e^8*f^2 - 72*a^4*b^6*c*e^6*f^4 + 88*a^6*b^4*c*e^4*f^6 - 52*a^8*b^
2*c*e^2*f^8))/((a*f + b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^
```

```
10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^
2*f^8)))/(2*a^(1/2)*c*f*(a*c)^(1/2)*(b^2*c*e^2 - a^2*c*f^2)^(1/2)) + (4*A^2
*a^(1/2)*b^2*f*(a*c)^(1/2)*(a^2*f^2 + 2*b^2*e^2)^2)/(c*e^2*(a*f + b*e)^4*(a
*f - b*e)^4*(b^2*c*e^2 - a^2*c*f^2)^(3/2)))*((a*c - b*c*x)^(1/2) - (a*c)^(1
/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - ((4*(4*A^2*b^8*c*e^4 + A^2*a^4*b^4*
c*f^4 + 4*A^2*a^2*b^6*c*e^2*f^2))/(b^10*e^10 - 4*a^2*b^8*e^8*f^2 + 6*a^4*b^
6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8) + (A^2*b^4*(a^2*f^2 + 2*b^
2*e^2)^2*(4*a^10*c^2*f^10 + 4*b^10*c^2*e^10 - 12*a^2*b^8*c^2*e^8*f^2 + 8*a^
4*b^6*c^2*e^6*f^4 + 8*a^6*b^4*c^2*e^4*f^6 - 12*a^8*b^2*c^2*e^2*f^8))/((a*f
+ b*e)^4*(a*f - b*e)^4*(a^2*c*f^2 - b^2*c*e^2)*(b^10*e^10 - 4*a^2*b^8*e^8*f
^2 + 6*a^4*b^6*e^6*f^4 - 4*a^6*b^4*e^4*f^6 + a^8*b^2*e^2*f^8)))/(2*a^(1/2)*
c*f*(a*c)^(1/2)*(b^2*c*e^2 - a^2*c*f^2)^(1/2)))*(b^8*e^10*(a^2*c*f^2 - b^2*
c*e^2) + a^8*e^2*f^8*(a^2*c*f^2 - b^2*c*e^2) - 4*a^2*b^6*e^8*f^2*(a^2*c*f^2
 - b^2*c*e^2) + 6*a^4*b^4*e^6*f^4*(a^2*c*f^2 - b^2*c*e^2) - 4*a^6*b^2*e^4*f
^6*(a^2*c*f^2 - b^2*c*e^2)))/(16*A^2*b^6*e^4 + 4*A^2*a^4*b^2*f^4 + 16*A^2*a
^2*b^4*e^2*f^2))))/(2*(a*f + b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(
1/2)) + (3*B*a^2*b^2*e*f*(2*atan((2*b^3*c^3*e^3 + 2*b*c^2*e*(a^2*c*f^2 - b^
2*c*e^2) + 2*a^2*b*c^3*e*f^2 + (3*a^(3/2)*f^3*(a*c)^(3/2)*((a*c - b*c*x)^(1
/2) - (a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3 + (2*b^3*c^2*e^3*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 - (3*a^(1/2)
*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))^3*(a^2*c*f^2 - b^2*c*e^2
))/((a + b*x)^(1/2) - a^(1/2))^3 - (a^(3/2)*c*f^3*(a*c)^(3/2)*((a*c - b*c*x
)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + (2*b*c*e*((a*c - b*c*
x)^(1/2) - (a*c)^(1/2))^2*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/
2))^2 + (a^(1/2)*c*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2))*(a^2*c
*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (10*a^2*b*c^2*e*f^2*((a*c
- b*c*x)^(1/2) - (a*c)^(1/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (7*a^(1/2)
*b^2*c^2*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^
(1/2) - a^(1/2)) - (a^(1/2)*b^2*c*e^2*f*(a*c)^(1/2)*((a*c - b*c*x)^(1/2) -
(a*c)^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^3)/(4*a^(1/2)*b*c^2*e*f*(a*c)^(
1/2)*(b^2*c*e^2 - a^2*c*f^2)^(1/2))) - 2*atan(((((a*c - b*c*x)^(1/2) - (a*c
)^(1/2))*(a^2*c*f^2 - b^2*c*e^2))/((a + b*x)^(1/2) - a^(1/2)) - (a^2*c*f^2*
((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((a + b*x)^(1/2) - a^(1/2)) + 2*a^(1/2
)*b*c*e*f*(a*c)^(1/2))/(2*b*c*e*(b^2*c*e^2 - a^2*c*f^2)^(1/2)))))/(2*(a*f +
 b*e)^2*(a*f - b*e)^2*(b^2*c*e^2 - a^2*c*f^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(f*x+e)**3/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Timed out
```

3.34 $\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{dx-1}\sqrt{dx+1}\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\cosh^{-1}(dx)}{2d^3}+\frac{cx^2\sqrt{dx-1}\sqrt{dx+1}}{3d^2}$$

Rubi [A] time = 0.15, antiderivative size = 151, normalized size of antiderivative = 1.74, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1610, 1809, 780, 217, 206}

$$-\frac{\left(1-d^2x^2\right)\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4\sqrt{dx-1}\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\sqrt{dx+1}}-\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(c*x^2*(1 - d^2*x^2))/(3*d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((2*(2*c + 3*
a*d^2) + 3*b*d^2*x)*(1 - d^2*x^2))/(6*d^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (
b*Sqrt[-1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 + d*
x]*Sqrt[1 + d*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
```

```
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{x\left(2c+3ad^2+3bd^2x\right)}{\sqrt{-1+d^2x^2}}\,dx}{3d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int -}{2d^2\sqrt{-1+dx}\sqrt{}}$$

$$= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Sub}}{2d^2\sqrt{-}}$$

$$= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tanh^{-1}}{2d^3\sqrt{-1+dx}}$$

Mathematica [A] time = 0.36, size = 149, normalized size = 1.71

$$\frac{\sqrt{-(dx-1)^2}\sqrt{dx+1}\left(3d^2(2a+bx)+2c\left(d^2x^2+2\right)\right)+6\sqrt{dx-1}\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)(d(2ad-b)+2c)-12\sqrt{1-dx}\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)(d(ad-b)+c)}{6d^4\sqrt{1-dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[-(-1 + d*x)^2]*Sqrt[1 + d*x]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2))
+ 6*(2*c + d*(-b + 2*a*d))*Sqrt[-1 + d*x]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] - 1
2*(c + d*(-b + a*d))*Sqrt[1 - d*x]*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(6*
d^4*Sqrt[1 - d*x])
```

IntegrateAlgebraic [B] time = 0.00, size = 230, normalized size = 2.64

$$\frac{-\frac{6ad^2(dx-1)^{5/2}}{(dx+1)^{5/2}}+\frac{12ad^2(dx-1)^{3/2}}{(dx+1)^{3/2}}-\frac{6ad^2\sqrt{dx-1}}{\sqrt{dx+1}}+\frac{3bd(dx-1)^{5/2}}{(dx+1)^{5/2}}-\frac{3bd\sqrt{dx-1}}{\sqrt{dx+1}}-\frac{6c(dx-1)^{5/2}}{(dx+1)^{5/2}}+\frac{4c(dx-1)^{3/2}}{(dx+1)^{3/2}}-\frac{6c\sqrt{dx-1}}{\sqrt{dx+1}}}{3d^4\left(\frac{dx-1}{dx+1}-1\right)^3}+\frac{b\tanh^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-6*c*(-1 + d*x)^(5/2))/(1 + d*x)^(5/2) + (3*b*d*(-1 + d*x)^(5/2))/(1 + d*
x)^(5/2) - (6*a*d^2*(-1 + d*x)^(5/2))/(1 + d*x)^(5/2) + (4*c*(-1 + d*x)^(3/
2))/(1 + d*x)^(3/2) + (12*a*d^2*(-1 + d*x)^(3/2))/(1 + d*x)^(3/2) - (6*c*Sq
rt[-1 + d*x])/Sqrt[1 + d*x] - (3*b*d*Sqrt[-1 + d*x])/Sqrt[1 + d*x] - (6*a*d
^2*Sqrt[-1 + d*x])/Sqrt[1 + d*x])/(3*d^4*(-1 + (-1 + d*x)/(1 + d*x))^3) + (
b*ArcTanh[Sqrt[-1 + d*x]/Sqrt[1 + d*x]])/d^3
```

fricas [A] time = 1.29, size = 73, normalized size = 0.84

$$-\frac{3\,bd\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)-\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\,\sqrt{dx-1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/6*(3*b*d*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - (2*c*d^2*x^2 + 3*b*d^
2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(d*x - 1))/d^4
```

giac [A] time = 1.46, size = 105, normalized size = 1.21

$$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\left((dx+1)\left(\frac{2\,(dx+1)c}{d^3}+\frac{3\,bd^{10}-4\,cd^9}{d^{12}}\right)+\frac{3\left(2\,ad^{11}-bd^{10}+2\,cd^9\right)}{d^{12}}\right)-\frac{6\,b\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*(2*(d*x + 1)*c/d^3 + (3*b*d^10
- 4*c*d^9)/d^12) + 3*(2*a*d^11 - b*d^10 + 2*c*d^9)/d^12) - 6*b*log(sqrt(d*x
 + 1) - sqrt(d*x - 1))/d^2)/d
```

maple [C] time = 0.00, size = 137, normalized size = 1.57

$$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(2\sqrt{d^2x^2-1}\,c\,d^2x^2\mathrm{csgn}(d)+3\sqrt{d^2x^2-1}\,b\,d^2x\,\mathrm{csgn}(d)+6\sqrt{d^2x^2-1}\,a\,d^2\mathrm{csgn}(d)+3bd\ln\left(\left(dx+\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+4\sqrt{d^2x^2-1}\,c\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{6\sqrt{d^2x^2-1}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] 1/6*(d*x-1)^(1/2)*(d*x+1)^(1/2)*(2*(d^2*x^2-1)^(1/2)*c*d^2*x^2*csgn(d)+3*(d
^2*x^2-1)^(1/2)*b*d^2*x*csgn(d)+6*(d^2*x^2-1)^(1/2)*a*d^2*csgn(d)+3*b*d*ln(
(d*x+(d^2*x^2-1)^(1/2)*csgn(d))*csgn(d))+4*(d^2*x^2-1)^(1/2)*c*csgn(d))/(d^
2*x^2-1)^(1/2)/d^4*csgn(d)
```

maxima [A] time = 1.02, size = 100, normalized size = 1.15

$$\frac{\sqrt{d^2x^2-1}\,cx^2}{3\,d^2}+\frac{\sqrt{d^2x^2-1}\,bx}{2\,d^2}+\frac{\sqrt{d^2x^2-1}\,a}{d^2}+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{2\,d^3}+\frac{2\,\sqrt{d^2x^2-1}\,c}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*sqrt(d^2*x^2 - 1)*c*x^2/d^2 + 1/2*sqrt(d^2*x^2 - 1)*b*x/d^2 + sqrt(d^2*
x^2 - 1)*a/d^2 + 1/2*b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3 + 2/3*sqrt(
d^2*x^2 - 1)*c/d^4
```

mupad [B] time = 14.76, size = 318, normalized size = 3.66

$$\frac{\sqrt{d\,x-1}\,\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,b\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}+\frac{14\,b\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,b\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}+\frac{2\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\sqrt{d\,x+1}-1}}{d^3-\frac{4\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{6\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}-\frac{4\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}+\frac{d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^8}{{\left(\sqrt{d\,x+1}-1\right)}^8}}+\frac{a\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2))/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (2*b*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((d*
x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*b*((d*x - 1)^(1/2) - 1i
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*b*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1
/2) - 1)^7 + (2*b*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*
d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(
```

```
1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((
d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8) + ((d*x - 1)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d) + (c*x^2)/(3*d^2) +
(2*c*x)/(3*d^3)))/(d*x + 1)^(1/2) + (a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

sympy [C] time = 80.46, size = 308, normalized size = 3.54

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1 \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}-\frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}+\frac{cG_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{icG_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*a*meijerg(((-1, -3/4, -1/2, -1/4, 0
, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*d**2) + b*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1
, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*b*mei
jerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0
)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) + c*meijerg(((-5/4, -
3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(d**2*x*
*2))/(4*pi**(3/2)*d**4) + I*c*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()),
((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)*d**4)
```

3.35 $\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=52

$$\frac{(2ad^2+c)\cosh^{-1}(dx)}{2d^3}+\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,(2b+cx)}{2d^2}$$

Rubi [B] time = 0.07, antiderivative size = 135, normalized size of antiderivative = 2.60, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {901, 1815, 641, 217, 206}

$$\frac{\sqrt{d^2x^2-1}\,(2ad^2+c)\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{d^2\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{cx(1-d^2x^2)}{2d^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) - (c*x*(1 - d^2*x^2
))/(2*d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((c + 2*a*d^2)*Sqrt[-1 + d^2*x^2]
*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 901

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^Fr
acPart[m])/(d*f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{c+2ad^2+2bd^2x}{\sqrt{-1+d^2x^2}}\,dx}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+}}}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\right.}{2d^2\sqrt{-1+dx}\sqrt{}}$$
$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\tanh^{-1}\left(\right.}{2d^3\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [B] time = 0.22, size = 126, normalized size = 2.42

$$\frac{4\sqrt{1-dx}\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)(d(ad-b)+c)+d\sqrt{-(dx-1)^2}\sqrt{dx+1}\,(2b+cx)+2\sqrt{dx-1}\,(2bd-c)\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)}{2d^3\sqrt{1-dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (d*(2*b + c*x)*Sqrt[-(-1 + d*x)^2]*Sqrt[1 + d*x] + 2*(-c + 2*b*d)*Sqrt[-1 +
 d*x]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] + 4*(c + d*(-b + a*d))*Sqrt[1 - d*x]*Ar
cTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(2*d^3*Sqrt[1 - d*x])
```

IntegrateAlgebraic [B] time = 0.00, size = 112, normalized size = 2.15

$$\frac{\left(2ad^2+c\right)\tanh^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)}{d^3} - \frac{\sqrt{dx-1}\left(\frac{2bd(dx-1)}{dx+1}-2bd-\frac{c(dx-1)}{dx+1}-c\right)}{d^3\sqrt{dx+1}\left(\frac{dx-1}{dx+1}-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((Sqrt[-1 + d*x]*(-c - 2*b*d - (c*(-1 + d*x))/(1 + d*x) + (2*b*d*(-1 + d*x
))/(1 + d*x)))/(d^3*Sqrt[1 + d*x]*(-1 + (-1 + d*x)/(1 + d*x))^2)) + ((c + 2
*a*d^2)*ArcTanh[Sqrt[-1 + d*x]/Sqrt[1 + d*x]])/d^3
```

fricas [A] time = 1.08, size = 61, normalized size = 1.17

$$\frac{(cdx+2\,bd)\sqrt{dx+1}\,\sqrt{dx-1}-\left(2\,ad^2+c\right)\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(d*x - 1) - (2*a*d^2 + c)*log(-d*x +
 sqrt(d*x + 1)*sqrt(d*x - 1)))/d^3
```

giac [A] time = 1.39, size = 80, normalized size = 1.54

$$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\left(\frac{(dx+1)c}{d^2}+\frac{2\,bd^5-cd^4}{d^6}\right)-\frac{2\left(2\,ad^2+c\right)\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*c/d^2 + (2*b*d^5 - c*d^4)/d^6)
- 2*(2*a*d^2 + c)*log(sqrt(d*x + 1) - sqrt(d*x - 1))/d^2)/d
```

maple [C] time = 0.00, size = 120, normalized size = 2.31

$$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(2a\,d^2\ln\left(\left(dx+\sqrt{d^2x^2-1}\,\operatorname{csgn}(d)\right)\operatorname{csgn}(d)\right)+\sqrt{d^2x^2-1}\,cdx\operatorname{csgn}(d)+2\sqrt{d^2x^2-1}\,bd\operatorname{csgn}(d)+c\ln\left(\left(dx+\sqrt{d^2x^2-1}\,\operatorname{csgn}(d)\right)\operatorname{csgn}(d)\right)\right)\operatorname{csgn}(d)}{2\sqrt{d^2x^2-1}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] 1/2*(d*x-1)^(1/2)*(d*x+1)^(1/2)*(2*a*d^2*ln((d*x+(d^2*x^2-1)^(1/2)*csgn(d))
*csgn(d))+(d^2*x^2-1)^(1/2)*c*d*x*csgn(d)+2*(d^2*x^2-1)^(1/2)*b*d*csgn(d)+c
*ln((d*x+(d^2*x^2-1)^(1/2)*csgn(d))*csgn(d)))/(d^2*x^2-1)^(1/2)/d^3*csgn(d)
```

maxima [B] time = 1.11, size = 90, normalized size = 1.73

$$\frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{d}+\frac{\sqrt{d^2x^2-1}\,cx}{2\,d^2}+\frac{\sqrt{d^2x^2-1}\,b}{d^2}+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + 1/2*sqrt(d^2*x^2 - 1)*c*x/d^2 +
sqrt(d^2*x^2 - 1)*b/d^2 + 1/2*c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3
```

mupad [B] time = 14.59, size = 312, normalized size = 6.00

$$\frac{b\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}+\frac{2\,c\,\mathrm{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{-d^2}}\right)}{\sqrt{-d^2}}-\frac{\frac{14\,c\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}+\frac{14\,c\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,c\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}+\frac{2\,c\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\sqrt{d\,x+1}-1}}{d^3-\frac{4\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{6\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}-\frac{4\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}+\frac{d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^8}{{\left(\sqrt{d\,x+1}-1\right)}^8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (2*c*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((d*
x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*c*((d*x - 1)^(1/2) - 1i
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*c*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1
/2) - 1)^7 + (2*c*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*
d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(
1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((
d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8) - (4*a*atan((d*((d*x - 1)^(1/2) - 1i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(
1/2))))/(-d^2)^(1/2) + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

sympy [C] time = 48.76, size = 277, normalized size = 5.33

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\-\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\-\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{cG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}-\frac{icG_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\-\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 1
/(d**2*x**2))/(4*pi**(3/2)*d) - I*a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)*d) + b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1
/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*b*meijerg(((-1, -3/4, -1
/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi
)/(d**2*x**2))/(4*pi**(3/2)*d**2) + c*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0
, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3
) - I*c*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2
, -1, -1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

3.36 $\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=55

$$a\tan^{-1}\left(\sqrt{dx-1}\,\sqrt{dx+1}\right)+\frac{b\cosh^{-1}(dx)}{d}+\frac{c\sqrt{dx-1}\,\sqrt{dx+1}}{d^2}$$

Rubi [B] time = 0.18, antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1610, 1809, 844, 217, 206, 266, 63, 205}

$$\frac{a\sqrt{d^2x^2-1}\,\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\,\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (a*Sqrt[-1 + d^2*
x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*Sqrt[-
1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 +
d*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] && !IntegerQ[m]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{ad^2+bd^2x}{x\sqrt{-1+d^2x^2}}\,dx}{d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+}}}{\sqrt{-1+dx}\sqrt{1+d}}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x, x^2\right)}{2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+\right.}{}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\text{Subs}}{d^2\sqrt{-1}}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{a\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tanh^{}}{d\sqrt{-1+dx}\sqrt{}}$$

Mathematica [B] time = 0.42, size = 128, normalized size = 2.33

$$\frac{\frac{ad^2\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)+cd^2x^2-2c\sqrt{1-d^2x^2}\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)-c}{\sqrt{dx-1}\sqrt{dx+1}} - 2(c-bd)\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)}{d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-c + c*d^2*x^2 - 2*c*Sqrt[1 - d^2*x^2]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] + a*
d^2*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 +
 d*x]) - 2*(c - b*d)*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d^2
```

IntegrateAlgebraic [A]   time = 0.00, size = 91, normalized size = 1.65

$$2a\tan^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)+\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)}{d}-\frac{2c\sqrt{dx-1}}{d^2\sqrt{dx+1}\left(\frac{dx-1}{dx+1}-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-2*c*Sqrt[-1 + d*x])/(d^2*Sqrt[1 + d*x]*(-1 + (-1 + d*x)/(1 + d*x))) + 2*a
*ArcTan[Sqrt[-1 + d*x]/Sqrt[1 + d*x]] + (2*b*ArcTanh[Sqrt[-1 + d*x]/Sqrt[1
+ d*x]])/d
```

fricas [A]   time = 0.63, size = 73, normalized size = 1.33

$$\frac{2\,ad^2\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)-bd\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+\sqrt{dx+1}\,\sqrt{dx-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (2*a*d^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - b*d*log(-d*x + sqrt(d
*x + 1)*sqrt(d*x - 1)) + sqrt(d*x + 1)*sqrt(d*x - 1)*c)/d^2
```

giac [A]   time = 1.37, size = 71, normalized size = 1.29

$$-2\,a\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)-\frac{b\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}+\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - b*log((sqrt(d*x + 1) -
 sqrt(d*x - 1))^2)/d + sqrt(d*x + 1)*sqrt(d*x - 1)*c/d^2
```

maple [C]   time = 0.00, size = 95, normalized size = 1.73

$$\frac{\left(-a\,d^2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\mathrm{csgn}(d)+bd\ln\left(\left(dx+\sqrt{(dx+1)(dx-1)}\ \mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+\sqrt{d^2x^2-1}\ c\,\mathrm{csgn}(d)\right)\sqrt{dx-1}\ \sqrt{dx+1}\ \mathrm{csgn}(d)}{\sqrt{d^2x^2-1}\ d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-a*d^2*arctan(1/(d^2*x^2-1)^(1/2))*csgn(d)+b*d*ln((d*x+((d*x+1)*(d*x-1))^(
1/2)*csgn(d))*csgn(d))+(d^2*x^2-1)^(1/2)*c*csgn(d))*(d*x-1)^(1/2)*(d*x+1)^(
1/2)/(d^2*x^2-1)^(1/2)/d^2*csgn(d)
```

maxima [A]   time = 2.34, size = 56, normalized size = 1.02

$$-a\arcsin\left(\frac{1}{d|x|}\right)+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{d}+\frac{\sqrt{d^2x^2-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -a*arcsin(1/(d*abs(x))) + b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(d
^2*x^2 - 1)*c/d^2
```

mupad [B] time = 5.39, size = 118, normalized size = 2.15

$$\frac{c\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}-\frac{4\,b\,\mathrm{atan}\left(\frac{d\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{-d^2}}\right)}{\sqrt{-d^2}}-a\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (c*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2 - (4*b*atan((d*((d*x - 1)^(1/2) - 1
i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - a*(log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/((
d*x + 1)^(1/2) - 1)))*1i
```

sympy [C] time = 47.37, size = 240, normalized size = 4.36

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\,\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\,\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\,\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\,\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{cG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\,\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{icG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\,\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1
/(d**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), (
(1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*p
i**(3/2)*d) + c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4
, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*c*meijerg(((-1, -3/4,
 -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

3.37 $\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=55

$$\frac{a\sqrt{dx-1}\sqrt{dx+1}}{x}+b\tan^{-1}\left(\sqrt{dx-1}\sqrt{dx+1}\right)+\frac{c\cosh^{-1}(dx)}{d}$$

Rubi [B] time = 0.18, antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1610, 1807, 844, 217, 206, 266, 63, 205}

$$-\frac{a\left(1-d^2x^2\right)}{x\sqrt{dx-1}\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\,\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\sqrt{dx+1}}+\frac{c\sqrt{d^2x^2-1}\,\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*(1 - d^2*x^2))/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (b*Sqrt[-1 + d^2*x^
2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (c*Sqrt[-1
+ d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 + d*
x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^2\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{b+cx}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+}}}{\sqrt{-1+dx}\sqrt{1+d}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx,x,x^2\right)}{2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c\sqrt{-1+\right.}{}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{c\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}}{d^2\sqrt{-1}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{c\sqrt{-1+d^2x^2}\tanh^{-}}{d\sqrt{-1+dx}\sqrt{}}$$

Mathematica [A] time = 0.18, size = 89, normalized size = 1.62

$$\frac{a\left(d^2x^2-1\right)+bx\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{x\sqrt{dx-1}\sqrt{dx+1}}+\frac{2c\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (a*(-1 + d^2*x^2) + b*x*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(x*S
qrt[-1 + d*x]*Sqrt[1 + d*x]) + (2*c*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

IntegrateAlgebraic [A] time = 0.00, size = 89, normalized size = 1.62

$$\frac{2ad\sqrt{dx-1}}{\sqrt{dx+1}\left(\frac{dx-1}{dx+1}+1\right)}+2b\tan^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)+\frac{2c\tanh^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (2*a*d*Sqrt[-1 + d*x])/(Sqrt[1 + d*x]*(1 + (-1 + d*x)/(1 + d*x))) + 2*b*Arc
Tan[Sqrt[-1 + d*x]/Sqrt[1 + d*x]] + (2*c*ArcTanh[Sqrt[-1 + d*x]/Sqrt[1 + d*
x]])/d
```

fricas [A] time = 1.03, size = 82, normalized size = 1.49

$$\frac{ad^2x+2\,bdx\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+\sqrt{dx+1}\,\sqrt{dx-1}\,ad-cx\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] (a*d^2*x + 2*b*d*x*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + sqrt(d*x +
1)*sqrt(d*x - 1)*a*d - c*x*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)))/(d*x)
```

giac [A] time = 1.52, size = 83, normalized size = 1.51

$$-\frac{2\,bd\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)-\frac{8\,ad^2}{\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4}+c\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -(2*b*d*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - 8*a*d^2/((sqrt(d*x
+ 1) - sqrt(d*x - 1))^4 + 4) + c*log((sqrt(d*x + 1) - sqrt(d*x - 1))^2))/d
```

maple [C] time = 0.00, size = 96, normalized size = 1.75

$$\frac{\left(-bdx\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\mathrm{csgn}(d)+\sqrt{d^2x^2-1}\,ad\,\mathrm{csgn}(d)+cx\ln\left(\left(dx+\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)\right)\sqrt{dx-1}\,\sqrt{dx+1}\,\mathrm{csgn}(d)}{\sqrt{d^2x^2-1}\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-b*d*x*arctan(1/(d^2*x^2-1)^(1/2))*csgn(d)+(d^2*x^2-1)^(1/2)*a*d*csgn(d)+c
*x*ln((d*x+(d^2*x^2-1)^(1/2)*csgn(d))*csgn(d)))*(d*x-1)^(1/2)*(d*x+1)^(1/2)
/(d^2*x^2-1)^(1/2)/d/x*csgn(d)
```

maxima [A] time = 2.35, size = 56, normalized size = 1.02

$$-b\arcsin\left(\frac{1}{d|x|}\right)+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{d}+\frac{\sqrt{d^2x^2-1}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -b*arcsin(1/(d*abs(x))) + c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(d
^2*x^2 - 1)*a/x
```

mupad [B] time = 5.15, size = 118, normalized size = 2.15

$$\frac{a\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}-\frac{4\,c\,\mathrm{atan}\left(\frac{d\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{-d^2}}\right)}{\sqrt{-d^2}}-b\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^2*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x - (4*c*atan((d*((d*x - 1)^(1/2) - 1i)
)/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - b*(log(((d*x - 1)^(
1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/((d*
x + 1)^(1/2) - 1)))*1i
```

sympy [C] time = 45.81, size = 216, normalized size = 3.93

$$-\frac{adG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iadG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{cG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{icG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**2/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0
,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)) + c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0
), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*c*meijerg(((-1/2, -1/4, 0, 1/4,
1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2)
)/(4*pi**(3/2)*d)
```

3.38 $\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=83

$$\frac{1}{2}\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{dx-1}\sqrt{dx+1}\right)+\frac{a\sqrt{dx-1}\sqrt{dx+1}}{2x^2}+\frac{b\sqrt{dx-1}\sqrt{dx+1}}{x}$$

Rubi [A] time = 0.19, antiderivative size = 129, normalized size of antiderivative = 1.55, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1610, 1807, 807, 266, 63, 205}

$$\frac{\sqrt{d^2x^2-1}\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\sqrt{dx+1}}-\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}-\frac{b\left(1-d^2x^2\right)}{x\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*(1 - d^2*x^2))/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2))
/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((2*c + a*d^2)*Sqrt[-1 + d^2*x^2]*ArcTa
n[Sqrt[-1 + d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^3\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2b+\left(2c+ad^2\right)x}{x^2\sqrt{-1+d^2x^2}}\,dx}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\int \frac{}{x\sqrt{-}}}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}}{4\sqrt{-1+dx}\sqrt{1}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}}{2d^2\sqrt{-1+d}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\tan^{-1}\left(\right.}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [A] time = 0.12, size = 82, normalized size = 0.99

$$\frac{\left(d^2x^2-1\right)(a+2bx)+x^2\sqrt{d^2x^2-1}\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((a + 2*b*x)*(-1 + d^2*x^2) + (2*c + a*d^2)*x^2*Sqrt[-1 + d^2*x^2]*ArcTan[S
qrt[-1 + d^2*x^2]])/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

IntegrateAlgebraic [A] time = 0.00, size = 107, normalized size = 1.29

$$\left(ad^2+2c\right)\tan^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)-\frac{d\sqrt{dx-1}\left(\frac{ad(dx-1)}{dx+1}-ad-\frac{2b(dx-1)}{dx+1}-2b\right)}{\sqrt{dx+1}\left(\frac{dx-1}{dx+1}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((d*Sqrt[-1 + d*x]*(-2*b - a*d - (2*b*(-1 + d*x))/(1 + d*x) + (a*d*(-1 + d
*x))/(1 + d*x)))/(Sqrt[1 + d*x]*(1 + (-1 + d*x)/(1 + d*x))^2)) + (2*c + a*d
^2)*ArcTan[Sqrt[-1 + d*x]/Sqrt[1 + d*x]]
```

fricas [A] time = 1.11, size = 69, normalized size = 0.83

$$\frac{2\,bdx^2 + 2\left(ad^2 + 2\,c\right)x^2\arctan\left(-dx + \sqrt{dx+1}\,\sqrt{dx-1}\right) + (2\,bx + a)\sqrt{dx+1}\,\sqrt{dx-1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*b*d*x^2 + 2*(a*d^2 + 2*c)*x^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x -
 1)) + (2*b*x + a)*sqrt(d*x + 1)*sqrt(d*x - 1))/x^2
```

giac [B] time = 1.44, size = 145, normalized size = 1.75

$$-\frac{\left(ad^3 + 2\,cd\right)\arctan\left(\frac{1}{2}\left(\sqrt{dx+1} - \sqrt{dx-1}\right)^2\right) + \frac{2\left(ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^6 - 4\,bd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4 - 4\,ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2 - 16\,bd^2\right)}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -((a*d^3 + 2*c*d)*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(a*d^3*
(sqrt(d*x + 1) - sqrt(d*x - 1))^6 - 4*b*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^4 - 4*a*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 16*b*d^2)/((sqrt(d*x + 1)
- sqrt(d*x - 1))^4 + 4)^2)/d
```

maple [C] time = 0.00, size = 103, normalized size = 1.24

$$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(a\,d^2x^2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right) + 2c\,x^2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right) - 2\sqrt{d^2x^2-1}\,bx - \sqrt{d^2x^2-1}\,a\right)\mathrm{csgn}(d)^2}{2\sqrt{d^2x^2-1}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(d*x-1)^(1/2)*(d*x+1)^(1/2)*(a*d^2*x^2*arctan(1/(d^2*x^2-1)^(1/2))+2*c
*x^2*arctan(1/(d^2*x^2-1)^(1/2))-2*(d^2*x^2-1)^(1/2)*b*x-(d^2*x^2-1)^(1/2)*
a)/(d^2*x^2-1)^(1/2)/x^2*csgn(d)^2
```

maxima [A] time = 2.47, size = 61, normalized size = 0.73

$$-\frac{1}{2}\,ad^2\arcsin\left(\frac{1}{d|x|}\right) - c\arcsin\left(\frac{1}{d|x|}\right) + \frac{\sqrt{d^2x^2-1}\,b}{x} + \frac{\sqrt{d^2x^2-1}\,a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*a*d^2*arcsin(1/(d*abs(x))) - c*arcsin(1/(d*abs(x))) + sqrt(d^2*x^2 - 1
)*b/x + 1/2*sqrt(d^2*x^2 - 1)*a/x^2
```

mupad [B] time = 12.77, size = 316, normalized size = 3.81

$$\frac{\frac{a\,d^2\,1\mathrm{i}}{32} + \frac{a\,d^2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2 1\mathrm{i}}{16\left(\sqrt{d\,x+1}-1\right)^2} - \frac{a\,d^2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4 15\mathrm{i}}{32\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2} + \frac{2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4} + \frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}} - c\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right) - \ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)1\mathrm{i} - \frac{a\,d^2\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)1\mathrm{i}}{2} + \frac{a\,d^2\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)1\mathrm{i}}{2} + \frac{b\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x} + \frac{a\,d^2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2 1\mathrm{i}}{32\left(\sqrt{d\,x+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^3*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((a*d^2*1i)/32 + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2) -
 1)^2) - (a*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4))
/((((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2) -
1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1/2)
- 1)^6) - c*(log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - lo
g(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))*1i - (a*d^2*log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*d^2*log(((d*x - 1)^(
1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2
))/x + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

sympy [C] time = 75.51, size = 212, normalized size = 2.55

$$-\frac{ad^2G_{6,6}^{5,3}\left(\begin{matrix} \frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2} \\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iad^2G_{6,6}^{2,6}\left(\begin{matrix} 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bdG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{ibdG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{cG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{icG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**3/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,
)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*a*d**2*meijerg(((1, 5/4, 3/2, 7/4, 2,
1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)) - b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4
, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*b*d*meijerg(((1/2, 3/4, 1, 5/
4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2
))/(4*pi**(3/2)) - c*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5
/4, 3/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*c*meijerg(((0, 1/4, 1/2,
3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x*
*2))/(4*pi**(3/2))
```

3.39 $\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=116

$$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,(2ad^2+3c)}{3x}+\frac{a\sqrt{dx-1}\,\sqrt{dx+1}}{3x^3}+\frac{1}{2}bd^2\tan^{-1}\left(\sqrt{dx-1}\,\sqrt{dx+1}\right)+\frac{b\sqrt{dx-1}\,\sqrt{dx+1}}{2x^2}$$

Rubi [A] time = 0.22, antiderivative size = 171, normalized size of antiderivative = 1.47, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1610, 1807, 835, 807, 266, 63, 205}

$$-\frac{(1-d^2x^2)(2ad^2+3c)}{3x\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{a(1-d^2x^2)}{3x^3\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{2x^2\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{bd^2\sqrt{d^2x^2-1}\,\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*(1 - d^2*x^2))/(3*x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2))
/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((3*c + 2*a*d^2)*(1 - d^2*x^2))/(3*
x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*d^2*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1
+ d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^4\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{3b+\left(3c+2ad^2\right)x}{x^3\sqrt{-1+d^2x^2}}\,dx}{3\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2\left(3c+2ad^2\right)+3bd^2x}{x^2\sqrt{-1+d^2x^2}}}{6\sqrt{-1+dx}\,\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(bd^2\right.}{}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(bd^2\right.}{}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{\right.}}{}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{bd^2}{}$$

Mathematica [A] time = 0.12, size = 94, normalized size = 0.81

$$\frac{\left(d^2x^2-1\right)\left(a\left(4d^2x^2+2\right)+3x(b+2cx)\right)+3bd^2x^3\sqrt{d^2x^2-1}\,\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{6x^3\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-1 + d^2*x^2)*(3*x*(b + 2*c*x) + a*(2 + 4*d^2*x^2)) + 3*b*d^2*x^3*Sqrt[-1
 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(6*x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]
)
```

IntegrateAlgebraic [A] time = 0.00, size = 168, normalized size = 1.45

$$\frac{d\sqrt{dx-1}\left(\frac{4ad^2(dx-1)}{dx+1}+\frac{6ad^2(dx-1)^2}{(dx+1)^2}+6ad^2-\frac{3bd(dx-1)^2}{(dx+1)^2}+3bd+\frac{12c(dx-1)}{dx+1}+\frac{6c(dx-1)^2}{(dx+1)^2}+6c\right)}{3\sqrt{dx+1}\left(\frac{dx-1}{dx+1}+1\right)^3}+bd^2\tan^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (d*Sqrt[-1 + d*x]*(6*c + 3*b*d + 6*a*d^2 + (6*c*(-1 + d*x)^2)/(1 + d*x)^2 -
 (3*b*d*(-1 + d*x)^2)/(1 + d*x)^2 + (6*a*d^2*(-1 + d*x)^2)/(1 + d*x)^2 + (1
2*c*(-1 + d*x))/(1 + d*x) + (4*a*d^2*(-1 + d*x))/(1 + d*x)))/(3*Sqrt[1 + d*
x]*(1 + (-1 + d*x)/(1 + d*x))^3) + b*d^2*ArcTan[Sqrt[-1 + d*x]/Sqrt[1 + d*x
]]
```

fricas [A] time = 1.07, size = 90, normalized size = 0.78

$$\frac{6\,bd^2x^3\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+2\left(2\,ad^3+3\,cd\right)x^3+\left(2\left(2\,ad^2+3\,c\right)x^2+3\,bx+2\,a\right)\sqrt{dx+1}\,\sqrt{dx-1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/6*(6*b*d^2*x^3*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + 2*(2*a*d^3 +
3*c*d)*x^3 + (2*(2*a*d^2 + 3*c)*x^2 + 3*b*x + 2*a)*sqrt(d*x + 1)*sqrt(d*x -
 1))/x^3
```

giac [B] time = 1.40, size = 197, normalized size = 1.70

$$-\frac{3\,bd^3\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(3\,bd^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{10}-12\,cd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^8-96\,ad^4\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-96\,cd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-48\,bd^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2-128\,ad^4-192\,cd^2\right)}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -1/3*(3*b*d^3*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(3*b*d^3*(s
qrt(d*x + 1) - sqrt(d*x - 1))^10 - 12*c*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^8 - 96*a*d^4*(sqrt(d*x + 1) - sqrt(d*x - 1))^4 - 96*c*d^2*(sqrt(d*x + 1) -
 sqrt(d*x - 1))^4 - 48*b*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 128*a*d^4
- 192*c*d^2)/((sqrt(d*x + 1) - sqrt(d*x - 1))^4 + 4)^3)/d
```

maple [C] time = 0.00, size = 123, normalized size = 1.06

$$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(3b\,d^2x^3\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)-4\sqrt{d^2x^2-1}\,a\,d^2x^2-6\sqrt{d^2x^2-1}\,c\,x^2-3\sqrt{d^2x^2-1}\,bx-2\sqrt{d^2x^2-1}\,a\right)\mathrm{csgn}(d)^2}{6\sqrt{d^2x^2-1}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/6*(d*x-1)^(1/2)*(d*x+1)^(1/2)*(3*b*d^2*x^3*arctan(1/(d^2*x^2-1)^(1/2))-4
*(d^2*x^2-1)^(1/2)*a*d^2*x^2-6*(d^2*x^2-1)^(1/2)*c*x^2-3*(d^2*x^2-1)^(1/2)*
b*x-2*(d^2*x^2-1)^(1/2)*a)/(d^2*x^2-1)^(1/2)/x^3*csgn(d)^2
```

maxima [A] time = 3.05, size = 86, normalized size = 0.74

$$-\frac{1}{2}\,bd^2\arcsin\left(\frac{1}{d|x|}\right)+\frac{2\sqrt{d^2x^2-1}\,ad^2}{3\,x}+\frac{\sqrt{d^2x^2-1}\,c}{x}+\frac{\sqrt{d^2x^2-1}\,b}{2\,x^2}+\frac{\sqrt{d^2x^2-1}\,a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*b*d^2*arcsin(1/(d*abs(x))) + 2/3*sqrt(d^2*x^2 - 1)*a*d^2/x + sqrt(d^2*
x^2 - 1)*c/x + 1/2*sqrt(d^2*x^2 - 1)*b/x^2 + 1/3*sqrt(d^2*x^2 - 1)*a/x^3
```

mupad [B] time = 11.82, size = 304, normalized size = 2.62

$$\frac{\frac{b\,d^2\,1\mathrm{i}}{32}+\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{16\,{\left(\sqrt{d\,x+1}-1\right)}^2}-\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4\,15\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^4}}{\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}+\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}}-\frac{b\,d^2\,\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)\,1\mathrm{i}}{2}+\frac{b\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\,1\mathrm{i}}{2}+\frac{c\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}+\frac{\sqrt{d\,x-1}\,\left(\frac{2\,a\,d^3\,x^3}{3}+\frac{2\,a\,d^2\,x^2}{3}+\frac{a\,d\,x}{3}+\frac{a}{3}\right)}{x^3\,\sqrt{d\,x+1}}+\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^4*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((b*d^2*1i)/32 + (b*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2) -
 1)^2) - (b*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4))
/(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2) -
1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1/2)
- 1)^6) - (b*d^2*log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*
1i)/2 + (b*d^2*log(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (c
*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x + ((d*x - 1)^(1/2)*(a/3 + (2*a*d^2*x^2)
/3 + (2*a*d^3*x^3)/3 + (a*d*x)/3))/(x^3*(d*x + 1)^(1/2)) + (b*d^2*((d*x - 1
)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

sympy [C] time = 128.74, size = 219, normalized size = 1.89

$$-\frac{a d^{3} G_{6, 6}^{5, 3}\left(\begin{matrix} \frac{9}{4}, \frac{11}{4}, 1 & \frac{5}{2}, \frac{5}{2}, 3 \\ 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}, 3 & 0 \end{matrix} \middle| {\frac{1}{d^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}}} - \frac{i a d^{3} G_{6, 6}^{2, 6}\left(\begin{matrix} \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}, 1 & \\ \frac{7}{4}, \frac{9}{4} & \frac{3}{2}, 2, 2, 0 \end{matrix} \middle| {\frac{e^{2 i \pi}}{d^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}}} - \frac{b d^{2} G_{6, 6}^{5, 3}\left(\begin{matrix} \frac{7}{4}, \frac{9}{4}, 1 & 2, 2, \frac{5}{2} \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} & 0 \end{matrix} \middle| {\frac{1}{d^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}}} + \frac{i b d^{2} G_{6, 6}^{2, 6}\left(\begin{matrix} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 & \\ \frac{5}{4}, \frac{7}{4} & 1, \frac{3}{2}, \frac{3}{2}, 0 \end{matrix} \middle| {\frac{e^{2 i \pi}}{d^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}}} - \frac{c d G_{6, 6}^{5, 3}\left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 & \frac{3}{2}, \frac{3}{2}, 2 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 & 0 \end{matrix} \middle| {\frac{1}{d^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}}} - \frac{i c d G_{6, 6}^{2, 6}\left(\begin{matrix} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 & \\ \frac{3}{4}, \frac{5}{4} & \frac{1}{2}, 1, 1, 0 \end{matrix} \middle| {\frac{e^{2 i \pi}}{d^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**4/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d**3*meijerg(((3/2, 7/4, 2, 9/4, 5
/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(
4*pi**(3/2)) - b*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2,
9/4, 5/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*d**2*meijerg(((1, 5/4,
 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(d
**2*x**2))/(4*pi**(3/2)) - c*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1,
 5/4, 3/2, 7/4, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*c*d*meijerg(((1
/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*
pi)/(d**2*x**2))/(4*pi**(3/2))
```

3.40 $\int \frac{a+bx+cx^2}{\sqrt{-1+x}\,\sqrt{1+x}\,(d+ex)^3}\,dx$

Optimal. Leaf size=199

$$-\frac{\sqrt{x-1}\,\sqrt{x+1}\,\left(ae^2-bde+cd^2\right)}{2e\left(d^2-e^2\right)(d+ex)^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{x+1}\,\sqrt{d+e}}{\sqrt{x-1}\,\sqrt{d-e}}\right)\left(d^2(2a+c)+e^2(a+2c)-3bde\right)}{(d-e)^{5/2}(d+e)^{5/2}}+\frac{\sqrt{x-1}\,\sqrt{x+1}\,\left(-d\right.}{2e\left(\right.}$$

Rubi [A] time = 0.33, antiderivative size = 242, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1610, 1651, 807, 725, 206}

$$-\frac{\left(1-x^2\right)\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right)\right)}{2e\sqrt{x-1}\,\sqrt{x+1}\,\left(d^2-e^2\right)^2(d+ex)}+\frac{\left(1-x^2\right)\left(ae^2-bde+cd^2\right)}{2e\sqrt{x-1}\,\sqrt{x+1}\,\left(d^2-e^2\right)(d+ex)^2}-\frac{\sqrt{x^2-1}\,\tanh^{-1}\left(\frac{dx+e}{\sqrt{x^2-1}\,\sqrt{d^2-e^2}}\right)\left(-a\left(2d^2+e^2\right)+3bde-c\left(d^2+2e^2\right)\right)}{2\sqrt{x-1}\,\sqrt{x+1}\,\left(d^2-e^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)^3),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(1 - x^2))/(2*e*(d^2 - e^2)*Sqrt[-1 + x]*Sqrt[1 +
x]*(d + e*x)^2) - ((c*(d^3 - 4*d*e^2) - e*(3*a*d*e - b*(d^2 + 2*e^2)))*(1 -
 x^2))/(2*e*(d^2 - e^2)^2*Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)) - ((3*b*d*e -
 a*(2*d^2 + e^2) - c*(d^2 + 2*e^2))*Sqrt[-1 + x^2]*ArcTanh[(e + d*x)/(Sqrt[
d^2 - e^2]*Sqrt[-1 + x^2])])/(2*(d^2 - e^2)^(5/2)*Sqrt[-1 + x]*Sqrt[1 + x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1651

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
```

```
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{-1+x}\sqrt{1+x}\,(d+ex)^3}\,dx = \frac{\sqrt{-1+x^2}\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x}\sqrt{1+x}}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\left(1-x^2\right)}{2e\left(d^2-e^2\right)\sqrt{-1+x}\sqrt{1+x}\,(d+ex)^2} - \frac{\sqrt{-1+x^2}\int \frac{-2(ad+cd-be)-\left(bd+\frac{cd^2}{e}-ae-2ce\right)}{(d+ex)^2\sqrt{-1+x^2}}}{2\left(d^2-e^2\right)\sqrt{-1+x}\sqrt{1+x}}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\left(1-x^2\right)}{2e\left(d^2-e^2\right)\sqrt{-1+x}\sqrt{1+x}\,(d+ex)^2} - \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right.\right.}{2e\left(d^2-e^2\right)^2\sqrt{-1+x}\sqrt{1+x}\,(d}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\left(1-x^2\right)}{2e\left(d^2-e^2\right)\sqrt{-1+x}\sqrt{1+x}\,(d+ex)^2} - \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right.\right.}{2e\left(d^2-e^2\right)^2\sqrt{-1+x}\sqrt{1+x}\,(d}$$

$$= \frac{\left(cd^2-bde+ae^2\right)\left(1-x^2\right)}{2e\left(d^2-e^2\right)\sqrt{-1+x}\sqrt{1+x}\,(d+ex)^2} - \frac{\left(c\left(d^3-4de^2\right)-e\left(3ade-b\left(d^2+2e^2\right)\right.\right.}{2e\left(d^2-e^2\right)^2\sqrt{-1+x}\sqrt{1+x}\,(d}$$

Mathematica [A] time = 0.76, size = 343, normalized size = 1.72

$$\frac{-(d+ex)\left(3de\sqrt{x-1}\sqrt{x+1}\sqrt{d-e}\sqrt{d+e}-2\left(2d^2+e^2\right)(d+ex)\tanh^{-1}\left(\frac{\sqrt{\frac{x-1}{x+1}}\sqrt{d-e}}{\sqrt{d+e}}\right)\right)\left(e(ae-bd)+cd^2\right)-e\sqrt{x-1}\sqrt{x+1}(d-e)^{3/2}(d+e)^{3/2}\left(e(ae-bd)+cd^2\right)+2e\sqrt{x-1}\sqrt{x+1}(d-e)^{3/2}(d+e)^{3/2}(d+ex)(2cd-be)-4d(d-e)(d+e)(d+ex)^2(2cd-be)\tanh^{-1}\left(\frac{\sqrt{\frac{x-1}{x+1}}\sqrt{d-e}}{\sqrt{d+e}}\right)+4c(d-e)^2(d+e)^2(d+ex)^2\tanh^{-1}\left(\frac{\sqrt{\frac{x-1}{x+1}}\sqrt{d-e}}{\sqrt{d+e}}\right)}{2e^2(d-e)^{5/2}(d+e)^{5/2}(d+ex)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)^3),x]
```

```
[Out] (-((d - e)^(3/2)*e*(d + e)^(3/2)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[-1 + x]*Sq
rt[1 + x]) + 2*(d - e)^(3/2)*e*(d + e)^(3/2)*(2*c*d - b*e)*Sqrt[-1 + x]*Sqr
t[1 + x]*(d + e*x) + 4*c*(d - e)^2*(d + e)^2*(d + e*x)^2*ArcTanh[(Sqrt[d -
e]*Sqrt[(-1 + x)/(1 + x)])/Sqrt[d + e]] - 4*d*(d - e)*(d + e)*(2*c*d - b*e)
*(d + e*x)^2*ArcTanh[(Sqrt[d - e]*Sqrt[(-1 + x)/(1 + x)])/Sqrt[d + e]] - (c
*d^2 + e*(-(b*d) + a*e))*(d + e*x)*(3*d*Sqrt[d - e]*e*Sqrt[d + e]*Sqrt[-1 +
 x]*Sqrt[1 + x] - 2*(2*d^2 + e^2)*(d + e*x)*ArcTanh[(Sqrt[d - e]*Sqrt[(-1 +
 x)/(1 + x)])/Sqrt[d + e]]))/(2*(d - e)^(5/2)*e^2*(d + e)^(5/2)*(d + e*x)^2
)
```

IntegrateAlgebraic [B] time = 0.67, size = 546, normalized size = 2.74

$$\frac{\tan^{-1}\left(\frac{\sqrt{x-1}\sqrt{-d-e}\sqrt{d-e}}{\sqrt{x+1}(d+e)}\right)\left(2ad^2+ae^2-3bde+cd^2+2ce^2\right)}{\sqrt{-d-e}(d-e)^{5/2}(d+e)^2}+\frac{-\frac{4ad^2e\sqrt{x-1}}{\sqrt{x+1}}+\frac{4ad^2e(x-1)^{3/2}}{(x+1)^{3/2}}-\frac{3ade^2\sqrt{x-1}}{\sqrt{x+1}}-\frac{3ade^2(x-1)^{3/2}}{(x+1)^{3/2}}+\frac{ae^3\sqrt{x-1}}{\sqrt{x+1}}-\frac{ae^3(x-1)^{3/2}}{(x+1)^{3/2}}+\frac{2bd^3\sqrt{x-1}}{\sqrt{x+1}}-\frac{2bd^3(x-1)^{3/2}}{(x+1)^{3/2}}+\frac{bd^2e\sqrt{x-1}}{\sqrt{x+1}}+\frac{bd^2e(x-1)^{3/2}}{(x+1)^{3/2}}+\frac{bde^2\sqrt{x-1}}{\sqrt{x+1}}-\frac{bde^2(x-1)^{3/2}}{(x+1)^{3/2}}+\frac{2be^3\sqrt{x-1}}{\sqrt{x+1}}+\frac{2be^3(x-1)^{3/2}}{(x+1)^{3/2}}+\frac{cd^3\sqrt{x-1}}{\sqrt{x+1}}+\frac{cd^3(x-1)^{3/2}}{(x+1)^{3/2}}-\frac{3cd^2e\sqrt{x-1}}{\sqrt{x+1}}+\frac{3cd^2e(x-1)^{3/2}}{(x+1)^{3/2}}-\frac{4cde^2\sqrt{x-1}}{\sqrt{x+1}}-\frac{4cde^2(x-1)^{3/2}}{(x+1)^{3/2}}}{(d-e)^2(d+e)^2\left(\frac{d(x-1)}{x+1}-d-\frac{e(x-1)}{x+1}-e\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(Sqrt[-1 + x]*Sqrt[1 + x]*(d + e*x)^3)
,x]
```

```
[Out] ((-2*b*d^3*(-1 + x)^(3/2))/(1 + x)^(3/2) + (c*d^3*(-1 + x)^(3/2))/(1 + x)^(
3/2) + (4*a*d^2*e*(-1 + x)^(3/2))/(1 + x)^(3/2) + (b*d^2*e*(-1 + x)^(3/2))/
(1 + x)^(3/2) + (3*c*d^2*e*(-1 + x)^(3/2))/(1 + x)^(3/2) - (3*a*d*e^2*(-1 +
 x)^(3/2))/(1 + x)^(3/2) - (b*d*e^2*(-1 + x)^(3/2))/(1 + x)^(3/2) - (4*c*d*
e^2*(-1 + x)^(3/2))/(1 + x)^(3/2) - (a*e^3*(-1 + x)^(3/2))/(1 + x)^(3/2) +
```

```
(2*b*e^3*(-1 + x)^(3/2))/(1 + x)^(3/2) + (2*b*d^3*Sqrt[-1 + x])/Sqrt[1 + x]
 + (c*d^3*Sqrt[-1 + x])/Sqrt[1 + x] - (4*a*d^2*e*Sqrt[-1 + x])/Sqrt[1 + x]
+ (b*d^2*e*Sqrt[-1 + x])/Sqrt[1 + x] - (3*c*d^2*e*Sqrt[-1 + x])/Sqrt[1 + x]
 - (3*a*d*e^2*Sqrt[-1 + x])/Sqrt[1 + x] + (b*d*e^2*Sqrt[-1 + x])/Sqrt[1 + x
] - (4*c*d*e^2*Sqrt[-1 + x])/Sqrt[1 + x] + (a*e^3*Sqrt[-1 + x])/Sqrt[1 + x]
 + (2*b*e^3*Sqrt[-1 + x])/Sqrt[1 + x])/((d - e)^2*(d + e)^2*(-d - e + (d*(-
1 + x))/(1 + x) - (e*(-1 + x))/(1 + x))^2) + ((2*a*d^2 + c*d^2 - 3*b*d*e +
a*e^2 + 2*c*e^2)*ArcTan[(Sqrt[-d - e]*Sqrt[d - e]*Sqrt[-1 + x])/((d + e)*Sq
rt[1 + x])])/(Sqrt[-d - e]*(d - e)^(5/2)*(d + e)^2)
```

fricas [**B**] time = 1.00, size = 1186, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fr
icas")
```

```
[Out] [1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c)*d^3*e
^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5
 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 + ((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 + (a
 + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^2 + 2*(
(2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(d^2 - e^2)*log((
d^2*x + d*e + (d^2 - e^2 + sqrt(d^2 - e^2)*d)*sqrt(x + 1)*sqrt(x - 1) + sqr
t(d^2 - e^2)*(d*x + e))/(e*x + d)) + (2*b*d^5*e^2 - (4*a + 3*c)*d^4*e^3 - b
*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 + b*d^4*e^3 -
 (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x)*sqrt(x +
 1)*sqrt(x - 1) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 + b*d^3*e^4
+ (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x)/(d^8*e^2 - 3*d^6*e^4 + 3*d^4*e^6 - d^
2*e^8 + (d^6*e^4 - 3*d^4*e^6 + 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3 - 3*d^5*e
^5 + 3*d^3*e^7 - d*e^9)*x), 1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*
d^4*e^3 + (3*a + 4*c)*d^3*e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a
 + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 - 2*((2*a +
c)*d^4*e^2 - 3*b*d^3*e^3 + (a + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e
^5 + (a + 2*c)*e^6)*x^2 + 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*
e^5)*x)*sqrt(-d^2 + e^2)*arctan(-(sqrt(-d^2 + e^2)*e*sqrt(x + 1)*sqrt(x - 1
) - sqrt(-d^2 + e^2)*(e*x + d))/(d^2 - e^2)) + (2*b*d^5*e^2 - (4*a + 3*c)*d
^4*e^3 - b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 + b
*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x
)*sqrt(x + 1)*sqrt(x - 1) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 +
b*d^3*e^4 + (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x)/(d^8*e^2 - 3*d^6*e^4 + 3*d^
4*e^6 - d^2*e^8 + (d^6*e^4 - 3*d^4*e^6 + 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3
 - 3*d^5*e^5 + 3*d^3*e^7 - d*e^9)*x)]
```

giac [**B**] time = 3.24, size = 605, normalized size = 3.04

$$-\frac{\left(2 a d^{2} + c d^{2} - 3 b d e + a e^{2} + 2 c e^{2}\right) \arctan\left(\frac{\left(\sqrt{x+1} - \sqrt{x-1}\right)^{2} e + 2 d}{2 \sqrt{-d^{2} + e^{2}}}\right)}{\left(d^{4} - 2 d^{2} e^{2} + e^{4}\right) \sqrt{-d^{2} + e^{2}}} + \frac{2 \left(2 c d^{4} \left(\sqrt{x+1} - \sqrt{x-1}\right)^{6} e + 4 c d^{5} \left(\sqrt{x+1} - \sqrt{x-1}\right)^{4} - 2 a d^{2} \left(\sqrt{x+1} - \sqrt{x-1}\right)^{6} e^{3} - 5 c d^{2} \left(\sqrt{x+1} - \sqrt{x-1}\right)^{6} e^{3} + 4 b d^{4} \left(\sqrt{x+1} - \sqrt{x-1}\right)^{4} e + 3 b d \left(\sqrt{x+1} - \sqrt{x-1}\right)^{6} e^{4} - 12 a d^{3} \left(\sqrt{x+1} - \sqrt{x-1}\right)^{4} e^{2} - 14 c d^{3} \left(\sqrt{x+1} - \sqrt{x-1}\right)^{4} e^{2} - a \left(\sqrt{x+1} - \sqrt{x-1}\right)^{6} e^{5} + 10 b d^{2} \left(\sqrt{x+1} - \sqrt{x-1}\right)^{4} e^{3} + 8 c d^{4} \left(\sqrt{x+1} - \sqrt{x-1}\right)^{2} e - 6 a d \left(\sqrt{x+1} - \sqrt{x-1}\right)^{4} e^{4} - 8 c d \left(\sqrt{x+1} - \sqrt{x-1}\right)^{4} e^{4} + 16 b d^{3} \left(\sqrt{x+1} - \sqrt{x-1}\right)^{2} e^{2} + 4 b \left(\sqrt{x+1} - \sqrt{x-1}\right)^{4} e^{5} - 40 a d^{2} \left(\sqrt{x+1} - \sqrt{x-1}\right)^{2} e^{3} - 44 c d^{2} \left(\sqrt{x+1} - \sqrt{x-1}\right)^{2} e^{3} + 20 b d \left(\sqrt{x+1} - \sqrt{x-1}\right)^{2} e^{4} + 8 c d^{3} e^{2} + 4 a \left(\sqrt{x+1} - \sqrt{x-1}\right)^{2} e^{5} + 8 b d^{2} e^{3} - 24 a d e^{4} - 32 c d e^{4} + 16 b e^{5}\right)}{\left(d^{4} e^{2} - 2 d^{2} e^{4} + e^{6}\right) \left(\left(\sqrt{x+1} - \sqrt{x-1}\right)^{4} e + 4 d \left(\sqrt{x+1} - \sqrt{x-1}\right)^{2} + 4 e\right)^{2}}$$

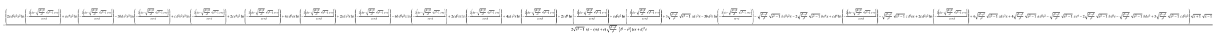
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -(2*a*d^2 + c*d^2 - 3*b*d*e + a*e^2 + 2*c*e^2)*arctan(1/2*((sqrt(x + 1) - s
qrt(x - 1))^2*e + 2*d)/sqrt(-d^2 + e^2))/((d^4 - 2*d^2*e^2 + e^4)*sqrt(-d^2
 + e^2)) + 2*(2*c*d^4*(sqrt(x + 1) - sqrt(x - 1))^6*e + 4*c*d^5*(sqrt(x + 1
) - sqrt(x - 1))^4 - 2*a*d^2*(sqrt(x + 1) - sqrt(x - 1))^6*e^3 - 5*c*d^2*(s
qrt(x + 1) - sqrt(x - 1))^6*e^3 + 4*b*d^4*(sqrt(x + 1) - sqrt(x - 1))^4*e +
 3*b*d*(sqrt(x + 1) - sqrt(x - 1))^6*e^4 - 12*a*d^3*(sqrt(x + 1) - sqrt(x -
 1))^4*e^2 - 14*c*d^3*(sqrt(x + 1) - sqrt(x - 1))^4*e^2 - a*(sqrt(x + 1) -
sqrt(x - 1))^6*e^5 + 10*b*d^2*(sqrt(x + 1) - sqrt(x - 1))^4*e^3 + 8*c*d^4*(
```

```
sqrt(x + 1) - sqrt(x - 1))^2*e - 6*a*d*(sqrt(x + 1) - sqrt(x - 1))^4*e^4 -
8*c*d*(sqrt(x + 1) - sqrt(x - 1))^4*e^4 + 16*b*d^3*(sqrt(x + 1) - sqrt(x -
1))^2*e^2 + 4*b*(sqrt(x + 1) - sqrt(x - 1))^4*e^5 - 40*a*d^2*(sqrt(x + 1) -
 sqrt(x - 1))^2*e^3 - 44*c*d^2*(sqrt(x + 1) - sqrt(x - 1))^2*e^3 + 20*b*d*(
sqrt(x + 1) - sqrt(x - 1))^2*e^4 + 8*c*d^3*e^2 + 4*a*(sqrt(x + 1) - sqrt(x
- 1))^2*e^5 + 8*b*d^2*e^3 - 24*a*d*e^4 - 32*c*d*e^4 + 16*b*e^5)/((d^4*e^2 -
 2*d^2*e^4 + e^6)*((sqrt(x + 1) - sqrt(x - 1))^4*e + 4*d*(sqrt(x + 1) - sqr
t(x - 1))^2 + 4*e)^2)
```

maple [B] time = 0.05, size = 1095, normalized size = 5.50



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(x-1)^(1/2)/(x+1)^(1/2),x)
```

```
[Out] -1/2*(3*x*a*d*e^3*((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)-2*x*b*e^4*((d^2-e^2)/
e^2)^(1/2)*(x^2-1)^(1/2)+2*ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*
x+e)/(e*x+d))*c*d^2*e^2-a*e^4*((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)+ln(-2*(-(
(d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x^2*a*e^4+2*ln(-2*(-((
d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x^2*c*e^4+ln(-2*(-((d^2
-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*a*d^2*e^2-3*ln(-2*(-((d^2-
e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*b*d^3*e+2*ln(-2*(-((d^2-e^2
)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*a*d^4+ln(-2*(-((d^2-e^2)/e^2)^
(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*c*d^4-x*b*d^2*e^2*((d^2-e^2)/e^2)^(1/
2)*(x^2-1)^(1/2)-x*c*d^3*e*((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)+4*x*c*d*e^3*
((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)+4*a*d^2*e^2*((d^2-e^2)/e^2)^(1/2)*(x^2-
1)^(1/2)-2*b*d^3*e*((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)-b*d*e^3*((d^2-e^2)/e
^2)^(1/2)*(x^2-1)^(1/2)+3*c*d^2*e^2*((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)+2*l
n(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x^2*a*d^2*e^2-
3*ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x^2*b*d*e^3
+ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x^2*c*d^2*e^
2+4*ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x*a*d^3*e
+2*ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x*a*d*e^3-
6*ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x*b*d^2*e^2
+2*ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x*c*d^3*e+
4*ln(-2*(-((d^2-e^2)/e^2)^(1/2)*(x^2-1)^(1/2)*e+d*x+e)/(e*x+d))*x*c*d*e^3)*
(x+1)^(1/2)*(x-1)^(1/2)/(x^2-1)^(1/2)/(d-e)/(d+e)/((d^2-e^2)/e^2)^(1/2)/(d^
2-e^2)/(e*x+d)^2/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-d>0)', see `assume?` for more det
ails)Is e-d positive, negative or zero?
```

mupad [B] time = 66.85, size = 7235, normalized size = 36.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((x - 1)^(1/2)*(x + 1)^(1/2)*(d + e*x)^3),x)
```

```
[Out] ((((x - 1)^(1/2) - 1i)^2*(2*c*e^3 + c*d^2*e)*12i)/(d^2*((x + 1)^(1/2) - 1)^
2*(d^4 + e^4 - 2*d^2*e^2)) - (2*(7*c*d^4 + 14*c*d^2*e^2)*((x - 1)^(1/2) - 1
i))/(7*d^3*((x + 1)^(1/2) - 1)*(d^4 + e^4 - 2*d^2*e^2)) + (((x - 1)^(1/2) -
 1i)^4*(2*c*e^3 - c*d^2*e)*24i)/(d^2*((x + 1)^(1/2) - 1)^4*(d^4 + e^4 - 2*d
^2*e^2)) - (2*(21*c*d^4 - 102*c*d^2*e^2)*((x - 1)^(1/2) - 1i)^5)/(3*d^3*((x
 + 1)^(1/2) - 1)^5*(d^4 + e^4 - 2*d^2*e^2)) - (2*(35*c*d^4 - 170*c*d^2*e^2)
*((x - 1)^(1/2) - 1i)^3)/(5*d^3*((x + 1)^(1/2) - 1)^3*(d^4 + e^4 - 2*d^2*e^
2)) + (c*((x - 1)^(1/2) - 1i)^7*(d^2*1i + e^2*2i)*2i)/(d*((x + 1)^(1/2) - 1
)^7*(d^4 + e^4 - 2*d^2*e^2)) + (12*c*e*((x - 1)^(1/2) - 1i)^6*(d^2*1i + e^2
*2i))/(d^2*((x + 1)^(1/2) - 1)^6*(d^4 + e^4 - 2*d^2*e^2)))/(((x - 1)^(1/2)
- 1i)^8/((x + 1)^(1/2) - 1)^8 - (e*((x - 1)^(1/2) - 1i)*8i)/(d*((x + 1)^(1/
2) - 1)) + (e*((x - 1)^(1/2) - 1i)^3*8i)/(d*((x + 1)^(1/2) - 1)^3) + (e*((x
 - 1)^(1/2) - 1i)^5*8i)/(d*((x + 1)^(1/2) - 1)^5) - (e*((x - 1)^(1/2) - 1i)
^7*8i)/(d*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^2*(4*d^2 + 16*e^2)
)/(d^2*((x + 1)^(1/2) - 1)^2) - (((x - 1)^(1/2) - 1i)^6*(4*d^2 + 16*e^2))/(
d^2*((x + 1)^(1/2) - 1)^6) + (((x - 1)^(1/2) - 1i)^4*(6*d^2 - 32*e^2))/(d^2
*((x + 1)^(1/2) - 1)^4) + 1) - ((2*((x - 1)^(1/2) - 1i)^3*(16*b*e^3 + 11*b*
d^2*e))/(d^2*((x + 1)^(1/2) - 1)^3*(d^4 + e^4 - 2*d^2*e^2)) - (6*b*e*((x -
1)^(1/2) - 1i)^7)/(((x + 1)^(1/2) - 1)^7*(d^4 + e^4 - 2*d^2*e^2)) - (6*b*e*
((x - 1)^(1/2) - 1i))/(((x + 1)^(1/2) - 1)*(d^4 + e^4 - 2*d^2*e^2)) + (((x
- 1)^(1/2) - 1i)^4*(2*b*e^4 - 2*b*d^4 + 3*b*d^2*e^2)*8i)/(d^3*((x + 1)^(1/2
) - 1)^4*(d^4 + e^4 - 2*d^2*e^2)) + (b*((x - 1)^(1/2) - 1i)^2*(2*d^4 + 2*e^
4 + 5*d^2*e^2)*4i)/(d^3*((x + 1)^(1/2) - 1)^2*(d^4 + e^4 - 2*d^2*e^2)) + (b
*((x - 1)^(1/2) - 1i)^6*(2*d^4 + 2*e^4 + 5*d^2*e^2)*4i)/(d^3*((x + 1)^(1/2)
 - 1)^6*(d^4 + e^4 - 2*d^2*e^2)) + (2*b*e*((x - 1)^(1/2) - 1i)^5*(11*d^2 +
16*e^2))/(d^2*((x + 1)^(1/2) - 1)^5*(d^4 + e^4 - 2*d^2*e^2)))/(((x - 1)^(1/
2) - 1i)^8/((x + 1)^(1/2) - 1)^8 - (e*((x - 1)^(1/2) - 1i)*8i)/(d*((x + 1)^
(1/2) - 1)) + (e*((x - 1)^(1/2) - 1i)^3*8i)/(d*((x + 1)^(1/2) - 1)^3) + (e*
((x - 1)^(1/2) - 1i)^5*8i)/(d*((x + 1)^(1/2) - 1)^5) - (e*((x - 1)^(1/2) -
1i)^7*8i)/(d*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^2*(4*d^2 + 16*e
^2))/(d^2*((x + 1)^(1/2) - 1)^2) - (((x - 1)^(1/2) - 1i)^6*(4*d^2 + 16*e^2)
)/(d^2*((x + 1)^(1/2) - 1)^6) + (((x - 1)^(1/2) - 1i)^4*(6*d^2 - 32*e^2))/(
d^2*((x + 1)^(1/2) - 1)^4) + 1) + ((2*(2*a*e^4 - 5*a*d^2*e^2)*((x - 1)^(1/2
) - 1i))/(d^3*((x + 1)^(1/2) - 1)*(d^4 + e^4 - 2*d^2*e^2)) - (((x - 1)^(1/2
) - 1i)^4*(2*a*e^5 - 9*a*d^2*e^3 + 4*a*d^4*e)*8i)/(d^4*((x + 1)^(1/2) - 1)^
4*(d^4 + e^4 - 2*d^2*e^2)) + (2*(2*a*e^4 - 5*a*d^2*e^2)*((x - 1)^(1/2) - 1i
)^7)/(d^3*((x + 1)^(1/2) - 1)^7*(d^4 + e^4 - 2*d^2*e^2)) - (2*(2*a*e^4 - 29
*a*d^2*e^2)*((x - 1)^(1/2) - 1i)^3)/(d^3*((x + 1)^(1/2) - 1)^3*(d^4 + e^4 -
 2*d^2*e^2)) - (2*(2*a*e^4 - 29*a*d^2*e^2)*((x - 1)^(1/2) - 1i)^5)/(d^3*((x
 + 1)^(1/2) - 1)^5*(d^4 + e^4 - 2*d^2*e^2)) + (e*((x - 1)^(1/2) - 1i)^2*(4*
a*d^4 - 2*a*e^4 + 7*a*d^2*e^2)*4i)/(d^4*((x + 1)^(1/2) - 1)^2*(d^4 + e^4 -
2*d^2*e^2)) + (e*((x - 1)^(1/2) - 1i)^6*(4*a*d^4 - 2*a*e^4 + 7*a*d^2*e^2)*4
i)/(d^4*((x + 1)^(1/2) - 1)^6*(d^4 + e^4 - 2*d^2*e^2)))/(((x - 1)^(1/2) - 1
i)^8/((x + 1)^(1/2) - 1)^8 - (e*((x - 1)^(1/2) - 1i)*8i)/(d*((x + 1)^(1/2)
- 1)) + (e*((x - 1)^(1/2) - 1i)^3*8i)/(d*((x + 1)^(1/2) - 1)^3) + (e*((x -
1)^(1/2) - 1i)^5*8i)/(d*((x + 1)^(1/2) - 1)^5) - (e*((x - 1)^(1/2) - 1i)^7*
8i)/(d*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^2*(4*d^2 + 16*e^2))/(
d^2*((x + 1)^(1/2) - 1)^2) - (((x - 1)^(1/2) - 1i)^6*(4*d^2 + 16*e^2))/(d^2
*((x + 1)^(1/2) - 1)^6) + (((x - 1)^(1/2) - 1i)^4*(6*d^2 - 32*e^2))/(d^2*((
x + 1)^(1/2) - 1)^4) + 1) - (c*atan(((c*(d^2 + 2*e^2)*((4*(c*e^7*8i - c*d^2
*e^5*12i + c*d^6*e*4i))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2
) + (4*((x - 1)^(1/2) - 1i)^2*(c*e^7*8i - c*d^2*e^5*12i + c*d^6*e*4i))/(((x
 + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) -
(c*(d^2 + 2*e^2)*((e*((x - 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1)) - (4
*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^10
 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2
*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/((
(x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)))
)/(2*(d + e)^(5/2)*(d - e)^(5/2)))*1i)/(2*(d + e)^(5/2)*(d - e)^(5/2)) + (c
```

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*(d^2 + 2*e^2)*((4*(c*e^7*8i - c*d^2*e^5*12i + c*d^6*e*4i))/(d^10 + d^2*e^8
 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(c*e^7*8i
 - c*d^2*e^5*12i + c*d^6*e*4i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*
d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) + (c*(d^2 + 2*e^2)*((e*((x - 1)^(1/2) - 1
i)*64i)/(d*((x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*
e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*
d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^
4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 -
4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2))))/(2*(d + e)^(5/2)*(d - e)^(5/2)))*1i)/
(2*(d + e)^(5/2)*(d - e)^(5/2)))/((8*(c^2*d^4 + 4*c^2*e^4 + 4*c^2*d^2*e^2))
/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) - (8*((x - 1)^(1/2) -
 1i)^2*(c^2*d^4 + 4*c^2*e^4 + 4*c^2*d^2*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10
+ d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) - (c*(d^2 + 2*e^2)*((4*(c*e
^7*8i - c*d^2*e^5*12i + c*d^6*e*4i))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^
4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(c*e^7*8i - c*d^2*e^5*12i + c*d^
6*e*4i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4
*d^8*e^2)) - (c*(d^2 + 2*e^2)*((e*((x - 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/
2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^
8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^
(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28
*d^8*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 -
 4*d^8*e^2))))/(2*(d + e)^(5/2)*(d - e)^(5/2))))/(2*(d + e)^(5/2)*(d - e)^(
5/2)) + (c*(d^2 + 2*e^2)*((4*(c*e^7*8i - c*d^2*e^5*12i + c*d^6*e*4i))/(d^10
 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2
*(c*e^7*8i - c*d^2*e^5*12i + c*d^6*e*4i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^
2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) + (c*(d^2 + 2*e^2)*((e*((x - 1)
^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2*e^
8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^
6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^2*e
^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10 +
d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2))))/(2*(d + e)^(5/2)*(d - e)^(5
/2))))/(2*(d + e)^(5/2)*(d - e)^(5/2))))*(d^2 + 2*e^2)*1i)/((d + e)^(5/2)*(
d - e)^(5/2)) - (a*atan(((a*(2*d^2 + e^2)*((4*(a*e^7*4i - a*d^4*e^3*12i + a
*d^6*e*8i))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x -
 1)^(1/2) - 1i)^2*(a*e^7*4i - a*d^4*e^3*12i + a*d^6*e*8i))/(((x + 1)^(1/2)
- 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) - (a*(2*d^2 +
e^2)*((e*((x - 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4
*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 -
 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 1
2*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2
) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2))))/(2*(d + e)
^(5/2)*(d - e)^(5/2)))*1i)/(2*(d + e)^(5/2)*(d - e)^(5/2)) + (a*(2*d^2 + e^
2)*((4*(a*e^7*4i - a*d^4*e^3*12i + a*d^6*e*8i))/(d^10 + d^2*e^8 - 4*d^4*e^6
 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(a*e^7*4i - a*d^4*e^3
*12i + a*d^6*e*8i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*
d^6*e^4 - 4*d^8*e^2)) + (a*(2*d^2 + e^2)*((e*((x - 1)^(1/2) - 1i)*64i)/(d*(
(x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*
e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (
4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d
^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 +
6*d^6*e^4 - 4*d^8*e^2))))/(2*(d + e)^(5/2)*(d - e)^(5/2)))*1i)/(2*(d + e)^(
5/2)*(d - e)^(5/2)))/((8*(4*a^2*d^4 + a^2*e^4 + 4*a^2*d^2*e^2))/(d^10 + d^2
*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) - (8*((x - 1)^(1/2) - 1i)^2*(4*a^
2*d^4 + a^2*e^4 + 4*a^2*d^2*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 -
4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) - (a*(2*d^2 + e^2)*((4*(a*e^7*4i - a*d^
4*e^3*12i + a*d^6*e*8i))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^
2) + (4*((x - 1)^(1/2) - 1i)^2*(a*e^7*4i - a*d^4*e^3*12i + a*d^6*e*8i))/(((
x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) -
 (a*(2*d^2 + e^2)*((e*((x - 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1)) - (
```

```
4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^1
0 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^
2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/(
((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2))
))/(2*(d + e)^(5/2)*(d - e)^(5/2))))/(2*(d + e)^(5/2)*(d - e)^(5/2)) + (a*(
2*d^2 + e^2)*((4*(a*e^7*4i - a*d^4*e^3*12i + a*d^6*e*8i))/(d^10 + d^2*e^8 -
 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(a*e^7*4i -
 a*d^4*e^3*12i + a*d^6*e*8i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^
4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) + (a*(2*d^2 + e^2)*((e*((x - 1)^(1/2) - 1i)
*64i)/(d*((x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^
6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^
8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*
e^6 + 72*d^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*
d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2))))/(2*(d + e)^(5/2)*(d - e)^(5/2))))/(2*(d
 + e)^(5/2)*(d - e)^(5/2))))*(2*d^2 + e^2)*1i)/((d + e)^(5/2)*(d - e)^(5/2)
) + (b*d*e*atan(((b*d*e*((4*(b*d^5*e^2*12i - b*d^3*e^4*24i + b*d*e^6*12i))/
(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) -
1i)^2*(b*d^5*e^2*12i - b*d^3*e^4*24i + b*d*e^6*12i))/(((x + 1)^(1/2) - 1)^2
*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) - (3*b*d*e*((e*((x -
 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2
*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6
*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^
2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10
 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2))))/(2*(d + e)^(5/2)*(d - e)
^(5/2)))*3i)/(2*(d + e)^(5/2)*(d - e)^(5/2)) + (b*d*e*((4*(b*d^5*e^2*12i -
b*d^3*e^4*24i + b*d*e^6*12i))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d
^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(b*d^5*e^2*12i - b*d^3*e^4*24i + b*d*e^
6*12i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*
d^8*e^2)) + (3*b*d*e*((e*((x - 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1))
- (4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(
d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1
i)^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2)
)/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^
2))))/(2*(d + e)^(5/2)*(d - e)^(5/2)))*3i)/(2*(d + e)^(5/2)*(d - e)^(5/2)))
/((72*b^2*d^2*e^2)/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) - (
72*b^2*d^2*e^2*((x - 1)^(1/2) - 1i)^2)/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e
^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) - (3*b*d*e*((4*(b*d^5*e^2*12i - b*
d^3*e^4*24i + b*d*e^6*12i))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8
*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(b*d^5*e^2*12i - b*d^3*e^4*24i + b*d*e^6*
12i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^
8*e^2)) - (3*b*d*e*((e*((x - 1)^(1/2) - 1i)*64i)/(d*((x + 1)^(1/2) - 1)) -
(4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 + 8*d^6*e^4 - 12*d^8*e^2))/(d^
10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)
^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6 + 72*d^6*e^4 - 28*d^8*e^2))/
(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)
)))/(2*(d + e)^(5/2)*(d - e)^(5/2))))/(2*(d + e)^(5/2)*(d - e)^(5/2)) + (3*
b*d*e*((4*(b*d^5*e^2*12i - b*d^3*e^4*24i + b*d*e^6*12i))/(d^10 + d^2*e^8 -
4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2) + (4*((x - 1)^(1/2) - 1i)^2*(b*d^5*e^2*1
2i - b*d^3*e^4*24i + b*d*e^6*12i))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 -
 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e^2)) + (3*b*d*e*((e*((x - 1)^(1/2) - 1i)*64
i)/(d*((x + 1)^(1/2) - 1)) - (4*(4*d^10 + 4*e^10 - 12*d^2*e^8 + 8*d^4*e^6 +
 8*d^6*e^4 - 12*d^8*e^2))/(d^10 + d^2*e^8 - 4*d^4*e^6 + 6*d^6*e^4 - 4*d^8*e
^2) + (4*((x - 1)^(1/2) - 1i)^2*(4*d^10 - 12*e^10 + 52*d^2*e^8 - 88*d^4*e^6
 + 72*d^6*e^4 - 28*d^8*e^2))/(((x + 1)^(1/2) - 1)^2*(d^10 + d^2*e^8 - 4*d^4
*e^6 + 6*d^6*e^4 - 4*d^8*e^2))))/(2*(d + e)^(5/2)*(d - e)^(5/2))))/(2*(d +
e)^(5/2)*(d - e)^(5/2))))*3i)/((d + e)^(5/2)*(d - e)^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Timed out
```

3.41 $\int (a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=1348

$$\frac{C(c+dx)^{3/2}(e+fx)^{3/2}(a+bx)^3}{6bdf}-\frac{(2aCdf-b(4Bdf-3C(de+cf)))(c+dx)^{3/2}(e+fx)^{3/2}(a+bx)^2}{20bd^2f^2}-\frac{(c+dx)^{3/2}}{}$$

Rubi [A] time = 2.37, antiderivative size = 1345, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1615, 153, 147, 50, 63, 217, 206}

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((d*e - c*f)*(8*a^2*d^2*f^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*
(2*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^
2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*
e*f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 + 28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f
^2 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f +
 5*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*
Sqrt[c + d*x]*Sqrt[e + f*x])/(512*d^5*f^5) + ((8*a^2*d^2*f^2*(C*(5*d^2*e^2
+ 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*
(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d
*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 +
28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f^2 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*
(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f
 + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(256*d^5*f^
4) + ((4*b*B*d*f - 2*a*C*d*f - 3*b*C*(d*e + c*f))*(a + b*x)^2*(c + d*x)^(3/
2)*(e + f*x)^(3/2))/(20*b*d^2*f^2) + (C*(a + b*x)^3*(c + d*x)^(3/2)*(e + f*
x)^(3/2))/(6*b*d*f) - ((c + d*x)^(3/2)*(e + f*x)^(3/2)*(64*a^3*C*d^3*f^3 -
8*a^2*b*d^2*f^2*(16*B*d*f - 7*C*(d*e + c*f)) - 8*a*b^2*d*f*(C*(35*d^2*e^2 +
 38*c*d*e*f + 35*c^2*f^2) + 10*d*f*(8*A*d*f - 5*B*(d*e + c*f))) + b^3*(7*C*
(15*d^3*e^3 + 17*c*d^2*e^2*f + 17*c^2*d*e*f^2 + 15*c^3*f^3) + 4*d*f*(50*A*d
*f*(d*e + c*f) - B*(35*d^2*e^2 + 38*c*d*e*f + 35*c^2*f^2))) + 6*b*d*f*(10*b
*d*f*(2*b*c*C*e + a*C*d*e + a*c*C*f - 4*A*b*d*f) - (4*a*d*f - 7*b*(d*e + c*
f))*(4*b*B*d*f - 2*a*C*d*f - 3*b*C*(d*e + c*f)))*x))/(960*b*d^4*f^4) - ((d*
e - c*f)^2*(8*a^2*d^2*f^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2
*A*d*f - B*(d*e + c*f))) - 8*a*b*d*f*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*
d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*
f + 5*c^2*f^2))) + b^2*(C*(21*d^4*e^4 + 28*c*d^3*e^3*f + 30*c^2*d^2*e^2*f^2
 + 28*c^3*d*e*f^3 + 21*c^4*f^4) + 4*d*f*(2*A*d*f*(5*d^2*e^2 + 6*c*d*e*f + 5
*c^2*f^2) - B*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3))))*Ar
cTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(512*d^(11/2)*f^(11
/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int (a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)dx = \frac{C(a+bx)^3(c+dx)^{3/2}(e+fx)^{3/2}}{6bdf} + \frac{\int (a+bx)^2\sqrt{c+dx}\,\sqrt{e}}{}$$

$$= \frac{(4bBdf-2aCdf-3bC(de+cf))(a+bx)^2(c+dx)^{3/2}(e+}{20bd^2f^2}$$

$$= \frac{(4bBdf-2aCdf-3bC(de+cf))(a+bx)^2(c+dx)^{3/2}(e+}{20bd^2f^2}$$

$$= \frac{\left(8a^2d^2f^2\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de\right.\right.}{}$$

$$= \frac{(de-cf)\left(8a^2d^2f^2\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2A\right.\right.}{}$$

$$= \frac{(de-cf)\left(8a^2d^2f^2\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2A\right.\right.}{}$$

$$= \frac{(de-cf)\left(8a^2d^2f^2\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2A\right.\right.}{}$$

$$= \frac{(de-cf)\left(8a^2d^2f^2\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2A\right.\right.}{}$$

<u>Mathematica</u> [B] time = 7.13, size = 3599, normalized size = 2.67

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (2*b^2*C*(d*e - c*f)^4*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/(
(d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(11/2)*((63/(128*
(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f
))))^5) + 21/(32*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (
c*d*f)/(d*e - c*f))))^4) + 63/(80*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e
)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3) + 9/(10*(1 + (d*f*(c + d*x))/((d*
e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c +
d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1))/4 +
(63*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c
+ d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt
[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e
 - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]
*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/(
(d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(2048*d^2*f^2*
(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f
)/(d*e - c*f))))^5)))/(3*d^5*f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c
*f)))^(9/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*b*(d*e - c*f)^3*(-4*b*C*e
 + b*B*f + 2*a*C*f)*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*
e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(9/2)*((3*(35/(64*(1
 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))
))^4) + 35/(48*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*
```

```
d*f)/(d*e - c*f))))^3) + 7/(8*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d
*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*(
(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1)))/10 + (21*(d*e - c*f)^2*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*
f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c
 + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*
e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e
 - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*
e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]))))/(512*d^2*f^2*(c + d*x)^2*(1 + (d
*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^4)
))/(3*d^4*f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(7/2)*Sqrt[(d
*(e + f*x))/(d*e - c*f)]) + (2*(d*e - c*f)^2*(6*b^2*C*e^2 - 3*b^2*B*e*f - 6
*a*b*C*e*f + A*b^2*f^2 + 2*a*b*B*f^2 + a^2*C*f^2)*(c + d*x)^(3/2)*Sqrt[e +
f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e
- c*f))))^(7/2)*((3*(5/(8*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e -
 c*f) - (c*d*f)/(d*e - c*f))))^3) + 5/(6*(1 + (d*f*(c + d*x))/((d*e - c*f)*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d
*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1)))/8 + (15*(d*e
 - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/
((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt
[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*
Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d
^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c
*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]))))/(256*d^2*f^2*(c + d*x)
^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
c*f))))^3)))/(3*d^3*f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(5/
2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*(-(b*e) + a*f)*(d*e - c*f)*(4*b*C*
e^2 - 3*b*B*e*f - 2*a*C*e*f + 2*A*b*f^2 + a*B*f^2)*(c + d*x)^(3/2)*Sqrt[e +
 f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e
 - c*f))))^(5/2)*((3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c
*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e
)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1))/2 + (3*(d*e - c*f)^2*((d^2*e)/
(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*
e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*A
rcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e -
 c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) -
 (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e -
 c*f) - (c*d*f)/(d*e - c*f)))]))))/(32*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*
x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2)))/(3*d^2*
f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(3/2)*Sqrt[(d*(e + f*x)
)/(d*e - c*f)]) + (2*(-(b*e) + a*f)^2*(C*e^2 - B*e*f + A*f^2)*(c + d*x)^(3/
2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (
c*d*f)/(d*e - c*f))))^(3/2)*(3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e
)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))) + (3*(d*e - c*f)^2*((d^2*e)/(d*e -
c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e
 - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[
(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) -
 (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f
)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) -
 (c*d*f)/(d*e - c*f)))]))))/(16*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d
*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))))))/(3*d*f^4*Sqrt[d/
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*f)
])
```

IntegrateAlgebraic [B] time = 5.30, size = 9831, normalized size = 7.29

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2
),x]
```

```
[Out] Result too large to show
```

fricas [A] time = 8.08, size = 3096, normalized size = 2.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/30720*(15*(21*C*b^2*d^6*e^6 - 14*(C*b^2*c*d^5 + 2*(2*C*a*b + B*b^2)*d^6)
*e^5*f - 5*(C*b^2*c^2*d^4 - 4*(2*C*a*b + B*b^2)*c*d^5 - 8*(C*a^2 + 2*B*a*b
+ A*b^2)*d^6)*e^4*f^2 - 4*(C*b^2*c^3*d^3 - 2*(2*C*a*b + B*b^2)*c^2*d^4 + 8*
(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 16*(B*a^2 + 2*A*a*b)*d^6)*e^3*f^3 - (5*C*
b^2*c^4*d^2 - 128*A*a^2*d^6 - 8*(2*C*a*b + B*b^2)*c^3*d^3 + 16*(C*a^2 + 2*B
*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2*A*a*b)*c*d^5)*e^2*f^4 - 2*(7*C*b^2*c^
5*d + 128*A*a^2*c*d^5 - 10*(2*C*a*b + B*b^2)*c^4*d^2 + 16*(C*a^2 + 2*B*a*b
+ A*b^2)*c^3*d^3 - 32*(B*a^2 + 2*A*a*b)*c^2*d^4)*e*f^5 + (21*C*b^2*c^6 + 12
8*A*a^2*c^2*d^4 - 28*(2*C*a*b + B*b^2)*c^5*d + 40*(C*a^2 + 2*B*a*b + A*b^2)
*c^4*d^2 - 64*(B*a^2 + 2*A*a*b)*c^3*d^3)*f^6)*sqrt(d*f)*log(8*d^2*f^2*x^2 +
 d^2*e^2 + 6*c*d*e*f + c^2*f^2 - 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x
 + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) + 4*(1280*C*b^2*d^6*f^6*x^5
+ 315*C*b^2*d^6*e^5*f - 105*(C*b^2*c*d^5 + 4*(2*C*a*b + B*b^2)*d^6)*e^4*f^2
 - 2*(41*C*b^2*c^2*d^4 - 80*(2*C*a*b + B*b^2)*c*d^5 - 300*(C*a^2 + 2*B*a*b
+ A*b^2)*d^6)*e^3*f^3 - 2*(41*C*b^2*c^3*d^3 - 68*(2*C*a*b + B*b^2)*c^2*d^4
+ 140*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 480*(B*a^2 + 2*A*a*b)*d^6)*e^2*f^4
- 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 32*(2*C*a*b + B*b^2)*c^3*d^3 + 56*(
C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 128*(B*a^2 + 2*A*a*b)*c*d^5)*e*f^5 + 15*
(21*C*b^2*c^5*d + 128*A*a^2*c*d^5 - 28*(2*C*a*b + B*b^2)*c^4*d^2 + 40*(C*a^
2 + 2*B*a*b + A*b^2)*c^3*d^3 - 64*(B*a^2 + 2*A*a*b)*c^2*d^4)*f^6 + 128*(C*b
^2*d^6*e*f^5 + (C*b^2*c*d^5 + 12*(2*C*a*b + B*b^2)*d^6)*f^6)*x^4 - 16*(9*C*
b^2*d^6*e^2*f^4 - 2*(C*b^2*c*d^5 + 6*(2*C*a*b + B*b^2)*d^6)*e*f^5 + 3*(3*C*
b^2*c^2*d^4 - 4*(2*C*a*b + B*b^2)*c*d^5 - 40*(C*a^2 + 2*B*a*b + A*b^2)*d^6)
*f^6)*x^3 + 8*(21*C*b^2*d^6*e^3*f^3 - (5*C*b^2*c*d^5 + 28*(2*C*a*b + B*b^2)
*d^6)*e^2*f^4 - (5*C*b^2*c^2*d^4 - 8*(2*C*a*b + B*b^2)*c*d^5 - 40*(C*a^2 +
2*B*a*b + A*b^2)*d^6)*e*f^5 + (21*C*b^2*c^3*d^3 - 28*(2*C*a*b + B*b^2)*c^2*
d^4 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 320*(B*a^2 + 2*A*a*b)*d^6)*f^6)*
x^2 - 2*(105*C*b^2*d^6*e^4*f^2 - 28*(C*b^2*c*d^5 + 5*(2*C*a*b + B*b^2)*d^6)
*e^3*f^3 - 2*(13*C*b^2*c^2*d^4 - 22*(2*C*a*b + B*b^2)*c*d^5 - 100*(C*a^2 +
2*B*a*b + A*b^2)*d^6)*e^2*f^4 - 4*(7*C*b^2*c^3*d^3 - 11*(2*C*a*b + B*b^2)*c
^2*d^4 + 20*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 80*(B*a^2 + 2*A*a*b)*d^6)*e*f
^5 + 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 28*(2*C*a*b + B*b^2)*c^3*d^3 + 4
0*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2*A*a*b)*c*d^5)*f^6)*x)*s
qrt(d*x + c)*sqrt(f*x + e))/(d^6*f^6), 1/15360*(15*(21*C*b^2*d^6*e^6 - 14*(
C*b^2*c*d^5 + 2*(2*C*a*b + B*b^2)*d^6)*e^5*f - 5*(C*b^2*c^2*d^4 - 4*(2*C*a*
b + B*b^2)*c*d^5 - 8*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e^4*f^2 - 4*(C*b^2*c^3*
d^3 - 2*(2*C*a*b + B*b^2)*c^2*d^4 + 8*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 16*
(B*a^2 + 2*A*a*b)*d^6)*e^3*f^3 - (5*C*b^2*c^4*d^2 - 128*A*a^2*d^6 - 8*(2*C*
a*b + B*b^2)*c^3*d^3 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2
*A*a*b)*c*d^5)*e^2*f^4 - 2*(7*C*b^2*c^5*d + 128*A*a^2*c*d^5 - 10*(2*C*a*b +
 B*b^2)*c^4*d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 32*(B*a^2 + 2*A*a*
b)*c^2*d^4)*e*f^5 + (21*C*b^2*c^6 + 128*A*a^2*c^2*d^4 - 28*(2*C*a*b + B*b^2
)*c^5*d + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^4*d^2 - 64*(B*a^2 + 2*A*a*b)*c^3*d
^3)*f^6)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x +
c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(1280
*C*b^2*d^6*f^6*x^5 + 315*C*b^2*d^6*e^5*f - 105*(C*b^2*c*d^5 + 4*(2*C*a*b +
B*b^2)*d^6)*e^4*f^2 - 2*(41*C*b^2*c^2*d^4 - 80*(2*C*a*b + B*b^2)*c*d^5 - 30
0*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e^3*f^3 - 2*(41*C*b^2*c^3*d^3 - 68*(2*C*a*
```

```
b + B*b^2)*c^2*d^4 + 140*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 480*(B*a^2 + 2*A
*a*b)*d^6)*e^2*f^4 - 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 32*(2*C*a*b + B*
b^2)*c^3*d^3 + 56*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 128*(B*a^2 + 2*A*a*b)
*c*d^5)*e*f^5 + 15*(21*C*b^2*c^5*d + 128*A*a^2*c*d^5 - 28*(2*C*a*b + B*b^2)
*c^4*d^2 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^3 - 64*(B*a^2 + 2*A*a*b)*c^2*
d^4)*f^6 + 128*(C*b^2*d^6*e*f^5 + (C*b^2*c*d^5 + 12*(2*C*a*b + B*b^2)*d^6)*
f^6)*x^4 - 16*(9*C*b^2*d^6*e^2*f^4 - 2*(C*b^2*c*d^5 + 6*(2*C*a*b + B*b^2)*d
^6)*e*f^5 + 3*(3*C*b^2*c^2*d^4 - 4*(2*C*a*b + B*b^2)*c*d^5 - 40*(C*a^2 + 2*
B*a*b + A*b^2)*d^6)*f^6)*x^3 + 8*(21*C*b^2*d^6*e^3*f^3 - (5*C*b^2*c*d^5 + 2
8*(2*C*a*b + B*b^2)*d^6)*e^2*f^4 - (5*C*b^2*c^2*d^4 - 8*(2*C*a*b + B*b^2)*c
*d^5 - 40*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e*f^5 + (21*C*b^2*c^3*d^3 - 28*(2*
C*a*b + B*b^2)*c^2*d^4 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 320*(B*a^2 +
2*A*a*b)*d^6)*f^6)*x^2 - 2*(105*C*b^2*d^6*e^4*f^2 - 28*(C*b^2*c*d^5 + 5*(2*
C*a*b + B*b^2)*d^6)*e^3*f^3 - 2*(13*C*b^2*c^2*d^4 - 22*(2*C*a*b + B*b^2)*c*
d^5 - 100*(C*a^2 + 2*B*a*b + A*b^2)*d^6)*e^2*f^4 - 4*(7*C*b^2*c^3*d^3 - 11*
(2*C*a*b + B*b^2)*c^2*d^4 + 20*(C*a^2 + 2*B*a*b + A*b^2)*c*d^5 + 80*(B*a^2
+ 2*A*a*b)*d^6)*e*f^5 + 5*(21*C*b^2*c^4*d^2 - 384*A*a^2*d^6 - 28*(2*C*a*b +
 B*b^2)*c^3*d^3 + 40*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^4 - 64*(B*a^2 + 2*A*a*
b)*c*d^5)*f^6)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^6*f^6)]
```

giac [B] time = 6.33, size = 4708, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/7680*(7680*((c*d*f - d^2*e)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x
+ c)*d*f - c*d*f + d^2*e)))/sqrt(d*f) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)
*sqrt(d*x + c))*A*a^2*c*abs(d)/d^2 + 320*(sqrt((d*x + c)*d*f - c*d*f + d^2*
e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^3*e)
/(d^7*f^4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4)) +
 3*(5*c^3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*f)*s
qrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))*C*a
^2*c*abs(d)/d^2 + 640*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2
*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^3*e)/(d^7*f^4)) + 3*(11
*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4)) + 3*(5*c^3*f^3 - 3*c
^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt
((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))*B*a*b*c*abs(d)/d^2 + 8
0*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x +
c)/d^3 - (25*c*d^11*f^6 - d^12*f^5*e)/(d^14*f^6)) + (163*c^2*d^11*f^6 - 14*
c*d^12*f^5*e - 5*d^13*f^4*e^2)/(d^14*f^6)) - 3*(93*c^3*d^11*f^6 - 15*c^2*d^
12*f^5*e - 9*c*d^13*f^4*e^2 - 5*d^14*f^3*e^3)/(d^14*f^6))*sqrt(d*x + c) - 3
*(35*c^4*f^4 - 20*c^3*d*f^3*e - 6*c^2*d^2*f^2*e^2 - 4*c*d^3*f*e^3 - 5*d^4*e
^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e))
)/(sqrt(d*f)*d^2*f^3))*C*a*b*c*abs(d)/d^2 + 320*(sqrt((d*x + c)*d*f - c*d*f
 + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6
*f^3*e)/(d^7*f^4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*
f^4)) + 3*(5*c^3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt
(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^
2))*A*b^2*c*abs(d)/d^2 + 40*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x +
c)*(4*(d*x + c)*(6*(d*x + c)/d^3 - (25*c*d^11*f^6 - d^12*f^5*e)/(d^14*f^6))
 + (163*c^2*d^11*f^6 - 14*c*d^12*f^5*e - 5*d^13*f^4*e^2)/(d^14*f^6)) - 3*(9
3*c^3*d^11*f^6 - 15*c^2*d^12*f^5*e - 9*c*d^13*f^4*e^2 - 5*d^14*f^3*e^3)/(d^
14*f^6))*sqrt(d*x + c) - 3*(35*c^4*f^4 - 20*c^3*d*f^3*e - 6*c^2*d^2*f^2*e^2
 - 4*c*d^3*f*e^3 - 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x
+ c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^3))*B*b^2*c*abs(d)/d^2 + 4*(sq
rt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(4*(d*x + c)*(6*(d*x + c)*(8*(d*x + c)
/d^4 - (41*c*d^19*f^8 - d^20*f^7*e)/(d^23*f^8)) + (513*c^2*d^19*f^8 - 26*c*
```

```
d^20*f^7*e - 7*d^21*f^6*e^2)/(d^23*f^8)) - 5*(447*c^3*d^19*f^8 - 37*c^2*d^2
0*f^7*e - 19*c*d^21*f^6*e^2 - 7*d^22*f^5*e^3)/(d^23*f^8))*(d*x + c) + 15*(1
93*c^4*d^19*f^8 - 28*c^3*d^20*f^7*e - 18*c^2*d^21*f^6*e^2 - 12*c*d^22*f^5*e
^3 - 7*d^23*f^4*e^4)/(d^23*f^8))*sqrt(d*x + c) + 15*(63*c^5*f^5 - 35*c^4*d*
f^4*e - 10*c^3*d^2*f^3*e^2 - 6*c^2*d^3*f^2*e^3 - 5*c*d^4*f*e^4 - 7*d^5*e^5)
*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(
sqrt(d*f)*d^3*f^4))*C*b^2*c*abs(d)/d^2 + 320*(sqrt((d*x + c)*d*f - c*d*f +
d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^
3*e)/(d^7*f^4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4
)) + 3*(5*c^3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*
f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))
*B*a^2*abs(d)/d + 40*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(
d*x + c)*(6*(d*x + c)/d^3 - (25*c*d^11*f^6 - d^12*f^5*e)/(d^14*f^6)) + (163
*c^2*d^11*f^6 - 14*c*d^12*f^5*e - 5*d^13*f^4*e^2)/(d^14*f^6)) - 3*(93*c^3*d
^11*f^6 - 15*c^2*d^12*f^5*e - 9*c*d^13*f^4*e^2 - 5*d^14*f^3*e^3)/(d^14*f^6)
)*sqrt(d*x + c) - 3*(35*c^4*f^4 - 20*c^3*d*f^3*e - 6*c^2*d^2*f^2*e^2 - 4*c*
d^3*f*e^3 - 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*
f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^3))*C*a^2*abs(d)/d + 640*(sqrt((d*x +
 c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*
c*d^5*f^4 - d^6*f^3*e)/(d^7*f^4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7
*f^2*e^2)/(d^7*f^4)) + 3*(5*c^3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3
)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/
(sqrt(d*f)*d*f^2))*A*a*b*abs(d)/d + 80*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)
*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 - (25*c*d^11*f^6 - d^12*f^5*e)/
(d^14*f^6)) + (163*c^2*d^11*f^6 - 14*c*d^12*f^5*e - 5*d^13*f^4*e^2)/(d^14*f
^6)) - 3*(93*c^3*d^11*f^6 - 15*c^2*d^12*f^5*e - 9*c*d^13*f^4*e^2 - 5*d^14*f
^3*e^3)/(d^14*f^6))*sqrt(d*x + c) - 3*(35*c^4*f^4 - 20*c^3*d*f^3*e - 6*c^2*
d^2*f^2*e^2 - 4*c*d^3*f*e^3 - 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) +
 sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^3))*B*a*b*abs(d)/d
+ 8*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(4*(d*x + c)*(6*(d*x + c)*(8*(d
*x + c)/d^4 - (41*c*d^19*f^8 - d^20*f^7*e)/(d^23*f^8)) + (513*c^2*d^19*f^8
- 26*c*d^20*f^7*e - 7*d^21*f^6*e^2)/(d^23*f^8)) - 5*(447*c^3*d^19*f^8 - 37*
c^2*d^20*f^7*e - 19*c*d^21*f^6*e^2 - 7*d^22*f^5*e^3)/(d^23*f^8))*(d*x + c)
+ 15*(193*c^4*d^19*f^8 - 28*c^3*d^20*f^7*e - 18*c^2*d^21*f^6*e^2 - 12*c*d^2
2*f^5*e^3 - 7*d^23*f^4*e^4)/(d^23*f^8))*sqrt(d*x + c) + 15*(63*c^5*f^5 - 35
*c^4*d*f^4*e - 10*c^3*d^2*f^3*e^2 - 6*c^2*d^3*f^2*e^3 - 5*c*d^4*f*e^4 - 7*d
^5*e^5)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2
*e)))/(sqrt(d*f)*d^3*f^4))*C*a*b*abs(d)/d + 40*(sqrt((d*x + c)*d*f - c*d*f
+ d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 - (25*c*d^11*f^6 - d^12
*f^5*e)/(d^14*f^6)) + (163*c^2*d^11*f^6 - 14*c*d^12*f^5*e - 5*d^13*f^4*e^2)
/(d^14*f^6)) - 3*(93*c^3*d^11*f^6 - 15*c^2*d^12*f^5*e - 9*c*d^13*f^4*e^2 -
5*d^14*f^3*e^3)/(d^14*f^6))*sqrt(d*x + c) - 3*(35*c^4*f^4 - 20*c^3*d*f^3*e
- 6*c^2*d^2*f^2*e^2 - 4*c*d^3*f*e^3 - 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*
x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^3))*A*b^2*a
bs(d)/d + 4*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(4*(d*x + c)*(6*(d*x +
c)*(8*(d*x + c)/d^4 - (41*c*d^19*f^8 - d^20*f^7*e)/(d^23*f^8)) + (513*c^2*d
^19*f^8 - 26*c*d^20*f^7*e - 7*d^21*f^6*e^2)/(d^23*f^8)) - 5*(447*c^3*d^19*f
^8 - 37*c^2*d^20*f^7*e - 19*c*d^21*f^6*e^2 - 7*d^22*f^5*e^3)/(d^23*f^8))*(d
*x + c) + 15*(193*c^4*d^19*f^8 - 28*c^3*d^20*f^7*e - 18*c^2*d^21*f^6*e^2 -
12*c*d^22*f^5*e^3 - 7*d^23*f^4*e^4)/(d^23*f^8))*sqrt(d*x + c) + 15*(63*c^5*
f^5 - 35*c^4*d*f^4*e - 10*c^3*d^2*f^3*e^2 - 6*c^2*d^3*f^2*e^3 - 5*c*d^4*f*e
^4 - 7*d^5*e^5)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d
*f + d^2*e)))/(sqrt(d*f)*d^3*f^4))*B*b^2*abs(d)/d + (sqrt((d*x + c)*d*f - c
*d*f + d^2*e)*(2*(4*(2*(d*x + c)*(8*(d*x + c)*(10*(d*x + c)/d^5 - (61*c*d^2
9*f^10 - d^30*f^9*e)/(d^34*f^10)) + 3*(417*c^2*d^29*f^10 - 14*c*d^30*f^9*e
- 3*d^31*f^8*e^2)/(d^34*f^10)) - (3481*c^3*d^29*f^10 - 183*c^2*d^30*f^9*e -
 77*c*d^31*f^8*e^2 - 21*d^32*f^7*e^3)/(d^34*f^10))*(d*x + c) + 5*(2279*c^4*
d^29*f^10 - 176*c^3*d^30*f^9*e - 106*c^2*d^31*f^8*e^2 - 56*c*d^32*f^7*e^3 -
 21*d^33*f^6*e^4)/(d^34*f^10))*(d*x + c) - 15*(793*c^5*d^29*f^10 - 105*c^4*
```

```
d^30*f^9*e - 70*c^3*d^31*f^8*e^2 - 50*c^2*d^32*f^7*e^3 - 35*c*d^33*f^6*e^4
- 21*d^34*f^5*e^5)/(d^34*f^10))*sqrt(d*x + c) - 15*(231*c^6*f^6 - 126*c^5*d
*f^5*e - 35*c^4*d^2*f^4*e^2 - 20*c^3*d^3*f^3*e^3 - 15*c^2*d^4*f^2*e^4 - 14*
c*d^5*f*e^5 - 21*d^6*e^6)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)
*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^4*f^5))*C*b^2*abs(d)/d + 1920*(sqrt((d
*x + c)*d*f - c*d*f + d^2*e)*(2*d*x + 2*c - (5*c*f^2 - d*f*e)/f^2)*sqrt(d*x
 + c) - (3*c^2*d*f^2 - 2*c*d^2*f*e - d^3*e^2)*log(abs(-sqrt(d*f)*sqrt(d*x +
 c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*f))*B*a^2*c*abs(d)/d
^3 + 3840*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*d*x + 2*c - (5*c*f^2 - d*
f*e)/f^2)*sqrt(d*x + c) - (3*c^2*d*f^2 - 2*c*d^2*f*e - d^3*e^2)*log(abs(-sq
rt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*f)
)*A*a*b*c*abs(d)/d^3 + 1920*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*d*x + 2
*c - (5*c*f^2 - d*f*e)/f^2)*sqrt(d*x + c) - (3*c^2*d*f^2 - 2*c*d^2*f*e - d^
3*e^2)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*
e)))/(sqrt(d*f)*f))*A*a^2*abs(d)/d^2)/d
```

maple [B] time = 0.05, size = 6728, normalized size = 4.99

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 details)Is c*f-d*e zero or nonzero?
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

3.42 $\int (a+bx)\sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=721

$$-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2Cd^2f^2+6bdfx(6aCdf-b(10Bdf-7C(cf+de)))-10abdf(8Bdf-5C(cf+de\right.}{240bd^3f^3}$$

Rubi [A] time = 0.96, antiderivative size = 719, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1615, 147, 50, 63, 217, 206}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((d*e - c*f)*(2*a*d*f*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d
*f - B*(d*e + c*f))) - b*(C*(7*d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*
c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^
2))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(128*d^4*f^4) + ((2*a*d*f*(C*(5*d^2*e^2
+ 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - b*(C*(7*d^3*e
^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f
) - B*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))))*(c + d*x)^(3/2)*Sqrt[e + f*x])
/(64*d^4*f^3) + (C*(a + b*x)^2*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(5*b*d*f) -
 ((c + d*x)^(3/2)*(e + f*x)^(3/2)*(48*a^2*C*d^2*f^2 - 10*a*b*d*f*(8*B*d*f -
 5*C*(d*e + c*f)) - b^2*(C*(35*d^2*e^2 + 38*c*d*e*f + 35*c^2*f^2) + 10*d*f*
(8*A*d*f - 5*B*(d*e + c*f))) - 6*b*d*f*(10*b*B*d*f - 6*a*C*d*f - 7*b*C*(d*e
 + c*f))*x))/(240*b*d^3*f^3) - ((d*e - c*f)^2*(2*a*d*f*(C*(5*d^2*e^2 + 6*c*
d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f))) - b*(C*(7*d^3*e^3 + 9
*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 7*c^3*f^3) + 2*d*f*(8*A*d*f*(d*e + c*f) - B*
(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[e + f*x])])/(128*d^(9/2)*f^(9/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
```

```
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int (a+bx)\sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)dx = \frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}+\frac{\int (a+bx)\sqrt{c+dx}\,\sqrt{e+fx}}{}$$

$$=\frac{C(a+bx)^2(c+dx)^{3/2}(e+fx)^{3/2}}{5bdf}-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(48a^2\right.}{}$$

$$=\frac{\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right.\right.}{}$$

$$=\frac{(de-cf)\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(\right.\right.}{}$$

$$=\frac{(de-cf)\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(\right.\right.}{}$$

$$=\frac{(de-cf)\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(\right.\right.}{}$$

$$=\frac{(de-cf)\left(2adf\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(\right.\right.}{}$$

Mathematica [B] time = 6.61, size = 2722, normalized size = 3.78

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (2*b*C*(d*e - c*f)^3*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d
*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(9/2)*((3*(35/(64*(
1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)
)))^4) + 35/(48*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c
*d*f)/(d*e - c*f))))^3) + 7/(8*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(
d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1)))/10 + (21*(d*e - c*f)^2
*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c
*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[
c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2
*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*
e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2
*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(512*d^2*f^2*(c + d*x)^2*(1 + (
d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^4
)))/(3*d^4*f^3*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(7/2)*Sqrt[(
d*(e + f*x))/(d*e - c*f)]) + (2*(d*e - c*f)^2*(-3*b*C*e + b*B*f + a*C*f)*(c
 + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e
 - c*f) - (c*d*f)/(d*e - c*f))))^(7/2)*((3*(5/(8*(1 + (d*f*(c + d*x))/((d*e
 - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3) + 5/(6*(1 + (d*f*(
c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (
1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)
)))^(-1)))/8 + (15*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)
)^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c
*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c +
d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(
Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d
*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))
/(256*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e -
 c*f) - (c*d*f)/(d*e - c*f))))^3)))/(3*d^3*f^3*(d/((d^2*e)/(d*e - c*f) - (c
*d*f)/(d*e - c*f)))^(5/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*(d*e - c*f)
*(3*b*C*e^2 - 2*b*B*e*f - 2*a*C*e*f + A*b*f^2 + a*B*f^2)*(c + d*x)^(3/2)*Sq
rt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f
)/(d*e - c*f))))^(5/2)*((3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d
*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*(
(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1))/2 + (3*(d*e - c*f)^2*((d
^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c +
d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/
(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e -
c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/
(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(32*d^2*f^2*(c + d*x)^2*(1 + (d*f*(
c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2)))/(
3*d^2*f^3*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(3/2)*Sqrt[(d*(e
+ f*x))/(d*e - c*f)]) + (2*(-(b*e) + a*f)*(C*e^2 - B*e*f + A*f^2)*(c + d*x)
^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f)
 - (c*d*f)/(d*e - c*f))))^(3/2)*(3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d
^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))) + (3*(d*e - c*f)^2*((d^2*e)/(d*
e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/
(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcS
inh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*
f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c
*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*
```

```
f) - (c*d*f)/(d*e - c*f)))]))))/(16*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))
/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))))))/(3*d*f^3*Sqr
t[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e -
c*f)])
```

IntegrateAlgebraic [B] time = 2.79, size = 4538, normalized size = 6.29

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),
x]
```

```
[Out] ((-105*b*C*d^5*e^5*f^4*Sqrt[e + f*x])/Sqrt[c + d*x] + (75*b*c*C*d^4*e^4*f^5
*Sqrt[e + f*x])/Sqrt[c + d*x] + (150*b*B*d^5*e^4*f^5*Sqrt[e + f*x])/Sqrt[c
+ d*x] + (150*a*C*d^5*e^4*f^5*Sqrt[e + f*x])/Sqrt[c + d*x] + (30*b*c^2*C*d^
3*e^3*f^6*Sqrt[e + f*x])/Sqrt[c + d*x] - (120*b*B*c*d^4*e^3*f^6*Sqrt[e + f*
x])/Sqrt[c + d*x] - (120*a*c*C*d^4*e^3*f^6*Sqrt[e + f*x])/Sqrt[c + d*x] - (
240*A*b*d^5*e^3*f^6*Sqrt[e + f*x])/Sqrt[c + d*x] - (240*a*B*d^5*e^3*f^6*Sqr
t[e + f*x])/Sqrt[c + d*x] + (30*b*c^3*C*d^2*e^2*f^7*Sqrt[e + f*x])/Sqrt[c +
 d*x] - (60*b*B*c^2*d^3*e^2*f^7*Sqrt[e + f*x])/Sqrt[c + d*x] - (60*a*c^2*C*
d^3*e^2*f^7*Sqrt[e + f*x])/Sqrt[c + d*x] + (240*A*b*c*d^4*e^2*f^7*Sqrt[e +
f*x])/Sqrt[c + d*x] + (240*a*B*c*d^4*e^2*f^7*Sqrt[e + f*x])/Sqrt[c + d*x] +
 (480*a*A*d^5*e^2*f^7*Sqrt[e + f*x])/Sqrt[c + d*x] + (75*b*c^4*C*d*e*f^8*Sq
rt[e + f*x])/Sqrt[c + d*x] - (120*b*B*c^3*d^2*e*f^8*Sqrt[e + f*x])/Sqrt[c +
 d*x] - (120*a*c^3*C*d^2*e*f^8*Sqrt[e + f*x])/Sqrt[c + d*x] + (240*A*b*c^2*
d^3*e*f^8*Sqrt[e + f*x])/Sqrt[c + d*x] + (240*a*B*c^2*d^3*e*f^8*Sqrt[e + f*
x])/Sqrt[c + d*x] - (960*a*A*c*d^4*e*f^8*Sqrt[e + f*x])/Sqrt[c + d*x] - (10
5*b*c^5*C*f^9*Sqrt[e + f*x])/Sqrt[c + d*x] + (150*b*B*c^4*d*f^9*Sqrt[e + f*
x])/Sqrt[c + d*x] + (150*a*c^4*C*d*f^9*Sqrt[e + f*x])/Sqrt[c + d*x] - (240*
A*b*c^3*d^2*f^9*Sqrt[e + f*x])/Sqrt[c + d*x] - (240*a*B*c^3*d^2*f^9*Sqrt[e
+ f*x])/Sqrt[c + d*x] + (480*a*A*c^2*d^3*f^9*Sqrt[e + f*x])/Sqrt[c + d*x] -
 (790*b*C*d^6*e^5*f^3*(e + f*x)^(3/2))/(c + d*x)^(3/2) + (2210*b*c*C*d^5*e^
4*f^4*(e + f*x)^(3/2))/(c + d*x)^(3/2) + (580*b*B*d^6*e^4*f^4*(e + f*x)^(3/
2))/(c + d*x)^(3/2) + (580*a*C*d^6*e^4*f^4*(e + f*x)^(3/2))/(c + d*x)^(3/2)
 - (1420*b*c^2*C*d^4*e^3*f^5*(e + f*x)^(3/2))/(c + d*x)^(3/2) - (2000*b*B*c
*d^5*e^3*f^5*(e + f*x)^(3/2))/(c + d*x)^(3/2) - (2000*a*c*C*d^5*e^3*f^5*(e
+ f*x)^(3/2))/(c + d*x)^(3/2) - (160*A*b*d^6*e^3*f^5*(e + f*x)^(3/2))/(c +
d*x)^(3/2) - (160*a*B*d^6*e^3*f^5*(e + f*x)^(3/2))/(c + d*x)^(3/2) - (140*b
*c^3*C*d^3*e^2*f^6*(e + f*x)^(3/2))/(c + d*x)^(3/2) + (1560*b*B*c^2*d^4*e^2
*f^6*(e + f*x)^(3/2))/(c + d*x)^(3/2) + (1560*a*c^2*C*d^4*e^2*f^6*(e + f*x)
^(3/2))/(c + d*x)^(3/2) + (1440*A*b*c*d^5*e^2*f^6*(e + f*x)^(3/2))/(c + d*x
)^(3/2) + (1440*a*B*c*d^5*e^2*f^6*(e + f*x)^(3/2))/(c + d*x)^(3/2) - (960*a
*A*d^6*e^2*f^6*(e + f*x)^(3/2))/(c + d*x)^(3/2) - (350*b*c^4*C*d^2*e*f^7*(e
 + f*x)^(3/2))/(c + d*x)^(3/2) + (560*b*B*c^3*d^3*e*f^7*(e + f*x)^(3/2))/(c
 + d*x)^(3/2) + (560*a*c^3*C*d^3*e*f^7*(e + f*x)^(3/2))/(c + d*x)^(3/2) - (
2400*A*b*c^2*d^4*e*f^7*(e + f*x)^(3/2))/(c + d*x)^(3/2) - (2400*a*B*c^2*d^4
*e*f^7*(e + f*x)^(3/2))/(c + d*x)^(3/2) + (1920*a*A*c*d^5*e*f^7*(e + f*x)^(
3/2))/(c + d*x)^(3/2) + (490*b*c^5*C*d*f^8*(e + f*x)^(3/2))/(c + d*x)^(3/2)
 - (700*b*B*c^4*d^2*f^8*(e + f*x)^(3/2))/(c + d*x)^(3/2) - (700*a*c^4*C*d^2
*f^8*(e + f*x)^(3/2))/(c + d*x)^(3/2) + (1120*A*b*c^3*d^3*f^8*(e + f*x)^(3/
2))/(c + d*x)^(3/2) + (1120*a*B*c^3*d^3*f^8*(e + f*x)^(3/2))/(c + d*x)^(3/2
) - (960*a*A*c^2*d^4*f^8*(e + f*x)^(3/2))/(c + d*x)^(3/2) + (896*b*C*d^7*e^
5*f^2*(e + f*x)^(5/2))/(c + d*x)^(5/2) - (640*b*c*C*d^6*e^4*f^3*(e + f*x)^(
5/2))/(c + d*x)^(5/2) - (1280*b*B*d^7*e^4*f^3*(e + f*x)^(5/2))/(c + d*x)^(5
/2) - (1280*a*C*d^7*e^4*f^3*(e + f*x)^(5/2))/(c + d*x)^(5/2) - (2560*b*c^2*
C*d^5*e^3*f^4*(e + f*x)^(5/2))/(c + d*x)^(5/2) + (2560*b*B*c*d^6*e^3*f^4*(e
 + f*x)^(5/2))/(c + d*x)^(5/2) + (2560*a*c*C*d^6*e^3*f^4*(e + f*x)^(5/2))/(
c + d*x)^(5/2) + (1280*A*b*d^7*e^3*f^4*(e + f*x)^(5/2))/(c + d*x)^(5/2) + (
1280*a*B*d^7*e^3*f^4*(e + f*x)^(5/2))/(c + d*x)^(5/2) + (2560*b*c^3*C*d^4*e
```

^2*f^5*(e + f*x)^(5/2))/(c + d*x)^(5/2) - (3840*A*b*c*d^6*e^2*f^5*(e + f*x)
^(5/2))/(c + d*x)^(5/2) - (3840*a*B*c*d^6*e^2*f^5*(e + f*x)^(5/2))/(c + d*x
)^(5/2) + (640*b*c^4*C*d^3*e*f^6*(e + f*x)^(5/2))/(c + d*x)^(5/2) - (2560*b
*B*c^3*d^4*e*f^6*(e + f*x)^(5/2))/(c + d*x)^(5/2) - (2560*a*c^3*C*d^4*e*f^6
*(e + f*x)^(5/2))/(c + d*x)^(5/2) + (3840*A*b*c^2*d^5*e*f^6*(e + f*x)^(5/2)
)/(c + d*x)^(5/2) + (3840*a*B*c^2*d^5*e*f^6*(e + f*x)^(5/2))/(c + d*x)^(5/2
) - (896*b*c^5*C*d^2*f^7*(e + f*x)^(5/2))/(c + d*x)^(5/2) + (1280*b*B*c^4*d
^3*f^7*(e + f*x)^(5/2))/(c + d*x)^(5/2) + (1280*a*c^4*C*d^3*f^7*(e + f*x)^(
5/2))/(c + d*x)^(5/2) - (1280*A*b*c^3*d^4*f^7*(e + f*x)^(5/2))/(c + d*x)^(5
/2) - (1280*a*B*c^3*d^4*f^7*(e + f*x)^(5/2))/(c + d*x)^(5/2) - (490*b*C*d^8
*e^5*f*(e + f*x)^(7/2))/(c + d*x)^(7/2) + (350*b*c*C*d^7*e^4*f^2*(e + f*x)^
(7/2))/(c + d*x)^(7/2) + (700*b*B*d^8*e^4*f^2*(e + f*x)^(7/2))/(c + d*x)^(7
/2) + (700*a*C*d^8*e^4*f^2*(e + f*x)^(7/2))/(c + d*x)^(7/2) + (140*b*c^2*C*
d^6*e^3*f^3*(e + f*x)^(7/2))/(c + d*x)^(7/2) - (560*b*B*c*d^7*e^3*f^3*(e +
f*x)^(7/2))/(c + d*x)^(7/2) - (560*a*c*C*d^7*e^3*f^3*(e + f*x)^(7/2))/(c +
d*x)^(7/2) - (1120*A*b*d^8*e^3*f^3*(e + f*x)^(7/2))/(c + d*x)^(7/2) - (1120
*a*B*d^8*e^3*f^3*(e + f*x)^(7/2))/(c + d*x)^(7/2) + (1420*b*c^3*C*d^5*e^2*f
^4*(e + f*x)^(7/2))/(c + d*x)^(7/2) - (1560*b*B*c^2*d^6*e^2*f^4*(e + f*x)^(
7/2))/(c + d*x)^(7/2) - (1560*a*c^2*C*d^6*e^2*f^4*(e + f*x)^(7/2))/(c + d*x
)^(7/2) + (2400*A*b*c*d^7*e^2*f^4*(e + f*x)^(7/2))/(c + d*x)^(7/2) + (2400*
a*B*c*d^7*e^2*f^4*(e + f*x)^(7/2))/(c + d*x)^(7/2) + (960*a*A*d^8*e^2*f^4*(
e + f*x)^(7/2))/(c + d*x)^(7/2) - (2210*b*c^4*C*d^4*e*f^5*(e + f*x)^(7/2))/
(c + d*x)^(7/2) + (2000*b*B*c^3*d^5*e*f^5*(e + f*x)^(7/2))/(c + d*x)^(7/2)
+ (2000*a*c^3*C*d^5*e*f^5*(e + f*x)^(7/2))/(c + d*x)^(7/2) - (1440*A*b*c^2*
d^6*e*f^5*(e + f*x)^(7/2))/(c + d*x)^(7/2) - (1440*a*B*c^2*d^6*e*f^5*(e + f
*x)^(7/2))/(c + d*x)^(7/2) - (1920*a*A*c*d^7*e*f^5*(e + f*x)^(7/2))/(c + d*
x)^(7/2) + (790*b*c^5*C*d^3*f^6*(e + f*x)^(7/2))/(c + d*x)^(7/2) - (580*b*B
*c^4*d^4*f^6*(e + f*x)^(7/2))/(c + d*x)^(7/2) - (580*a*c^4*C*d^4*f^6*(e + f
*x)^(7/2))/(c + d*x)^(7/2) + (160*A*b*c^3*d^5*f^6*(e + f*x)^(7/2))/(c + d*x
)^(7/2) + (160*a*B*c^3*d^5*f^6*(e + f*x)^(7/2))/(c + d*x)^(7/2) + (960*a*A*
c^2*d^6*f^6*(e + f*x)^(7/2))/(c + d*x)^(7/2) + (105*b*C*d^9*e^5*(e + f*x)^(
9/2))/(c + d*x)^(9/2) - (75*b*c*C*d^8*e^4*f*(e + f*x)^(9/2))/(c + d*x)^(9/2
) - (150*b*B*d^9*e^4*f*(e + f*x)^(9/2))/(c + d*x)^(9/2) - (150*a*C*d^9*e^4*
f*(e + f*x)^(9/2))/(c + d*x)^(9/2) - (30*b*c^2*C*d^7*e^3*f^2*(e + f*x)^(9/2
))/(c + d*x)^(9/2) + (120*b*B*c*d^8*e^3*f^2*(e + f*x)^(9/2))/(c + d*x)^(9/2
) + (120*a*c*C*d^8*e^3*f^2*(e + f*x)^(9/2))/(c + d*x)^(9/2) + (240*A*b*d^9*
e^3*f^2*(e + f*x)^(9/2))/(c + d*x)^(9/2) + (240*a*B*d^9*e^3*f^2*(e + f*x)^(
9/2))/(c + d*x)^(9/2) - (30*b*c^3*C*d^6*e^2*f^3*(e + f*x)^(9/2))/(c + d*x)^
(9/2) + (60*b*B*c^2*d^7*e^2*f^3*(e + f*x)^(9/2))/(c + d*x)^(9/2) + (60*a*c^
2*C*d^7*e^2*f^3*(e + f*x)^(9/2))/(c + d*x)^(9/2) - (240*A*b*c*d^8*e^2*f^3*(
e + f*x)^(9/2))/(c + d*x)^(9/2) - (240*a*B*c*d^8*e^2*f^3*(e + f*x)^(9/2))/(
c + d*x)^(9/2) - (480*a*A*d^9*e^2*f^3*(e + f*x)^(9/2))/(c + d*x)^(9/2) - (7
5*b*c^4*C*d^5*e*f^4*(e + f*x)^(9/2))/(c + d*x)^(9/2) + (120*b*B*c^3*d^6*e*f
^4*(e + f*x)^(9/2))/(c + d*x)^(9/2) + (120*a*c^3*C*d^6*e*f^4*(e + f*x)^(9/2
))/(c + d*x)^(9/2) - (240*A*b*c^2*d^7*e*f^4*(e + f*x)^(9/2))/(c + d*x)^(9/2
) - (240*a*B*c^2*d^7*e*f^4*(e + f*x)^(9/2))/(c + d*x)^(9/2) + (960*a*A*c*d^
8*e*f^4*(e + f*x)^(9/2))/(c + d*x)^(9/2) + (105*b*c^5*C*d^4*f^5*(e + f*x)^(
9/2))/(c + d*x)^(9/2) - (150*b*B*c^4*d^5*f^5*(e + f*x)^(9/2))/(c + d*x)^(9/
2) - (150*a*c^4*C*d^5*f^5*(e + f*x)^(9/2))/(c + d*x)^(9/2) + (240*A*b*c^3*d
^6*f^5*(e + f*x)^(9/2))/(c + d*x)^(9/2) + (240*a*B*c^3*d^6*f^5*(e + f*x)^(9
/2))/(c + d*x)^(9/2) - (480*a*A*c^2*d^7*f^5*(e + f*x)^(9/2))/(c + d*x)^(9/2
))/(1920*d^4*f^4*(f - (d*(e + f*x))/(c + d*x))^5) + ((7*b*C*d^5*e^5 - 5*b*c
*C*d^4*e^4*f - 10*b*B*d^5*e^4*f - 10*a*C*d^5*e^4*f - 2*b*c^2*C*d^3*e^3*f^2
+ 8*b*B*c*d^4*e^3*f^2 + 8*a*c*C*d^4*e^3*f^2 + 16*A*b*d^5*e^3*f^2 + 16*a*B*d
^5*e^3*f^2 - 2*b*c^3*C*d^2*e^2*f^3 + 4*b*B*c^2*d^3*e^2*f^3 + 4*a*c^2*C*d^3*
e^2*f^3 - 16*A*b*c*d^4*e^2*f^3 - 16*a*B*c*d^4*e^2*f^3 - 32*a*A*d^5*e^2*f^3
- 5*b*c^4*C*d*e*f^4 + 8*b*B*c^3*d^2*e*f^4 + 8*a*c^3*C*d^2*e*f^4 - 16*A*b*c^
2*d^3*e*f^4 - 16*a*B*c^2*d^3*e*f^4 + 64*a*A*c*d^4*e*f^4 + 7*b*c^5*C*f^5 - 1
0*b*B*c^4*d*f^5 - 10*a*c^4*C*d*f^5 + 16*A*b*c^3*d^2*f^5 + 16*a*B*c^3*d^2*f^

```
5 - 32*a*A*c^2*d^3*f^5)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d
*x])])/(128*d^(9/2)*f^(9/2))
```

fricas [A] time = 2.96, size = 1620, normalized size = 2.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [-1/7680*(15*(7*C*b*d^5*e^5 - 5*(C*b*c*d^4 + 2*(C*a + B*b)*d^5)*e^4*f - 2*(
C*b*c^2*d^3 - 4*(C*a + B*b)*c*d^4 - 8*(B*a + A*b)*d^5)*e^3*f^2 - 2*(C*b*c^3
*d^2 + 16*A*a*d^5 - 2*(C*a + B*b)*c^2*d^3 + 8*(B*a + A*b)*c*d^4)*e^2*f^3 -
(5*C*b*c^4*d - 64*A*a*c*d^4 - 8*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^
3)*e*f^4 + (7*C*b*c^5 - 32*A*a*c^2*d^3 - 10*(C*a + B*b)*c^4*d + 16*(B*a + A
*b)*c^3*d^2)*f^5)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f
^2 - 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2
*e*f + c*d*f^2)*x) - 4*(384*C*b*d^5*f^5*x^4 - 105*C*b*d^5*e^4*f + 10*(4*C*b
*c*d^4 + 15*(C*a + B*b)*d^5)*e^3*f^2 + 2*(17*C*b*c^2*d^3 - 35*(C*a + B*b)*c
*d^4 - 120*(B*a + A*b)*d^5)*e^2*f^3 + 10*(4*C*b*c^3*d^2 + 48*A*a*d^5 - 7*(C
*a + B*b)*c^2*d^3 + 16*(B*a + A*b)*c*d^4)*e*f^4 - 15*(7*C*b*c^4*d - 32*A*a*
c*d^4 - 10*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^3)*f^5 + 48*(C*b*d^5*
e*f^4 + (C*b*c*d^4 + 10*(C*a + B*b)*d^5)*f^5)*x^3 - 8*(7*C*b*d^5*e^2*f^3 -
2*(C*b*c*d^4 + 5*(C*a + B*b)*d^5)*e*f^4 + (7*C*b*c^2*d^3 - 10*(C*a + B*b)*c
*d^4 - 80*(B*a + A*b)*d^5)*f^5)*x^2 + 2*(35*C*b*d^5*e^3*f^2 - (11*C*b*c*d^4
 + 50*(C*a + B*b)*d^5)*e^2*f^3 - (11*C*b*c^2*d^3 - 20*(C*a + B*b)*c*d^4 - 8
0*(B*a + A*b)*d^5)*e*f^4 + 5*(7*C*b*c^3*d^2 + 96*A*a*d^5 - 10*(C*a + B*b)*c
^2*d^3 + 16*(B*a + A*b)*c*d^4)*f^5)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^
5), -1/3840*(15*(7*C*b*d^5*e^5 - 5*(C*b*c*d^4 + 2*(C*a + B*b)*d^5)*e^4*f -
2*(C*b*c^2*d^3 - 4*(C*a + B*b)*c*d^4 - 8*(B*a + A*b)*d^5)*e^3*f^2 - 2*(C*b*
c^3*d^2 + 16*A*a*d^5 - 2*(C*a + B*b)*c^2*d^3 + 8*(B*a + A*b)*c*d^4)*e^2*f^3
 - (5*C*b*c^4*d - 64*A*a*c*d^4 - 8*(C*a + B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2
*d^3)*e*f^4 + (7*C*b*c^5 - 32*A*a*c^2*d^3 - 10*(C*a + B*b)*c^4*d + 16*(B*a
+ A*b)*c^3*d^2)*f^5)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)
*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x
)) - 2*(384*C*b*d^5*f^5*x^4 - 105*C*b*d^5*e^4*f + 10*(4*C*b*c*d^4 + 15*(C*a
 + B*b)*d^5)*e^3*f^2 + 2*(17*C*b*c^2*d^3 - 35*(C*a + B*b)*c*d^4 - 120*(B*a
+ A*b)*d^5)*e^2*f^3 + 10*(4*C*b*c^3*d^2 + 48*A*a*d^5 - 7*(C*a + B*b)*c^2*d^
3 + 16*(B*a + A*b)*c*d^4)*e*f^4 - 15*(7*C*b*c^4*d - 32*A*a*c*d^4 - 10*(C*a
+ B*b)*c^3*d^2 + 16*(B*a + A*b)*c^2*d^3)*f^5 + 48*(C*b*d^5*e*f^4 + (C*b*c*d
^4 + 10*(C*a + B*b)*d^5)*f^5)*x^3 - 8*(7*C*b*d^5*e^2*f^3 - 2*(C*b*c*d^4 + 5
*(C*a + B*b)*d^5)*e*f^4 + (7*C*b*c^2*d^3 - 10*(C*a + B*b)*c*d^4 - 80*(B*a +
 A*b)*d^5)*f^5)*x^2 + 2*(35*C*b*d^5*e^3*f^2 - (11*C*b*c*d^4 + 50*(C*a + B*b
)*d^5)*e^2*f^3 - (11*C*b*c^2*d^3 - 20*(C*a + B*b)*c*d^4 - 80*(B*a + A*b)*d^
5)*e*f^4 + 5*(7*C*b*c^3*d^2 + 96*A*a*d^5 - 10*(C*a + B*b)*c^2*d^3 + 16*(B*a
 + A*b)*c*d^4)*f^5)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^5)]
```

giac [B] time = 3.39, size = 2643, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/1920*(1920*((c*d*f - d^2*e)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x
+ c)*d*f - c*d*f + d^2*e)))/sqrt(d*f) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)
*sqrt(d*x + c))*A*a*c*abs(d)/d^2 + 80*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*
sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^3*e)/(d
^7*f^4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4)) + 3*
```

```
(5*c^3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*f)*sqrt
(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))*C*a*c*
abs(d)/d^2 + 80*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x
+ c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^3*e)/(d^7*f^4)) + 3*(11*c^2*d
^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4)) + 3*(5*c^3*f^3 - 3*c^2*d*f
^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x
+ c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))*B*b*c*abs(d)/d^2 + 10*(sqrt(
(d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 -
 (25*c*d^11*f^6 - d^12*f^5*e)/(d^14*f^6)) + (163*c^2*d^11*f^6 - 14*c*d^12*f
^5*e - 5*d^13*f^4*e^2)/(d^14*f^6)) - 3*(93*c^3*d^11*f^6 - 15*c^2*d^12*f^5*e
 - 9*c*d^13*f^4*e^2 - 5*d^14*f^3*e^3)/(d^14*f^6))*sqrt(d*x + c) - 3*(35*c^4
*f^4 - 20*c^3*d*f^3*e - 6*c^2*d^2*f^2*e^2 - 4*c*d^3*f*e^3 - 5*d^4*e^4)*log(
abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(
d*f)*d^2*f^3))*C*b*c*abs(d)/d^2 + 80*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*s
qrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^3*e)/(d^
7*f^4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4)) + 3*(
5*c^3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(
d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))*B*a*abs
(d)/d + 10*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(
6*(d*x + c)/d^3 - (25*c*d^11*f^6 - d^12*f^5*e)/(d^14*f^6)) + (163*c^2*d^11*
f^6 - 14*c*d^12*f^5*e - 5*d^13*f^4*e^2)/(d^14*f^6)) - 3*(93*c^3*d^11*f^6 -
15*c^2*d^12*f^5*e - 9*c*d^13*f^4*e^2 - 5*d^14*f^3*e^3)/(d^14*f^6))*sqrt(d*x
 + c) - 3*(35*c^4*f^4 - 20*c^3*d*f^3*e - 6*c^2*d^2*f^2*e^2 - 4*c*d^3*f*e^3
- 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f
+ d^2*e)))/(sqrt(d*f)*d^2*f^3))*C*a*abs(d)/d + 80*(sqrt((d*x + c)*d*f - c*d
*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d
^6*f^3*e)/(d^7*f^4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^
7*f^4)) + 3*(5*c^3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sq
rt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*
f^2))*A*b*abs(d)/d + 10*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(
4*(d*x + c)*(6*(d*x + c)/d^3 - (25*c*d^11*f^6 - d^12*f^5*e)/(d^14*f^6)) + (
163*c^2*d^11*f^6 - 14*c*d^12*f^5*e - 5*d^13*f^4*e^2)/(d^14*f^6)) - 3*(93*c^
3*d^11*f^6 - 15*c^2*d^12*f^5*e - 9*c*d^13*f^4*e^2 - 5*d^14*f^3*e^3)/(d^14*f
^6))*sqrt(d*x + c) - 3*(35*c^4*f^4 - 20*c^3*d*f^3*e - 6*c^2*d^2*f^2*e^2 - 4
*c*d^3*f*e^3 - 5*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)
*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^3))*B*b*abs(d)/d + (sqrt((d*x + c)
*d*f - c*d*f + d^2*e)*(2*(4*(d*x + c)*(6*(d*x + c)*(8*(d*x + c)/d^4 - (41*c
*d^19*f^8 - d^20*f^7*e)/(d^23*f^8)) + (513*c^2*d^19*f^8 - 26*c*d^20*f^7*e -
 7*d^21*f^6*e^2)/(d^23*f^8)) - 5*(447*c^3*d^19*f^8 - 37*c^2*d^20*f^7*e - 19
*c*d^21*f^6*e^2 - 7*d^22*f^5*e^3)/(d^23*f^8))*(d*x + c) + 15*(193*c^4*d^19*
f^8 - 28*c^3*d^20*f^7*e - 18*c^2*d^21*f^6*e^2 - 12*c*d^22*f^5*e^3 - 7*d^23*
f^4*e^4)/(d^23*f^8))*sqrt(d*x + c) + 15*(63*c^5*f^5 - 35*c^4*d*f^4*e - 10*c
^3*d^2*f^3*e^2 - 6*c^2*d^3*f^2*e^3 - 5*c*d^4*f*e^4 - 7*d^5*e^5)*log(abs(-sq
rt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^
3*f^4))*C*b*abs(d)/d + 480*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*d*x + 2*
c - (5*c*f^2 - d*f*e)/f^2)*sqrt(d*x + c) - (3*c^2*d*f^2 - 2*c*d^2*f*e - d^3
*e^2)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e
)))/(sqrt(d*f)*f))*B*a*c*abs(d)/d^3 + 480*(sqrt((d*x + c)*d*f - c*d*f + d^2
*e)*(2*d*x + 2*c - (5*c*f^2 - d*f*e)/f^2)*sqrt(d*x + c) - (3*c^2*d*f^2 - 2*
c*d^2*f*e - d^3*e^2)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f
- c*d*f + d^2*e)))/(sqrt(d*f)*f))*A*b*c*abs(d)/d^3 + 480*(sqrt((d*x + c)*d*
f - c*d*f + d^2*e)*(2*d*x + 2*c - (5*c*f^2 - d*f*e)/f^2)*sqrt(d*x + c) - (3
*c^2*d*f^2 - 2*c*d^2*f*e - d^3*e^2)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt
((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*f))*A*a*abs(d)/d^2)/d
```

maple [B] time = 0.02, size = 3571, normalized size = 4.95

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x)
```

```
[Out] -1/3840*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(150*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x
+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^5*e^4*f+480*A*ln(1/
2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/
2))*a*c^2*d^3*f^5+150*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(
d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^4*d*f^5+150*C*ln(1/2*(2*d*f*x+2*(d*f*x
^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^5*e^4*f+210
*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*c^4*f^4+210*C*(d*f)^(1/2)*
(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*d^4*e^4-240*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+
c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^5*e^3*f^2-240*
B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d
*f)^(1/2))*a*c^3*d^2*f^5-240*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^
(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^5*e^3*f^2+150*B*ln(1/2*(2*d*f*x
+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^4*
d*f^5+480*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c
*f+d*e)/(d*f)^(1/2))*a*d^5*e^2*f^3-240*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d
*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d^2*f^5-105*C*ln(1/
2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/
2))*b*c^5*f^5-105*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)
^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^5*e^5-96*C*x^3*b*c*d^3*f^4*(d*f*x^2+c*f*x+
d*e*x+c*e)^(1/2)*(d*f)^(1/2)-96*C*x^3*b*d^4*e*f^3*(d*f*x^2+c*f*x+d*e*x+c*e)
^(1/2)*(d*f)^(1/2)-160*B*x^2*b*c*d^3*f^4*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d
*f)^(1/2)-1920*A*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*a*d^4*f^4+24
0*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/
(d*f)^(1/2))*a*c^2*d^3*e*f^4+240*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c
*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^4*e^2*f^3-120*B*ln(1/2*(2
*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
b*c^3*d^2*e*f^4-60*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^3*e^2*f^3-960*A*ln(1/2*(2*d*f*x+2*(d*
f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^4*e*f^
4+240*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d
*e)/(d*f)^(1/2))*b*c^2*d^3*e*f^4+240*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e
*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^4*e^2*f^3-120*C*ln(1/
2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/
2))*a*c^3*d^2*e*f^4-60*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^3*e^2*f^3-120*C*ln(1/2*(2*d*f*x+2
*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^4*
e^3*f^2+75*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+
c*f+d*e)/(d*f)^(1/2))*b*c^4*d*e*f^4+30*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d
*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d^2*e^2*f^3+44*C*(d
*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b*c*d^3*e^2*f^2-80*B*(d*f)^(1/2
)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b*c*d^3*e*f^3-80*C*(d*f)^(1/2)*(d*f*x^2
+c*f*x+d*e*x+c*e)^(1/2)*x*a*c*d^3*e*f^3+44*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e
*x+c*e)^(1/2)*x*b*c^2*d^2*e*f^3-32*C*x^2*b*c*d^3*e*f^3*(d*f*x^2+c*f*x+d*e*x
+c*e)^(1/2)*(d*f)^(1/2)+200*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x
*a*c^2*d^2*f^4+200*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*a*d^4*e^
2*f^2-140*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b*c^3*d*f^4-140*C
*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b*d^4*e^3*f-320*A*(d*f)^(1/2
)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*c*d^3*e*f^3-320*B*(d*f)^(1/2)*(d*f*x^2+
c*f*x+d*e*x+c*e)^(1/2)*a*c*d^3*e*f^3+140*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x
+c*e)^(1/2)*a*c^2*d^2*e*f^3-320*A*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/
2)*x*b*c*d^3*f^4-320*A*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b*d^4*
e*f^3-320*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*a*c*d^3*f^4-320*B
*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*a*d^4*e*f^3-80*C*(d*f)^(1/2)
*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*c*d^3*e^3*f+200*B*(d*f)^(1/2)*(d*f*x^2+c
*f*x+d*e*x+c*e)^(1/2)*x*b*c^2*d^2*f^4+200*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*
x+c*e)^(1/2)*x*b*d^4*e^2*f^2+140*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1
/2)*b*c^2*d^2*e*f^3+140*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a*c*d
^3*e^2*f^2-80*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*c^3*d*e*f^3-6
```

```
8*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*c^2*d^2*e^2*f^2+140*B*(d*
f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*c*d^3*e^2*f^2-160*B*x^2*b*d^4*e*
f^3*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)-160*C*x^2*a*c*d^3*f^4*(d*f*
x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)-160*C*x^2*a*d^4*e*f^3*(d*f*x^2+c*f*x
+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+112*C*x^2*b*c^2*d^2*f^4*(d*f*x^2+c*f*x+d*e*x+
c*e)^(1/2)*(d*f)^(1/2)+112*C*x^2*b*d^4*e^2*f^2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1
/2)*(d*f)^(1/2)-768*C*x^4*b*d^4*f^4*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(
1/2)-960*B*x^3*b*d^4*f^4*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)-960*C*
x^3*a*d^4*f^4*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)-1280*A*x^2*b*d^4*
f^4*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)-1280*B*x^2*a*d^4*f^4*(d*f*x
^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)-120*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*
x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^4*e^3*f^2+30*C*l
n(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)
^(1/2))*b*c^2*d^3*e^3*f^2+75*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^
(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^4*e^4*f-960*A*(d*f)^(1/2)*(d*
f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a*c*d^3*f^4-960*A*(d*f)^(1/2)*(d*f*x^2+c*f*x+d
*e*x+c*e)^(1/2)*a*d^4*e*f^3+480*A*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/
2)*b*c^2*d^2*f^4+480*A*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*d^4*e^
2*f^2+480*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a*c^2*d^2*f^4+480*B
*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a*d^4*e^2*f^2-300*B*(d*f)^(1/2
)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b*c^3*d*f^4-300*B*(d*f)^(1/2)*(d*f*x^2+c*
f*x+d*e*x+c*e)^(1/2)*b*d^4*e^3*f-300*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e
)^(1/2)*a*c^3*d*f^4-300*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a*d^4
*e^3*f)/(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)/d^4/f^4/(d*f)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 details)Is c*f-d*e zero or nonzero?
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx) \sqrt{c + dx} \sqrt{e + fx} \left(A + Bx + Cx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
```

```
[Out] Integral((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

3.43 $\int \sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)dx$

Optimal. Leaf size=330

$$-\frac{(de-cf)^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)}{64d^{7/2}f^{7/2}}+\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(8d\right.}{}$$

Rubi [A] time = 0.30, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)}{32d^3f^2}+\frac{\sqrt{c+dx}\sqrt{e+fx}(de-cf)\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)}{64d^3f^3}-\frac{(de-cf)^2\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)}{64d^{7/2}f^{7/2}}-\frac{(c+dx)^{3/2}(e+fx)^{3/2}(-8Bdf+11cCf+5Cde)}{24d^2f^2}+\frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((d*e - c*f)*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d
*e + c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(64*d^3*f^3) + ((C*(5*d^2*e^2 + 6*
c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e + c*f)))*(c + d*x)^(3/2)*Sqr
t[e + f*x])/(32*d^3*f^2) - ((5*C*d*e + 11*c*C*f - 8*B*d*f)*(c + d*x)^(3/2)*
(e + f*x)^(3/2))/(24*d^2*f^2) + (C*(c + d*x)^(5/2)*(e + f*x)^(3/2))/(4*d^2*
f) - ((d*e - c*f)^2*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f
 - B*(d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])]
)/(64*d^(7/2)*f^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)dx = \frac{C(c+dx)^{5/2}(e+fx)^{3/2}}{4d^2f} + \frac{\int \sqrt{c+dx}\,\sqrt{e+fx}\,\left(\frac{1}{2}\left(-5cCde-3c^2Cf \cdots\right.\right.}{\cdots}$$

$$= -\frac{(5Cde+11cCf-8Bdf)(c+dx)^{3/2}(e+fx)^{3/2}}{24d^2f^2} + \frac{C(c+dx)^{5/2}(e+\cdots}{4d^2f}$$

$$= \frac{\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)(c+dx)^{3/\cdots}}{32d^3f^2}$$

$$= \frac{(de-cf)\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)\cdots}{64d^3f^3}$$

$$= \frac{(de-cf)\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)\cdots}{64d^3f^3}$$

$$= \frac{(de-cf)\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)\cdots}{64d^3f^3}$$

$$= \frac{(de-cf)\left(C\left(5d^2e^2+6cdef+5c^2f^2\right)+8df(2Adf-B(de+cf))\right)\cdots}{64d^3f^3}$$

Mathematica [A] time = 1.72, size = 306, normalized size = 0.93

$$\frac{d\sqrt{f}\sqrt{c+dx}(e+fx)\left(8df\left(6Adf(cf+d(e+2fx))+B\left(-3c^2f^2+2cdf(e+fx)+d^2\left(-3e^2+2efx+8f^2x^2\right)\right)\right)+C\left(15c^3f^3-c^2df^2(7e+10fx)+cd^2f\left(-7e^2+4efx+8f^2x^2\right)+d^3\left(15e^3-10e^2fx+8ef^2x^2+48f^3x^3\right)\right)\right)-3(de-cf)^{5/2}\sqrt{\frac{d(e+fx)}{de-cf}}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)\left(8df(2Adf-B(cf+de))+C\left(5c^2f^2+6cdef+5d^2e^2\right)\right)}{192d^4f^{7/2}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] (d*Sqrt[f]*Sqrt[c + d*x]*(e + f*x)*(C*(15*c^3*f^3 - c^2*d*f^2*(7*e + 10*f*x
) + c*d^2*f*(-7*e^2 + 4*e*f*x + 8*f^2*x^2) + d^3*(15*e^3 - 10*e^2*f*x + 8*e
*f^2*x^2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(c*f + d*(e + 2*f*x)) + B*(-3*c^2*
f^2 + 2*c*d*f*(e + f*x) + d^2*(-3*e^2 + 2*e*f*x + 8*f^2*x^2)))) - 3*(d*e -
c*f)^(5/2)*(C*(5*d^2*e^2 + 6*c*d*e*f + 5*c^2*f^2) + 8*d*f*(2*A*d*f - B*(d*e
 + c*f)))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/S
qrt[d*e - c*f]])/(192*d^4*f^(7/2)*Sqrt[e + f*x])
```

IntegrateAlgebraic [A] time = 0.98, size = 643, normalized size = 1.95

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2),x]
```

```
[Out] ((d*e - c*f)^2*Sqrt[e + f*x]*(15*C*d^2*e^2*f^3 + 18*c*C*d*e*f^4 - 24*B*d^2*
e*f^4 + 15*c^2*C*f^5 - 24*B*c*d*f^5 + 48*A*d^2*f^5 + (73*C*d^3*e^2*f^2*(e +
 f*x))/(c + d*x) - (66*c*C*d^2*e*f^3*(e + f*x))/(c + d*x) - (40*B*d^3*e*f^3
*(e + f*x))/(c + d*x) - (55*c^2*C*d*f^4*(e + f*x))/(c + d*x) + (88*B*c*d^2*
f^4*(e + f*x))/(c + d*x) - (48*A*d^3*f^4*(e + f*x))/(c + d*x) - (55*C*d^4*e
^2*f*(e + f*x)^2)/(c + d*x)^2 - (66*c*C*d^3*e*f^2*(e + f*x)^2)/(c + d*x)^2
+ (88*B*d^4*e*f^2*(e + f*x)^2)/(c + d*x)^2 + (73*c^2*C*d^2*f^3*(e + f*x)^2)
/(c + d*x)^2 - (40*B*c*d^3*f^3*(e + f*x)^2)/(c + d*x)^2 - (48*A*d^4*f^3*(e
+ f*x)^2)/(c + d*x)^2 + (15*C*d^5*e^2*(e + f*x)^3)/(c + d*x)^3 + (18*c*C*d^
4*e*f*(e + f*x)^3)/(c + d*x)^3 - (24*B*d^5*e*f*(e + f*x)^3)/(c + d*x)^3 + (
15*c^2*C*d^3*f^2*(e + f*x)^3)/(c + d*x)^3 - (24*B*c*d^4*f^2*(e + f*x)^3)/(c
 + d*x)^3 + (48*A*d^5*f^2*(e + f*x)^3)/(c + d*x)^3))/(192*d^3*f^3*Sqrt[c +
d*x]*(-f + (d*(e + f*x))/(c + d*x))^4) + ((d*e - c*f)^2*(-5*C*d^2*e^2 - 6*c
*C*d*e*f + 8*B*d^2*e*f - 5*c^2*C*f^2 + 8*B*c*d*f^2 - 16*A*d^2*f^2)*ArcTanh[
(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(64*d^(7/2)*f^(7/2))
```

fricas [A] time = 0.97, size = 840, normalized size = 2.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*C*d^4*e^4 - 4*(C*c*d^3 + 2*B*d^4)*e^3*f - 2*(C*c^2*d^2 - 4*B*c
*d^3 - 8*A*d^4)*e^2*f^2 - 4*(C*c^3*d - 2*B*c^2*d^2 + 8*A*c*d^3)*e*f^3 + (5*
C*c^4 - 8*B*c^3*d + 16*A*c^2*d^2)*f^4)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^
2 + 6*c*d*e*f + c^2*f^2 - 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*s
qrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) + 4*(48*C*d^4*f^4*x^3 + 15*C*d^4*e^
3*f - (7*C*c*d^3 + 24*B*d^4)*e^2*f^2 - (7*C*c^2*d^2 - 16*B*c*d^3 - 48*A*d^4
)*e*f^3 + 3*(5*C*c^3*d - 8*B*c^2*d^2 + 16*A*c*d^3)*f^4 + 8*(C*d^4*e*f^3 + (
C*c*d^3 + 8*B*d^4)*f^4)*x^2 - 2*(5*C*d^4*e^2*f^2 - 2*(C*c*d^3 + 4*B*d^4)*e*
f^3 + (5*C*c^2*d^2 - 8*B*c*d^3 - 48*A*d^4)*f^4)*x)*sqrt(d*x + c)*sqrt(f*x +
 e))/(d^4*f^4), 1/384*(3*(5*C*d^4*e^4 - 4*(C*c*d^3 + 2*B*d^4)*e^3*f - 2*(C*
c^2*d^2 - 4*B*c*d^3 - 8*A*d^4)*e^2*f^2 - 4*(C*c^3*d - 2*B*c^2*d^2 + 8*A*c*d
^3)*e*f^3 + (5*C*c^4 - 8*B*c^3*d + 16*A*c^2*d^2)*f^4)*sqrt(-d*f)*arctan(1/2
*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2
+ c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(48*C*d^4*f^4*x^3 + 15*C*d^4*e^3*f
- (7*C*c*d^3 + 24*B*d^4)*e^2*f^2 - (7*C*c^2*d^2 - 16*B*c*d^3 - 48*A*d^4)*e*
f^3 + 3*(5*C*c^3*d - 8*B*c^2*d^2 + 16*A*c*d^3)*f^4 + 8*(C*d^4*e*f^3 + (C*c*
d^3 + 8*B*d^4)*f^4)*x^2 - 2*(5*C*d^4*e^2*f^2 - 2*(C*c*d^3 + 4*B*d^4)*e*f^3
+ (5*C*c^2*d^2 - 8*B*c*d^3 - 48*A*d^4)*f^4)*x)*sqrt(d*x + c)*sqrt(f*x + e))
/(d^4*f^4)]
```

giac [B] time = 2.33, size = 1103, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(192*((c*d*f - d^2*e)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x +
c)*d*f - c*d*f + d^2*e)))/sqrt(d*f) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)*s
qrt(d*x + c))*A*c*abs(d)/d^2 + 8*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(
```

```
d*x + c)*(2*(d*x + c)*(4*(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^3*e)/(d^7*f^
4)) + 3*(11*c^2*d^5*f^4 - 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4)) + 3*(5*c^
3*f^3 - 3*c^2*d*f^2*e - c*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d*x
+ c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))*C*c*abs(d)/
d^2 + 8*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*
(d*x + c)/d^2 - (13*c*d^5*f^4 - d^6*f^3*e)/(d^7*f^4)) + 3*(11*c^2*d^5*f^4 -
 2*c*d^6*f^3*e - d^7*f^2*e^2)/(d^7*f^4)) + 3*(5*c^3*f^3 - 3*c^2*d*f^2*e - c
*d^2*f*e^2 - d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f
 - c*d*f + d^2*e)))/(sqrt(d*f)*d*f^2))*B*abs(d)/d + (sqrt((d*x + c)*d*f - c
*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*x + c)/d^3 - (25*c*d^11*f^6 -
 d^12*f^5*e)/(d^14*f^6)) + (163*c^2*d^11*f^6 - 14*c*d^12*f^5*e - 5*d^13*f^4
*e^2)/(d^14*f^6)) - 3*(93*c^3*d^11*f^6 - 15*c^2*d^12*f^5*e - 9*c*d^13*f^4*e
^2 - 5*d^14*f^3*e^3)/(d^14*f^6))*sqrt(d*x + c) - 3*(35*c^4*f^4 - 20*c^3*d*f
^3*e - 6*c^2*d^2*f^2*e^2 - 4*c*d^3*f*e^3 - 5*d^4*e^4)*log(abs(-sqrt(d*f)*sq
rt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^3))*C*
abs(d)/d + 48*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*d*x + 2*c - (5*c*f^2
- d*f*e)/f^2)*sqrt(d*x + c) - (3*c^2*d*f^2 - 2*c*d^2*f*e - d^3*e^2)*log(abs
(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f
)*f))*B*c*abs(d)/d^3 + 48*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*d*x + 2*c
 - (5*c*f^2 - d*f*e)/f^2)*sqrt(d*x + c) - (3*c^2*d*f^2 - 2*c*d^2*f*e - d^3*
e^2)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)
))/(sqrt(d*f)*f))*A*abs(d)/d^2)/d
```

maple [B] time = 0.02, size = 1431, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x)
```

```
[Out] -1/384*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(48*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c
*e)^(1/2)*d^3*e^2*f+48*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^4*e^2*f^2+15*C*ln(1/2*(2*d*f*x+2*(d*f*x
^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^4*f^4+15*C*ln
(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^
(1/2))*d^4*e^4+48*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*c^2*d*f^3-1
2*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/
(d*f)^(1/2))*c^3*d*e*f^3-6*C*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1
/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d^2*e^2*f^2-12*C*ln(1/2*(2*d*f*x+
2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^3*e
^3*f-96*A*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*c*d^2*f^3-96*A*(d*f)^
(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*d^3*e*f^2-96*A*ln(1/2*(2*d*f*x+2*(d*f
*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c*d^3*e*f^3-1
92*A*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*d^3*f^3+24*B*ln(1/2*(2*d
*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^
2*d^2*e*f^3+24*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1
/2)+c*f+d*e)/(d*f)^(1/2))*c*d^3*e^2*f^2-96*C*x^3*d^3*f^3*(d*f*x^2+c*f*x+d*e
*x+c*e)^(1/2)*(d*f)^(1/2)-24*B*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x+d*e*x+c*e)^
(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^3*d*f^4-24*B*ln(1/2*(2*d*f*x+2*(d
*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*d^4*e^3*f-3
0*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*c^3*f^3-30*C*(d*f)^(1/2)*(d
*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*d^3*e^3+48*A*ln(1/2*(2*d*f*x+2*(d*f*x^2+c*f*x
+d*e*x+c*e)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*c^2*d^2*f^4-128*B*x^2*d
^3*f^3*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2)-8*C*(d*f)^(1/2)*(d*f*x^2
+c*f*x+d*e*x+c*e)^(1/2)*x*c*d^2*e*f^2-32*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x
+c*e)^(1/2)*x*c*d^2*f^3-32*B*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*
d^3*e*f^2+20*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*c^2*d*f^3+20*C
*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*d^3*e^2*f-32*B*(d*f)^(1/2)*(
d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*c*d^2*e*f^2+14*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+
d*e*x+c*e)^(1/2)*c^2*d*e*f^2+14*C*(d*f)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/
```

```
2)*c*d^2*e^2*f-16*C*x^2*c*d^2*f^3*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/
2)-16*C*x^2*d^3*e*f^2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2))/(d*f*x^2
+c*f*x+d*e*x+c*e)^(1/2)/d^3/f^3/(d*f)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 details)Is c*f-d*e zero or nonzero?
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2),x)
```

```
[Out] \text{Hanged}
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2), x)
```

3.44 $\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{a+bx}\,dx$

Optimal. Leaf size=450

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)\left(16a^3Cd^3f^3-8a^2bd^2f^2(2Bdf+cCf+Cde)-2ab^2df\left(C(de-cf)^2-4df(2Adf+Bcf+\right.\right.}{8b^4d^{5/2}f^{5/2}}$$

Rubi [A] time = 1.37, antiderivative size = 453, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1615, 154, 157, 63, 217, 206, 93, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)\left(-8a^2bd^2f^2(2Bdf+cCf+Cde)+16a^3Cd^3f^3-2ab^2df\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)+b^3\left(-\left(C(de-cf)^2(cf+de)-2df\left(B(de-cf)^2-4Adf(cf+de)\right)\right)\right)\right)}{8b^4d^{5/2}f^{5/2}}+\frac{\sqrt{c+dx}\,\sqrt{e+fx}\left(\frac{(4adf-bcf+bde)(2aCdf+b(-2Bdf+cCf+Cde))}{bdf}-4aC(cf+de)+8Abdf\right)}{8b^2df}-\frac{2\sqrt{bc-ad}\,\sqrt{be-af}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}\,\sqrt{be-af}}{\sqrt{e+fx}\,\sqrt{bc-ad}}\right)}{b^4}-\frac{\sqrt{c+dx}\,(e+fx)^{3/2}(2aCdf+b(-2Bdf+cCf+Cde))}{4b^2df^2}+\frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x),x]
```

```
[Out] ((8*A*b*d*f - 4*a*C*(d*e + c*f) + ((b*d*e - b*c*f + 4*a*d*f)*(2*a*C*d*f + b
*(C*d*e + c*C*f - 2*B*d*f)))/(b*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/(8*b^2*d
*f) - ((2*a*C*d*f + b*(C*d*e + c*C*f - 2*B*d*f))*Sqrt[c + d*x]*(e + f*x)^(3
/2))/(4*b^2*d*f^2) + (C*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(3*b*d*f) - ((16*a
^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(C*d*e + c*C*f + 2*B*d*f) - 2*a*b^2*d*f*(C*(
d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)) - b^3*(C*(d*e - c*f)^2*(d*e
 + c*f) - 2*d*f*(B*(d*e - c*f)^2 - 4*A*d*f*(d*e + c*f))))*ArcTanh[(Sqrt[f]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(8*b^4*d^(5/2)*f^(5/2)) - (2*(A*b^
2 - a*(b*B - a*C))*Sqrt[b*c - a*d]*Sqrt[b*e - a*f]*ArcTanh[(Sqrt[b*e - a*f]
*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/b^4
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
```

```
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{a+bx}dx = \frac{C(c+dx)^{3/2}(e+fx)^{3/2}}{3bdf} + \frac{\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(\frac{3}{2}b(2Abdf-aC(de+cf))-\frac{3}{2}b(2aCdf}{a+bx}}{3b^2df}$$

$$= -\frac{(2aCdf+b(Cde+cCf-2Bdf))\sqrt{c+dx}(e+fx)^{3/2}}{4b^2df^2} + \frac{C(c+dx)}{3}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cd}{8b^3d^2f^2}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cd}{8b^3d^2f^2}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cd}{8b^3d^2f^2}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cd}{8b^3d^2f^2}$$

$$= \frac{(4bdf(2Abdf-aC(de+cf))+(bde-bcf+4adf)(2aCdf+b(Cd}{8b^3d^2f^2}$$

Mathematica [B] time = 6.21, size = 1936, normalized size = 4.30

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x),x]
```

```
[Out] (2*(A*b^2 - a*b*B + a^2*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))
/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2)*(1/(2*(1
+ (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))
)) + (Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*ArcSi
nh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f
) - (c*d*f)/(d*e - c*f)])])/(2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*(1 + (d*f*(c +
 d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2))))/
(b^3*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))
/(d*e - c*f)]) + (2*C*(d*e - c*f)*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(
c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(5/2)*
((3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*
e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (
c*d*f)/(d*e - c*f))))^(-1))/2 + (3*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*
d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) -
(c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*S
qrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(
d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c
*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(
d*e - c*f)))])))/(32*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2)))/(3*b*d^2*f*(d/((d^2*e)/(
d*e - c*f) - (c*d*f)/(d*e - c*f)))^(3/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) +
 (2*(-(b*C*e) + b*B*f - a*C*f)*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c +
 d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2)*(3/
```

```
(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
c*f))))) + (3*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*(
(2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))
 - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])
/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[
d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c
 + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]))))/(16*
d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f)
- (c*d*f)/(d*e - c*f)))))))/(3*b^2*d*f*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f
)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*f)]) - ((A*b^2 - a*b*B + a^2*C)
*(-(b*c) + a*d)*((2*Sqrt[f]*Sqrt[d*e - c*f]*Sqrt[d/((d^2*e)/(d*e - c*f) - (
c*d*f)/(d*e - c*f))]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[(
d*(e + f*x))/(d*e - c*f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e
 - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(b*d^(3/2)*Sqrt[
e + f*x]) - (2*Sqrt[-(b*e) + a*f]*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x]
)/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(b*Sqrt[-(b*c) + a*d])))/b^3
```

IntegrateAlgebraic [B] time = 1.79, size = 950, normalized size = 2.11

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x
),x]
```

```
[Out] ((d*e - c*f)*Sqrt[e + f*x]*(3*b^2*C*d^2*e^2*f^2 - 6*b^2*B*d^2*e*f^3 + 6*a*b
*C*d^2*e*f^3 - 3*b^2*c^2*C*f^4 + 6*b^2*B*c*d*f^4 - 6*a*b*c*C*d*f^4 + 24*A*b
^2*d^2*f^4 - 24*a*b*B*d^2*f^4 + 24*a^2*C*d^2*f^4 + (8*b^2*C*d^3*e^2*f*(e +
f*x))/(c + d*x) - (16*b^2*c*C*d^2*e*f^2*(e + f*x))/(c + d*x) + (8*b^2*c^2*C
*d*f^3*(e + f*x))/(c + d*x) - (48*A*b^2*d^3*f^3*(e + f*x))/(c + d*x) + (48*
a*b*B*d^3*f^3*(e + f*x))/(c + d*x) - (48*a^2*C*d^3*f^3*(e + f*x))/(c + d*x)
 - (3*b^2*C*d^4*e^2*(e + f*x)^2)/(c + d*x)^2 + (6*b^2*B*d^4*e*f*(e + f*x)^2
)/(c + d*x)^2 - (6*a*b*C*d^4*e*f*(e + f*x)^2)/(c + d*x)^2 + (3*b^2*c^2*C*d^
2*f^2*(e + f*x)^2)/(c + d*x)^2 - (6*b^2*B*c*d^3*f^2*(e + f*x)^2)/(c + d*x)^
2 + (6*a*b*c*C*d^3*f^2*(e + f*x)^2)/(c + d*x)^2 + (24*A*b^2*d^4*f^2*(e + f*
x)^2)/(c + d*x)^2 - (24*a*b*B*d^4*f^2*(e + f*x)^2)/(c + d*x)^2 + (24*a^2*C*
d^4*f^2*(e + f*x)^2)/(c + d*x)^2))/(24*b^3*d^2*f^2*Sqrt[c + d*x]*(-f + (d*(
e + f*x))/(c + d*x))^3) + (2*(A*b^2 - a*b*B + a^2*C)*Sqrt[b*c - a*d]*Sqrt[-
(b*e) + a*f]*ArcTan[(Sqrt[b*c - a*d]*Sqrt[-(b*e) + a*f]*Sqrt[e + f*x])/((b*
e - a*f)*Sqrt[c + d*x])])/b^4 + ((b^3*C*d^3*e^3 - b^3*c*C*d^2*e^2*f - 2*b^3
*B*d^3*e^2*f + 2*a*b^2*C*d^3*e^2*f - b^3*c^2*C*d*e*f^2 + 4*b^3*B*c*d^2*e*f^
2 - 4*a*b^2*c*C*d^2*e*f^2 + 8*A*b^3*d^3*e*f^2 - 8*a*b^2*B*d^3*e*f^2 + 8*a^2
*b*C*d^3*e*f^2 + b^3*c^3*C*f^3 - 2*b^3*B*c^2*d*f^3 + 2*a*b^2*c^2*C*d*f^3 +
8*A*b^3*c*d^2*f^3 - 8*a*b^2*B*c*d^2*f^3 + 8*a^2*b*c*C*d^2*f^3 - 16*a*A*b^2*
d^3*f^3 + 16*a^2*b*B*d^3*f^3 - 16*a^3*C*d^3*f^3)*ArcTanh[(Sqrt[d]*Sqrt[e +
f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(8*b^4*d^(5/2)*f^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 1.1
```

maple [B] time = 0.05, size = 4227, normalized size = 9.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x)
```

```
[Out] -1/48*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(48*C*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+
b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*
x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^4*d^3*f^3-3*C*ln(1/2*(2*d*f*
x+c*f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*((a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*c^3*f^3-3*C*ln(1/2*(2*d*f*x+c*
f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*((a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*d^3*e^3+48*B*(d*f)^(1/2)*ln((-2*a*
d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*
x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*c*d^2*f^
3-48*A*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d
*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*
e)/(b*x+a))*a*b^3*c*d^2*f^3-48*A*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x
+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^
(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^3*d^3*e*f^2+48*B*(d*f)^(1/2)*ln((
-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*
(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d^3
*e*f^2-24*B*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*
x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b^4*d^2*f^2+24*B*ln(1/2*(2*d*f*x+c*f+d*e+2*(d*
f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*
b*d*e+b^2*c*e)/b^2)^(1/2)*a*b^3*c*d^2*f^3+24*B*ln(1/2*(2*d*f*x+c*f+d*e+2*(d
*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a
*b*d*e+b^2*c*e)/b^2)^(1/2)*a*b^3*d^3*e*f^2-16*C*x^2*b^4*d^2*f^2*(d*f*x^2+c*
f*x+d*e*x+c*e)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f)^(1
/2)-24*C*ln(1/2*(2*d*f*x+c*f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1
/2))/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a^2*b^2*d^3
*e*f^2-6*C*ln(1/2*(2*d*f*x+c*f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^
(1/2))/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a*b^3*c^2
*d*f^3+48*B*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*
x^2+c*f*x+d*e*x+c*e)^(1/2)*a*b^3*d^2*f^2-12*B*(d*f)^(1/2)*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b^4*c*d*f^2-12
*B*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x
+d*e*x+c*e)^(1/2)*b^4*d^2*e*f-48*C*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^
2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a^2*b^2*d^2*f^2-6*C*ln(1/
2*(2*d*f*x+c*f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2))/(d*f)^(1/
2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a*b^3*d^3*e^2*f+3*C*ln(1/
2*(2*d*f*x+c*f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2))/(d*f)^(1/
2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*c^2*d*e*f^2+3*C*ln(1/
2*(2*d*f*x+c*f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2))/(d*f)^(1/
2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*c*d^2*e^2*f-12*B*ln(1
/2*(2*d*f*x+c*f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2))/(d*f)^(1
/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*c*d^2*e*f^2-48*C*(d*
f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e
)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a)
)*a^3*b*c*d^2*f^3-48*C*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d
*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*
```

```
c*f-a*d*e+2*b*c*e)/(b*x+a))*a^3*b*d^3*e*f^2-24*C*ln(1/2*(2*d*f*x+c*f+d*e+2*
(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a^2*b^2*c*d^2*f^3+48*A*(d*f)^(1/2)*ln((-2*a*d*
f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^
2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b^2*d^3*f^3+48
*A*ln(1/2*(2*d*f*x+c*f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2))/(
d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a*b^3*d^3*f^3-24*
A*ln(1/2*(2*d*f*x+c*f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2))/(d
*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*c*d^2*f^3-24*A
*ln(1/2*(2*d*f*x+c*f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2))/(d*
f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*b^4*d^3*e*f^2-48*B*
(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x
+a))*a^3*b*d^3*f^3-48*B*ln(1/2*(2*d*f*x+c*f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)
^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)*a^2*b^2*d^3*f^3+6*B*ln(1/2*(2*d*f*x+c*f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)
^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)*b^4*c^2*d*f^3+6*B*ln(1/2*(2*d*f*x+c*f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(
1/2)*(d*f)^(1/2))/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2
)*b^4*d^3*e^2*f+48*A*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*
f-a*d*e+2*b*c*e)/(b*x+a))*b^4*c*d^2*e*f^2+48*C*ln(1/2*(2*d*f*x+c*f+d*e+2*(d
*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a
*b*d*e+b^2*c*e)/b^2)^(1/2)*a^3*b*d^3*f^3-48*A*(d*f)^(1/2)*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b^4*d^2*f^2+6*
C*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+
d*e*x+c*e)^(1/2)*b^4*c^2*f^2+6*C*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b^4*d^2*e^2-4*C*(d*f)^(1/2)
*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1
/2)*b^4*c*d*e*f-4*C*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/
2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b^4*c*d*f^2-4*C*(d*f)^(1/2)*((a^2*d*f-
a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*b^4*d
^2*e*f+12*C*ln(1/2*(2*d*f*x+c*f+d*e+2*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*(d*f)
^(1/2))/(d*f)^(1/2))*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*a*b^3*c*
d^2*e*f^2+12*C*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d
*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a*b^3*c*d*f^2+12*C*(d*f)^(1/2)*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*a*b^3*d^2*e
*f+48*C*(d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c
*e)/(b*x+a))*a^2*b^2*c*d^2*e*f^2+24*C*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*x*a*b^3*d^2*f^2-48*B*(
d*f)^(1/2)*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c
*e)/b^2)^(1/2)*(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+
a))*a*b^3*c*d^2*e*f^2)/(d*f*x^2+c*f*x+d*e*x+c*e)^(1/2)/b^5/d^2/f^2/(d*f)^(1
/2)/((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*a*d*f-b*c*f>0)', see `assume?` fo
r more details)Is 2*a*d*f-b*c*f                                         -b*d
*e    zero or nonzero?
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x),x)
[Out] \text{Hanged}
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a),x)
[Out] Integral(sqrt(c + d*x)*sqrt(e + f*x)*(A + B*x + C*x**2)/(a + b*x), x)
```

3.45 $\int \frac{\sqrt{c+dx}\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^2}\,dx$

Optimal. Leaf size=521

$$\frac{\sqrt{c+dx}\,(e+fx)^{3/2}\left(3a^2Cdf-ab(2Bdf+cCf+Cde)+b^2(2Adf+cCe)\right)}{2b^2f(bc-ad)(be-af)}+\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(24a^2Cd^2f^2-8a\right.}{}$$

Rubi [A] time = 1.70, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1613, 154, 157, 63, 217, 206, 93, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(24a^2Cd^2f^2-8abdf(2Bdf+cCf+Cde)+b^2\left(-\left(C(de-cf)^2-4df(2Adf+Bcf+Bde)\right)\right)\right)}{4b^4d^{3/2}f^{3/2}}+\frac{\sqrt{c+dx}\sqrt{e+fx}\left(12a^2Cdf^2-abf(8Bdf+cCf+7Cde)+b^2(4df(Af+Be)-Ce(de-cf))\right)}{4b^3df(be-af)}+\frac{\sqrt{c+dx}(e+fx)^{3/2}\left(3a^2Cdf-ab(2Bdf+cCf+Cde)+b^2(2Adf+cCe)\right)}{2b^2f(bc-ad)(be-af)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(-a^2b(4Bdf+5C(cf+de))+6a^3Cdf+ab^2(2Adf+3Bcf+3Bde+4cCe)-b^3(Acf+Ade+2Bce)\right)}{b^4\sqrt{bc-ad}\sqrt{be-af}}-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^2,x]
```

```
[Out] ((12*a^2*C*d*f^2 - a*b*f*(7*C*d*e + c*C*f + 8*B*d*f) + b^2*(4*d*f*(B*e + A*
f) - C*e*(d*e - c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b^3*d*f*(b*e - a*f))
 + ((3*a^2*C*d*f + b^2*(c*C*e + 2*A*d*f) - a*b*(C*d*e + c*C*f + 2*B*d*f))*S
qrt[c + d*x]*(e + f*x)^(3/2))/(2*b^2*(b*c - a*d)*f*(b*e - a*f)) - ((A*b^2 -
 a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c - a*d)*(b*e - a*f)
*(a + b*x)) + ((24*a^2*C*d^2*f^2 - 8*a*b*d*f*(C*d*e + c*C*f + 2*B*d*f) - b^
2*(C*(d*e - c*f)^2 - 4*d*f*(B*d*e + B*c*f + 2*A*d*f)))*ArcTanh[(Sqrt[f]*Sqr
t[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(4*b^4*d^(3/2)*f^(3/2)) + ((6*a^3*C*d
*f - b^3*(2*B*c*e + A*d*e + A*c*f) + a*b^2*(4*c*C*e + 3*B*d*e + 3*B*c*f + 2
*A*d*f) - a^2*b*(4*B*d*f + 5*C*(d*e + c*f)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[
c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b^4*Sqrt[b*c - a*d]*Sqrt[b*e -
 a*f])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)}{(a+bx)^2}dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{b(bc-ad)(be-af)(a+bx)} - \int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\left(-\frac{3a^2C(de+cf}{\cdots}\right.}{\cdots}$$

$$= \frac{\left(3a^2Cdf+b^2(cCe+2Adf)-ab(Cde+cCf+2Bdf)\right)\sqrt{c+dx}\,(e+f}{2b^2(bc-ad)f(be-af)}$$

$$= \frac{\left(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-Ce(de-\right.}{4b^3df(be-af)}$$

$$= \frac{\left(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-Ce(de-\right.}{4b^3df(be-af)}$$

$$= \frac{\left(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-Ce(de-\right.}{4b^3df(be-af)}$$

$$= \frac{\left(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-Ce(de-\right.}{4b^3df(be-af)}$$

$$= \frac{\left(12a^2Cdf^2-abf(7Cde+cCf+8Bdf)+b^2(4df(Be+Af)-Ce(de-\right.}{4b^3df(be-af)}$$

Mathematica [B] time = 6.37, size = 2532, normalized size = 4.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^2,x]
```

```
[Out] -(((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c - a*d)*
(b*e - a*f)*(a + b*x))) + (2*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x]*(1 +
 (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))
^(3/2)*(1/(2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*
f)/(d*e - c*f))))) + (Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d
*e - c*f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d
^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x
]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c
*f))))^(3/2))))/(b^3*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sq
rt[(d*(e + f*x))/(d*e - c*f)]) + (2*C*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d
*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3
/2)*(3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/
(d*e - c*f))))) + (3*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*
f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
 c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c
+ d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])
/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 +
(d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])
))/(16*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e
- c*f) - (c*d*f)/(d*e - c*f))))))/(3*b^2*d*Sqrt[d/((d^2*e)/(d*e - c*f) - (
c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*(b*B - 2*a*C)*(b
```

```
*c - a*d)*((Sqrt[f]*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*ArcSinh
[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(b*d*Sqrt[e + f*x]) - (Sqrt[-(b*
e) + a*f]*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sq
rt[e + f*x])])/(b*Sqrt[-(b*c) + a*d])))/b^3 - ((A*b^2 - a*b*B + a^2*C)*((-4
*f*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)
/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2)*(3/(4*(1 + (d*f*(c + d*x))/((d*
e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))) + (3*(d*e - c*f)^2*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*
f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c
 + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*
e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e
 - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*
e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(16*d^2*f^2*(c + d*x)^2*(1 + (d*
f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))))))/
(3*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(
d*e - c*f)]) + ((2*a*b*d*f + (b*(-2*a*d*f - b*(d*e + c*f)))/2)*((2*Sqrt[c +
 d*x]*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f)
- (c*d*f)/(d*e - c*f))))^(3/2)*(1/(2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^
2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))) + (Sqrt[d*e - c*f]*Sqrt[(d^2*e)/
(d*e - c*f) - (c*d*f)/(d*e - c*f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/
(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(2*Sqrt
[d]*Sqrt[f]*Sqrt[c + d*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e -
 c*f) - (c*d*f)/(d*e - c*f))))^(3/2))))/(b*Sqrt[d/((d^2*e)/(d*e - c*f) - (c
*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*f)]) - ((-(b*c) + a*d)*((2*
Sqrt[f]*Sqrt[d*e - c*f]*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]
*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[(d*(e + f*x))/(d*e -
c*f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)
/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(b*d^(3/2)*Sqrt[e + f*x]) - (2*Sqrt[
-(b*e) + a*f]*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d
]*Sqrt[e + f*x])])/(b*Sqrt[-(b*c) + a*d])))/b))/b))/(b^2*(b*c - a*d)*(b*e -
 a*f))
```

<u>IntegrateAlgebraic</u> [A] time = 2.76, size = 942, normalized size = 1.81

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x
)^2,x]
```

```
[Out] ((d*e - c*f)*Sqrt[e + f*x]*(-(b^2*C*d*e^2*f) + b^2*c*C*e*f^2 + 4*b^2*B*d*e*
f^2 - 7*a*b*C*d*e*f^2 - a*b*c*C*f^3 + 4*A*b^2*d*f^3 - 8*a*b*B*d*f^3 + 12*a^
2*C*d*f^3 - (b^2*C*d^2*e^2*(e + f*x))/(c + d*x) + (2*b^2*c*C*d*e*f*(e + f*x
))/(c + d*x) - (4*b^2*B*d^2*e*f*(e + f*x))/(c + d*x) + (8*a*b*C*d^2*e*f*(e
+ f*x))/(c + d*x) - (b^2*c^2*C*f^2*(e + f*x))/(c + d*x) - (4*b^2*B*c*d*f^2*
(e + f*x))/(c + d*x) + (8*a*b*c*C*d*f^2*(e + f*x))/(c + d*x) - (8*A*b^2*d^2
*f^2*(e + f*x))/(c + d*x) + (16*a*b*B*d^2*f^2*(e + f*x))/(c + d*x) - (24*a^
2*C*d^2*f^2*(e + f*x))/(c + d*x) + (b^2*c*C*d^2*e*(e + f*x)^2)/(c + d*x)^2
- (a*b*C*d^3*e*(e + f*x)^2)/(c + d*x)^2 - (b^2*c^2*C*d*f*(e + f*x)^2)/(c +
d*x)^2 + (4*b^2*B*c*d^2*f*(e + f*x)^2)/(c + d*x)^2 - (7*a*b*c*C*d^2*f*(e +
f*x)^2)/(c + d*x)^2 + (4*A*b^2*d^3*f*(e + f*x)^2)/(c + d*x)^2 - (8*a*b*B*d^
3*f*(e + f*x)^2)/(c + d*x)^2 + (12*a^2*C*d^3*f*(e + f*x)^2)/(c + d*x)^2))/(
4*b^3*d*f*Sqrt[c + d*x]*(-f + (d*(e + f*x))/(c + d*x))^2*(-(b*e) + a*f + (b
*c*(e + f*x))/(c + d*x) - (a*d*(e + f*x))/(c + d*x))) + ((-2*b^3*B*c*e + 4*
a*b^2*c*C*e - A*b^3*d*e + 3*a*b^2*B*d*e - 5*a^2*b*C*d*e - A*b^3*c*f + 3*a*b
^2*B*c*f - 5*a^2*b*c*C*f + 2*a*A*b^2*d*f - 4*a^2*b*B*d*f + 6*a^3*C*d*f)*Arc
Tan[(Sqrt[b*c - a*d]*Sqrt[-(b*e) + a*f]*Sqrt[e + f*x])/((b*e - a*f)*Sqrt[c
+ d*x])])/(b^4*Sqrt[b*c - a*d]*Sqrt[-(b*e) + a*f]) + ((-(b^2*C*d^2*e^2) + 2
*b^2*c*C*d*e*f + 4*b^2*B*d^2*e*f - 8*a*b*C*d^2*e*f - b^2*c^2*C*f^2 + 4*b^2*
B*c*d*f^2 - 8*a*b*c*C*d*f^2 + 8*A*b^2*d^2*f^2 - 16*a*b*B*d^2*f^2 + 24*a^2*C
```

```
*d^2*f^2)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(4*b^4*
d^(3/2)*f^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [B] time = 13.12, size = 1585, normalized size = 3.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm=
"giac")
```

```
[Out] 1/4*sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*C*abs(d)
/(b^2*d^3) - (C*b^7*c*d^3*f^2*abs(d) + 8*C*a*b^6*d^4*f^2*abs(d) - 4*B*b^7*d
^4*f^2*abs(d) - C*b^7*d^4*f*abs(d)*e)/(b^9*d^6*f^2)) - (5*sqrt(d*f)*C*a^2*b
*c*f*abs(d) - 3*sqrt(d*f)*B*a*b^2*c*f*abs(d) + sqrt(d*f)*A*b^3*c*f*abs(d) -
 6*sqrt(d*f)*C*a^3*d*f*abs(d) + 4*sqrt(d*f)*B*a^2*b*d*f*abs(d) - 2*sqrt(d*f
)*A*a*b^2*d*f*abs(d) - 4*sqrt(d*f)*C*a*b^2*c*abs(d)*e + 2*sqrt(d*f)*B*b^3*c
*abs(d)*e + 5*sqrt(d*f)*C*a^2*b*d*abs(d)*e - 3*sqrt(d*f)*B*a*b^2*d*abs(d)*e
 + sqrt(d*f)*A*b^3*d*abs(d)*e)*arctan(-1/2*(b*c*d*f - 2*a*d^2*f + b*d^2*e -
 (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*b)/(sqrt
(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d))/(sqrt(a*b*c*d*f
^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*b^4*d) - 2*(sqrt(d*f)*C*a^2*b
*c^2*d*f^2*abs(d) - sqrt(d*f)*B*a*b^2*c^2*d*f^2*abs(d) + sqrt(d*f)*A*b^3*c^
2*d*f^2*abs(d) - 2*sqrt(d*f)*C*a^2*b*c*d^2*f*abs(d)*e + 2*sqrt(d*f)*B*a*b^2
*c*d^2*f*abs(d)*e - 2*sqrt(d*f)*A*b^3*c*d^2*f*abs(d)*e - sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b*c*f*abs(d
) + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e
))^2*B*a*b^2*c*f*abs(d) - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
c)*d*f - c*d*f + d^2*e))^2*A*b^3*c*f*abs(d) + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^3*d*f*abs(d) - 2*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^
2*b*d*f*abs(d) + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
- c*d*f + d^2*e))^2*A*a*b^2*d*f*abs(d) + sqrt(d*f)*C*a^2*b*d^3*abs(d)*e^2 -
 sqrt(d*f)*B*a*b^2*d^3*abs(d)*e^2 + sqrt(d*f)*A*b^3*d^3*abs(d)*e^2 - sqrt(d
*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2
*b*d*abs(d)*e + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c
*d*f + d^2*e))^2*B*a*b^2*d*abs(d)*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^3*d*abs(d)*e)/((b*c^2*d^2*f^2 -
2*b*c*d^3*f*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d
^2*e))^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f
+ d^2*e))^2*a*d^2*f + b*d^4*e^2 - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
c)*d*f - c*d*f + d^2*e))^2*b*d^2*e + (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^4*b)*b^4) + 1/8*(sqrt(d*f)*C*b^2*c^2*f^2*abs(d) +
 8*sqrt(d*f)*C*a*b*c*d*f^2*abs(d) - 4*sqrt(d*f)*B*b^2*c*d*f^2*abs(d) - 24*s
qrt(d*f)*C*a^2*d^2*f^2*abs(d) + 16*sqrt(d*f)*B*a*b*d^2*f^2*abs(d) - 8*sqrt(
d*f)*A*b^2*d^2*f^2*abs(d) - 2*sqrt(d*f)*C*b^2*c*d*f*abs(d)*e + 8*sqrt(d*f)*
C*a*b*d^2*f*abs(d)*e - 4*sqrt(d*f)*B*b^2*d^2*f*abs(d)*e + sqrt(d*f)*C*b^2*d
^2*abs(d)*e^2)*log((sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f +
d^2*e))^2)/(b^4*d^3*f^2)
```

maple [**B**] time = 0.05, size = 5051, normalized size = 9.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x)
```

```
[Out] result too large to display
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^2,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*a*d*f-b*c*f>0)', see `assume?` fo
r more details)Is 2*a*d*f-b*c*f                                        -b*d
*e    zero or nonzero?
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^2,x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.46 $\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\left(A+Bx+Cx^2\right)}{(a+bx)^3}\,dx$

Optimal. Leaf size=658

$$-\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\left(12a^3Cdf^2-a^2bf(4Bdf+11cCf+17Cde)+ab^2(Bf(3cf+5de)+4Ce(4cf+de))-b^3\left(c\left(-\right.\right.\right.}{4b^3(bc-ad)(be-af)^2}$$

Rubi [A] time = 2.68, antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1613, 149, 154, 157, 63, 217, 206, 93, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\,\sqrt{be-af}}{\sqrt{e+fx}\,\sqrt{bc-ad}}\right)\left(3a^2b^2\left(4Bdf(cf+de)+C\left(5c^2f^2+22cdef+5d^2e^2\right)\right)-8a^3bdf(Bdf+5C(cf+de))+24a^4Cd^2f^2-3ab^3\left(c^2f(Bf+8Ce)+2cde(3Bf+4Ce)+Bd^2e^2\right)-b^4\left(c^2\left(-\left(-Af^2+4Bef+8Ce^2\right)\right)-2cde(Af+2Be)+Ad^2e^2\right)\right)}{4b^4(bc-ad)^{3/2}(be-af)^{3/2}}-\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\left(-a^2bf(4Bdf+11cCf+17Cde)+12a^3Cdf^2+ab^2(Bf(3cf+5de)+4Ce(4cf+de))-b^3\left(cf(4Be-Af)+Adef+4cCe^2\right)\right)}{4b^3(bc-ad)(be-af)^2}+\frac{\sqrt{c+dx}\,(e+fx)^{3/2}\left(-a^2b(2Bdf+7C(cf+de))+6a^3Cdf+ab^2(-2Adf+3Bcf+3Bde+8cCe)-b^3(-Acf-Ade+4Bce)\right)}{4b^2(a+bx)(bc-ad)(be-af)^2}-\frac{(c+dx)^{3/2}(e+fx)^{3/2}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)(6aCdf-b(2Bdf+cCf+Cde))}{b^4\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^3,x]
```

```
[Out] -((12*a^3*C*d*f^2 - a^2*b*f*(17*C*d*e + 11*c*C*f + 4*B*d*f) - b^3*(4*c*C*e^
2 + A*d*e*f + c*f*(4*B*e - A*f)) + a*b^2*(B*f*(5*d*e + 3*c*f) + 4*C*e*(d*e
+ 4*c*f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b^3*(b*c - a*d)*(b*e - a*f)^2) +
 ((6*a^3*C*d*f - b^3*(4*B*c*e - A*d*e - A*c*f) + a*b^2*(8*c*C*e + 3*B*d*e +
 3*B*c*f - 2*A*d*f) - a^2*b*(2*B*d*f + 7*C*(d*e + c*f)))*Sqrt[c + d*x]*(e +
 f*x)^(3/2))/(4*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)) - ((A*b^2 - a*(b*B
 - a*C))*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(2*b*(b*c - a*d)*(b*e - a*f)*(a +
 b*x)^2) - ((6*a*C*d*f - b*(C*d*e + c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^4*Sqrt[d]*Sqrt[f]) - ((24*a^4*C*d^2
*f^2 - 3*a*b^3*(B*d^2*e^2 + c^2*f*(8*C*e + B*f) + 2*c*d*e*(4*C*e + 3*B*f))
- 8*a^3*b*d*f*(B*d*f + 5*C*(d*e + c*f)) - b^4*(A*d^2*e^2 - 2*c*d*e*(2*B*e +
 A*f) - c^2*(8*C*e^2 + 4*B*e*f - A*f^2)) + 3*a^2*b^2*(4*B*d*f*(d*e + c*f) +
 C*(5*d^2*e^2 + 22*c*d*e*f + 5*c^2*f^2)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c +
 d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(4*b^4*(b*c - a*d)^(3/2)*(b*e - a*
f)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\left(A+Bx+Cx^2\right)}{(a+bx)^3}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}(e+fx)^{3/2}}{2b(bc-ad)(be-af)(a+bx)^2} - \int \frac{\sqrt{c+dx}\,\sqrt{e+fx}\left(-\frac{3a^2C(de+cf}{}\right.}{}$$

$$= \frac{\left(6a^3Cdf - b^3(4Bce - Ade - Acf) + ab^2(8cCe + 3Bde + 3Bcf - 2Adf\right.}{4b^2(bc-ad)(be-af)^2(a}$$

$$= -\frac{\left(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3\left(4cCe^2 + Adef + c\right.\right.}{4b^3(bc -}$$

$$= -\frac{\left(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3\left(4cCe^2 + Adef + c\right.\right.}{4b^3(bc -}$$

$$= -\frac{\left(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3\left(4cCe^2 + Adef + c\right.\right.}{4b^3(bc -}$$

$$= -\frac{\left(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3\left(4cCe^2 + Adef + c\right.\right.}{4b^3(bc -}$$

$$= -\frac{\left(12a^3Cdf^2 - a^2bf(17Cde + 11cCf + 4Bdf) - b^3\left(4cCe^2 + Adef + c\right.\right.}{4b^3(bc -}$$

Mathematica [B] time = 6.44, size = 2150, normalized size = 3.27

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x)^3,x]
```

```
[Out] -1/2*((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*(e + f*x)^(3/2))/(b^2*(b*e - a*
f)*(a + b*x)^2) - ((b*B - 2*a*C)*(c + d*x)^(3/2)*(e + f*x)^(3/2))/(b*(b*c -
 a*d)*(b*e - a*f)*(a + b*x)) + (2*C*Sqrt[c + d*x]*Sqrt[e + f*x]*(1 + (d*f*(
c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(3/2)*
(1/(2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e
 - c*f))))) + (Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*
f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(
d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*(1 +
(d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^
(3/2))))/(b^3*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(
e + f*x))/(d*e - c*f)]) + (2*C*(b*c - a*d)*((Sqrt[f]*Sqrt[d*e - c*f]*Sqrt[(
d*(e + f*x))/(d*e - c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])
/(b*d*Sqrt[e + f*x]) - (Sqrt[-(b*e) + a*f]*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt
[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(b*Sqrt[-(b*c) + a*d])))/b^
3 - ((A*b^2 - a*(b*B - a*C))*(d*e - c*f)*((Sqrt[c + d*x]*Sqrt[e + f*x])/((b
*c - a*d)*(a + b*x)) - ((d*e - c*f)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*
x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/((-(b*c) + a*d)^(3/2)*Sqrt[-(b*e)
+ a*f])))/(4*b^2*(b*e - a*f)) - ((b*B - 2*a*C)*((-4*f*(c + d*x)^(3/2)*Sqrt[
e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(
d*e - c*f))))^(3/2)*(3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e -
 c*f) - (c*d*f)/(d*e - c*f))))) + (3*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (
```

```
c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f)
- (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]
*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)
/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
 c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)
/(d*e - c*f)))])))/(16*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f
)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))))))/(3*Sqrt[d/((d^2*e)/(d*e
- c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + ((2*a*b*d
*f + (b*(-2*a*d*f - b*(d*e + c*f)))/2)*((2*Sqrt[c + d*x]*Sqrt[e + f*x]*(1 +
 (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))
^(3/2)*(1/(2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*
f)/(d*e - c*f))))) + (Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d
*e - c*f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d
^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x
]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c
*f))))^(3/2))))/(b*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt
[(d*(e + f*x))/(d*e - c*f)]) - ((-(b*c) + a*d)*((2*Sqrt[f]*Sqrt[d*e - c*f]*
Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d^2*e)/(d*e - c*f
) - (c*d*f)/(d*e - c*f)]*Sqrt[(d*(e + f*x))/(d*e - c*f)]*ArcSinh[(Sqrt[d]*S
qrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(
d*e - c*f)])])/(b*d^(3/2)*Sqrt[e + f*x]) - (2*Sqrt[-(b*e) + a*f]*ArcTanh[(S
qrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(b*Sq
rt[-(b*c) + a*d])))/b))/b))/(b^2*(b*c - a*d)*(b*e - a*f))
```

IntegrateAlgebraic [B] time = 6.13, size = 1687, normalized size = 2.56

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x]*Sqrt[e + f*x]*(A + B*x + C*x^2))/(a + b*x
)^3,x]
```

```
[Out] -1/4*((-(d*e) + c*f)*Sqrt[e + f*x]*(4*b^4*c*C*e^3 - 4*a*b^3*C*d*e^3 + 4*b^4
*B*c*e^2*f - 20*a*b^3*c*C*e^2*f + A*b^4*d*e^2*f - 5*a*b^3*B*d*e^2*f + 21*a^
2*b^2*C*d*e^2*f - A*b^4*c*e*f^2 - 7*a*b^3*B*c*e*f^2 + 27*a^2*b^2*c*C*e*f^2
- a*A*b^3*d*e*f^2 + 9*a^2*b^2*B*d*e*f^2 - 29*a^3*b*C*d*e*f^2 + a*A*b^3*c*f^
3 + 3*a^2*b^2*B*c*f^3 - 11*a^3*b*c*C*f^3 - 4*a^3*b*B*d*f^3 + 12*a^4*C*d*f^3
 - (8*b^4*c^2*C*e^2*(e + f*x))/(c + d*x) - (4*b^4*B*c*d*e^2*(e + f*x))/(c +
 d*x) + (24*a*b^3*c*C*d*e^2*(e + f*x))/(c + d*x) - (A*b^4*d^2*e^2*(e + f*x)
)/(c + d*x) + (5*a*b^3*B*d^2*e^2*(e + f*x))/(c + d*x) - (17*a^2*b^2*C*d^2*e
^2*(e + f*x))/(c + d*x) - (4*b^4*B*c^2*e*f*(e + f*x))/(c + d*x) + (24*a*b^3
*c^2*C*e*f*(e + f*x))/(c + d*x) + (2*A*b^4*c*d*e*f*(e + f*x))/(c + d*x) + (
14*a*b^3*B*c*d*e*f*(e + f*x))/(c + d*x) - (62*a^2*b^2*c*C*d*e*f*(e + f*x))/
(c + d*x) - (12*a^2*b^2*B*d^2*e*f*(e + f*x))/(c + d*x) + (40*a^3*b*C*d^2*e*
f*(e + f*x))/(c + d*x) - (A*b^4*c^2*f^2*(e + f*x))/(c + d*x) + (5*a*b^3*B*c
^2*f^2*(e + f*x))/(c + d*x) - (17*a^2*b^2*c^2*C*f^2*(e + f*x))/(c + d*x) -
(12*a^2*b^2*B*c*d*f^2*(e + f*x))/(c + d*x) + (40*a^3*b*c*C*d*f^2*(e + f*x))
/(c + d*x) + (8*a^3*b*B*d^2*f^2*(e + f*x))/(c + d*x) - (24*a^4*C*d^2*f^2*(e
 + f*x))/(c + d*x) + (4*b^4*c^3*C*e*(e + f*x)^2)/(c + d*x)^2 + (4*b^4*B*c^2
*d*e*(e + f*x)^2)/(c + d*x)^2 - (20*a*b^3*c^2*C*d*e*(e + f*x)^2)/(c + d*x)^
2 - (A*b^4*c*d^2*e*(e + f*x)^2)/(c + d*x)^2 - (7*a*b^3*B*c*d^2*e*(e + f*x)^
2)/(c + d*x)^2 + (27*a^2*b^2*c*C*d^2*e*(e + f*x)^2)/(c + d*x)^2 + (a*A*b^3*
d^3*e*(e + f*x)^2)/(c + d*x)^2 + (3*a^2*b^2*B*d^3*e*(e + f*x)^2)/(c + d*x)^
2 - (11*a^3*b*C*d^3*e*(e + f*x)^2)/(c + d*x)^2 - (4*a*b^3*c^3*C*f*(e + f*x)
^2)/(c + d*x)^2 + (A*b^4*c^2*d*f*(e + f*x)^2)/(c + d*x)^2 - (5*a*b^3*B*c^2*
d*f*(e + f*x)^2)/(c + d*x)^2 + (21*a^2*b^2*c^2*C*d*f*(e + f*x)^2)/(c + d*x)
^2 - (a*A*b^3*c*d^2*f*(e + f*x)^2)/(c + d*x)^2 + (9*a^2*b^2*B*c*d^2*f*(e +
f*x)^2)/(c + d*x)^2 - (29*a^3*b*c*C*d^2*f*(e + f*x)^2)/(c + d*x)^2 - (4*a^3
*b*B*d^3*f*(e + f*x)^2)/(c + d*x)^2 + (12*a^4*C*d^3*f*(e + f*x)^2)/(c + d*x
)^2))/(b^3*(b*c - a*d)*(b*e - a*f)*Sqrt[c + d*x]*(-f + (d*(e + f*x))/(c + d
```

```
*x))*(-(b*e) + a*f + (b*c*(e + f*x))/(c + d*x) - (a*d*(e + f*x))/(c + d*x))
^2) + ((-8*b^4*c^2*C*e^2 - 4*b^4*B*c*d*e^2 + 24*a*b^3*c*C*d*e^2 + A*b^4*d^2
*e^2 + 3*a*b^3*B*d^2*e^2 - 15*a^2*b^2*C*d^2*e^2 - 4*b^4*B*c^2*e*f + 24*a*b^
3*c^2*C*e*f - 2*A*b^4*c*d*e*f + 18*a*b^3*B*c*d*e*f - 66*a^2*b^2*c*C*d*e*f -
 12*a^2*b^2*B*d^2*e*f + 40*a^3*b*C*d^2*e*f + A*b^4*c^2*f^2 + 3*a*b^3*B*c^2*
f^2 - 15*a^2*b^2*c^2*C*f^2 - 12*a^2*b^2*B*c*d*f^2 + 40*a^3*b*c*C*d*f^2 + 8*
a^3*b*B*d^2*f^2 - 24*a^4*C*d^2*f^2)*ArcTan[(Sqrt[b*c - a*d]*Sqrt[-(b*e) + a
*f]*Sqrt[e + f*x])/((b*e - a*f)*Sqrt[c + d*x])]])/(4*b^4*(b*c - a*d)^(3/2)*(
b*e - a*f)*Sqrt[-(b*e) + a*f]) + ((b*C*d*e + b*c*C*f + 2*b*B*d*f - 6*a*C*d*
f)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])]])/(b^4*Sqrt[d]*S
qrt[f])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [B] time = 39.57, size = 8347, normalized size = 12.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm=
"giac")
```

```
[Out] 1/4*(15*sqrt(d*f)*C*a^2*b^2*c^2*f^2*abs(d) - 3*sqrt(d*f)*B*a*b^3*c^2*f^2*ab
s(d) - sqrt(d*f)*A*b^4*c^2*f^2*abs(d) - 40*sqrt(d*f)*C*a^3*b*c*d*f^2*abs(d)
 + 12*sqrt(d*f)*B*a^2*b^2*c*d*f^2*abs(d) + 24*sqrt(d*f)*C*a^4*d^2*f^2*abs(d
) - 8*sqrt(d*f)*B*a^3*b*d^2*f^2*abs(d) - 24*sqrt(d*f)*C*a*b^3*c^2*f*abs(d)*
e + 4*sqrt(d*f)*B*b^4*c^2*f*abs(d)*e + 66*sqrt(d*f)*C*a^2*b^2*c*d*f*abs(d)*
e - 18*sqrt(d*f)*B*a*b^3*c*d*f*abs(d)*e + 2*sqrt(d*f)*A*b^4*c*d*f*abs(d)*e
- 40*sqrt(d*f)*C*a^3*b*d^2*f*abs(d)*e + 12*sqrt(d*f)*B*a^2*b^2*d^2*f*abs(d)
*e + 8*sqrt(d*f)*C*b^4*c^2*abs(d)*e^2 - 24*sqrt(d*f)*C*a*b^3*c*d*abs(d)*e^2
 + 4*sqrt(d*f)*B*b^4*c*d*abs(d)*e^2 + 15*sqrt(d*f)*C*a^2*b^2*d^2*abs(d)*e^2
 - 3*sqrt(d*f)*B*a*b^3*d^2*abs(d)*e^2 - sqrt(d*f)*A*b^4*d^2*abs(d)*e^2)*arc
tan(-1/2*(b*c*d*f - 2*a*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c
*d*f*e + a*b*d^2*f*e)*d))/((a*b^5*c*f - a^2*b^4*d*f - b^6*c*e + a*b^5*d*e)*
sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d) + 1/2*(9*sqr
t(d*f)*C*a^2*b^3*c^5*d^3*f^5*abs(d) - 5*sqrt(d*f)*B*a*b^4*c^5*d^3*f^5*abs(d
) + sqrt(d*f)*A*b^5*c^5*d^3*f^5*abs(d) - 10*sqrt(d*f)*C*a^3*b^2*c^4*d^4*f^5
*abs(d) + 6*sqrt(d*f)*B*a^2*b^3*c^4*d^4*f^5*abs(d) - 2*sqrt(d*f)*A*a*b^4*c^
4*d^4*f^5*abs(d) - 8*sqrt(d*f)*C*a*b^4*c^5*d^3*f^4*abs(d)*e + 4*sqrt(d*f)*B
*b^5*c^5*d^3*f^4*abs(d)*e - 27*sqrt(d*f)*C*a^2*b^3*c^4*d^4*f^4*abs(d)*e + 1
5*sqrt(d*f)*B*a*b^4*c^4*d^4*f^4*abs(d)*e - 3*sqrt(d*f)*A*b^5*c^4*d^4*f^4*ab
s(d)*e + 40*sqrt(d*f)*C*a^3*b^2*c^3*d^5*f^4*abs(d)*e - 24*sqrt(d*f)*B*a^2*b
^3*c^3*d^5*f^4*abs(d)*e + 8*sqrt(d*f)*A*a*b^4*c^3*d^5*f^4*abs(d)*e - 27*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*
a^2*b^3*c^4*d^2*f^4*abs(d) + 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^4*c^4*d^2*f^4*abs(d) - 3*sqrt(d*f)*(
sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^5*c^4*
d^2*f^4*abs(d) + 80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^2*C*a^3*b^2*c^3*d^3*f^4*abs(d) - 44*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^3*c^3*d^3*f
^4*abs(d) + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d
*f + d^2*e))^2*A*a*b^4*c^3*d^3*f^4*abs(d) - 56*sqrt(d*f)*(sqrt(d*f)*sqrt(d*
```

x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^4*b*c^2*d^4*f^4*abs(d) + 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^3*b^2*c^2*d^4*f^4*abs(d) - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a^2*b^3*c^2*d^4*f^4*abs(d) + 32*sqrt(d*f)*C*a*b^4*c^4*d^4*f^3*abs(d)*e^2 - 16*sqrt(d*f)*B*b^5*c^4*d^4*f^3*abs(d)*e^2 + 18*sqrt(d*f)*C*a^2*b^3*c^3*d^5*f^3*abs(d)*e^2 - 10*sqrt(d*f)*B*a*b^4*c^3*d^5*f^3*abs(d)*e^2 + 2*sqrt(d*f)*A*b^5*c^3*d^5*f^3*abs(d)*e^2 - 60*sqrt(d*f)*C*a^3*b^2*c^2*d^6*f^3*abs(d)*e^2 + 36*sqrt(d*f)*B*a^2*b^3*c^2*d^6*f^3*abs(d)*e^2 - 12*sqrt(d*f)*A*a*b^4*c^2*d^6*f^3*abs(d)*e^2 + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a*b^4*c^4*d^2*f^3*abs(d)*e - 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*b^5*c^4*d^2*f^3*abs(d)*e - 44*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b^3*c^3*d^3*f^3*abs(d)*e + 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^4*c^3*d^3*f^3*abs(d)*e + 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^5*c^3*d^3*f^3*abs(d)*e - 80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^3*b^2*c^2*d^4*f^3*abs(d)*e + 44*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^3*c^2*d^4*f^3*abs(d)*e - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a*b^4*c^2*d^4*f^3*abs(d)*e + 112*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^4*b*c*d^5*f^3*abs(d)*e - 64*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^3*b^2*c*d^5*f^3*abs(d)*e + 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a^2*b^3*c*d^5*f^3*abs(d)*e + 27*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^2*b^3*c^3*d*f^3*abs(d) - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a*b^4*c^3*d*f^3*abs(d) + 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*b^5*c^3*d*f^3*abs(d) - 102*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^3*b^2*c^2*d^2*f^3*abs(d) + 58*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a^2*b^3*c^2*d^2*f^3*abs(d) - 14*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*a*b^4*c^2*d^2*f^3*abs(d) + 152*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^4*b*c*d^3*f^3*abs(d) - 88*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a^3*b^2*c*d^3*f^3*abs(d) + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*a^2*b^3*c*d^3*f^3*abs(d) - 80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^5*d^4*f^3*abs(d) + 48*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a^4*b*d^4*f^3*abs(d) - 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*a^3*b^2*d^4*f^3*abs(d) - 48*sqrt(d*f)*C*a*b^4*c^3*d^5*f^2*abs(d)*e^3 + 24*sqrt(d*f)*B*b^5*c^3*d^5*f^2*abs(d)*e^3 + 18*sqrt(d*f)*C*a^2*b^3*c^2*d^6*f^2*abs(d)*e^3 - 10*sqrt(d*f)*B*a*b^4*c^2*d^6*f^2*abs(d)*e^3 + 2*sqrt(d*f)*A*b^5*c^2*d^6*f^2*abs(d)*e^3 + 40*sqrt(d*f)*C*a^3*b^2*c*d^7*f^2*abs(d)*e^3 - 24*sqrt(d*f)*B*a^2*b^3*c*d^7*f^2*abs(d)*e^3 + 8*sqrt(d*f)*A*a*b^4*c*d^7*f^2*abs(d)*e^3 - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a*b^4*c^3*d^3*f^2*abs(d)*e^2 + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*b^5*c^3*d^3*f^2*abs(d)*e^2 + 142*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b^3*c^2*d^4*f^2*abs(d)*e^2 - 70*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^4*c^2*d^4*f^2*abs(d)*e^2 - 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^5*c^2*d^4*f^2*abs(d)*e^2 - 80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^3*b^2*c*d^5*f^2*abs(d)*e^2 + 44*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^3*c*d^5*f^2*abs(d)*e^2 - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a*b^4*c*d^5*f^2*abs(d)*e^2 - 56*sqrt(d*f)*(sqr

t(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^4*b*d^6*f
^2*abs(d)*e^2 + 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
- c*d*f + d^2*e))^2*B*a^3*b^2*d^6*f^2*abs(d)*e^2 - 8*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a^2*b^3*d^6*f^2*abs
(d)*e^2 - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*
f + d^2*e))^4*C*a*b^4*c^3*d*f^2*abs(d)*e + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*b^5*c^3*d*f^2*abs(d)*e + 1
09*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e)
)^4*C*a^2*b^3*c^2*d^2*f^2*abs(d)*e - 57*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a*b^4*c^2*d^2*f^2*abs(d)*e + 5*s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*
A*b^5*c^2*d^2*f^2*abs(d)*e - 228*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
(d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^3*b^2*c*d^3*f^2*abs(d)*e + 124*sqrt(d
f)(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a^2
*b^3*c*d^3*f^2*abs(d)*e - 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x
 + c)*d*f - c*d*f + d^2*e))^4*A*a*b^4*c*d^3*f^2*abs(d)*e + 152*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^4*b*d^4
*f^2*abs(d)*e - 88*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
- c*d*f + d^2*e))^4*B*a^3*b^2*d^4*f^2*abs(d)*e + 24*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*a^2*b^3*d^4*f^2*abs(
d)*e - 9*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f +
d^2*e))^6*C*a^2*b^3*c^2*f^2*abs(d) + 5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
 sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*B*a*b^4*c^2*f^2*abs(d) - sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*A*b^5*c^2
*f^2*abs(d) + 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
c*d*f + d^2*e))^6*C*a^3*b^2*c*d*f^2*abs(d) - 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d
*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*B*a^2*b^3*c*d*f^2*abs(d) +
 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e)
)^6*A*a*b^4*c*d*f^2*abs(d) - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^6*C*a^4*b*d^2*f^2*abs(d) + 16*sqrt(d*f)*(sqr
t(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*B*a^3*b^2*d^2
*f^2*abs(d) - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c
*d*f + d^2*e))^6*A*a^2*b^3*d^2*f^2*abs(d) + 32*sqrt(d*f)*C*a*b^4*c^2*d^6*f*
abs(d)*e^4 - 16*sqrt(d*f)*B*b^5*c^2*d^6*f*abs(d)*e^4 - 27*sqrt(d*f)*C*a^2*b
^3*c*d^7*f*abs(d)*e^4 + 15*sqrt(d*f)*B*a*b^4*c*d^7*f*abs(d)*e^4 - 3*sqrt(d*
f)*A*b^5*c*d^7*f*abs(d)*e^4 - 10*sqrt(d*f)*C*a^3*b^2*d^8*f*abs(d)*e^4 + 6*s
qrt(d*f)*B*a^2*b^3*d^8*f*abs(d)*e^4 - 2*sqrt(d*f)*A*a*b^4*d^8*f*abs(d)*e^4
- 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*
e))^2*C*a*b^4*c^2*d^4*f*abs(d)*e^3 + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*b^5*c^2*d^4*f*abs(d)*e^3 - 44*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C
*a^2*b^3*c*d^5*f*abs(d)*e^3 + 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
(d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^4*c*d^5*f*abs(d)*e^3 + 4*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^5*c*d
^5*f*abs(d)*e^3 + 80*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*
f - c*d*f + d^2*e))^2*C*a^3*b^2*d^6*f*abs(d)*e^3 - 44*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^3*d^6*f*abs(
d)*e^3 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f
+ d^2*e))^2*A*a*b^4*d^6*f*abs(d)*e^3 - 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a*b^4*c^2*d^2*f*abs(d)*e^2 + 8
*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^
4*B*b^5*c^2*d^2*f*abs(d)*e^2 + 109*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^2*b^3*c*d^3*f*abs(d)*e^2 - 57*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a*
b^4*c*d^3*f*abs(d)*e^2 + 5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^4*A*b^5*c*d^3*f*abs(d)*e^2 - 102*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^3*b^2*d^4*f
*abs(d)*e^2 + 58*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
c*d*f + d^2*e))^4*B*a^2*b^3*d^4*f*abs(d)*e^2 - 14*sqrt(d*f)*(sqrt(d*f)*sqrt

```
(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*a*b^4*d^4*f*abs(d)*e^2
 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*
e))^6*C*a*b^4*c^2*f*abs(d)*e - 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
(d*x + c)*d*f - c*d*f + d^2*e))^6*B*b^5*c^2*f*abs(d)*e - 38*sqrt(d*f)*(sqrt
(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*C*a^2*b^3*c*d*
f*abs(d)*e + 22*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c
*d*f + d^2*e))^6*B*a*b^4*c*d*f*abs(d)*e - 6*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
 c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*A*b^5*c*d*f*abs(d)*e + 32*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*C*a
^3*b^2*d^2*f*abs(d)*e - 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^6*B*a^2*b^3*d^2*f*abs(d)*e + 8*sqrt(d*f)*(sqrt(d*
f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*A*a*b^4*d^2*f*abs
(d)*e - 8*sqrt(d*f)*C*a*b^4*c*d^7*abs(d)*e^5 + 4*sqrt(d*f)*B*b^5*c*d^7*abs(
d)*e^5 + 9*sqrt(d*f)*C*a^2*b^3*d^8*abs(d)*e^5 - 5*sqrt(d*f)*B*a*b^4*d^8*abs
(d)*e^5 + sqrt(d*f)*A*b^5*d^8*abs(d)*e^5 + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a*b^4*c*d^5*abs(d)*e^4 - 1
2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))
^2*B*b^5*c*d^5*abs(d)*e^4 - 27*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d
*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b^3*d^6*abs(d)*e^4 + 15*sqrt(d*f)*(sq
rt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^4*d^6*
abs(d)*e^4 - 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*
d*f + d^2*e))^2*A*b^5*d^6*abs(d)*e^4 - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c
) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a*b^4*c*d^3*abs(d)*e^3 + 12*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B
*b^5*c*d^3*abs(d)*e^3 + 27*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^4*C*a^2*b^3*d^4*abs(d)*e^3 - 15*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a*b^4*d^4*abs(
d)*e^3 + 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f
+ d^2*e))^4*A*b^5*d^4*abs(d)*e^3 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt((d*x + c)*d*f - c*d*f + d^2*e))^6*C*a*b^4*c*d*abs(d)*e^2 - 4*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*B*b^5*c*d
*abs(d)*e^2 - 9*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c
*d*f + d^2*e))^6*C*a^2*b^3*d^2*abs(d)*e^2 + 5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*B*a*b^4*d^2*abs(d)*e^2 - sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*A*
b^5*d^2*abs(d)*e^2)/((a*b^5*c*f - a^2*b^4*d*f - b^6*c*e + a*b^5*d*e)*(b*c^2
*d^2*f^2 - 2*b*c*d^3*f*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
- c*d*f + d^2*e))^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d
*f - c*d*f + d^2*e))^2*a*d^2*f + b*d^4*e^2 - 2*(sqrt(d*f)*sqrt(d*x + c) - s
qrt((d*x + c)*d*f - c*d*f + d^2*e))^2*b*d^2*e + (sqrt(d*f)*sqrt(d*x + c) -
sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*b)^2) + sqrt((d*x + c)*d*f - c*d*f +
 d^2*e)*sqrt(d*x + c)*C*abs(d)/(b^3*d^2) - 1/2*(sqrt(d*f)*C*b*c*f*abs(d) -
6*sqrt(d*f)*C*a*d*f*abs(d) + 2*sqrt(d*f)*B*b*d*f*abs(d) + sqrt(d*f)*C*b*d*a
bs(d)*e)*log((sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e)
)^2)/(b^4*d^2*f)
```

maple [B] time = 0.07, size = 12065, normalized size = 18.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(b*x+a)^3,x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for mo
re details)Is (a*d-b*c)    *(a*f-b*e) zero or nonzero?
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^(1/2)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x)^3,x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)*(f*x+e)**(1/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.47 $\int \frac{(a+bx)^2\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx$

Optimal. Leaf size=1032

$$\frac{C(c+dx)^{3/2}\sqrt{e+fx}\,(a+bx)^3}{5bdf}-\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(c+dx)^{3/2}\sqrt{e+fx}\,(a+bx)^2}{40bd^2f^2}-\frac{(c+dx)^{3/2}}{}$$

Rubi [A] time = 1.79, antiderivative size = 1032, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1615, 153, 147, 50, 63, 217, 206}

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] -((16*a^2*d^2*f^2*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d
*e*f + c^2*f^2)) + 4*a*b*d*f*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^
2 + 5*c^3*f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f +
c^2*f^2))) - b^2*(C*(63*d^4*e^4 + 28*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 12*
c^3*d*e*f^3 + 7*c^4*f^4) + 2*d*f*(8*A*d*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)
 - B*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3))))*Sqrt[c +
d*x]*Sqrt[e + f*x])/(128*d^4*f^5) - ((4*a*C*d*f + b*(9*C*d*e + 7*c*C*f - 10
*B*d*f))*(a + b*x)^2*(c + d*x)^(3/2)*Sqrt[e + f*x])/(40*b*d^2*f^2) + (C*(a
+ b*x)^3*(c + d*x)^(3/2)*Sqrt[e + f*x])/(5*b*d*f) - ((c + d*x)^(3/2)*Sqrt[e
 + f*x]*(96*a^3*C*d^3*f^3 + 8*a^2*b*d^2*f^2*(23*C*d*e + 9*c*C*f - 30*B*d*f)
 + 20*a*b^2*d*f*(8*d*f*(5*B*d*e + 3*B*c*f - 6*A*d*f) - C*(35*d^2*e^2 + 22*c
*d*e*f + 15*c^2*f^2)) + b^3*(C*(315*d^3*e^3 + 203*c*d^2*e^2*f + 145*c^2*d*e
*f^2 + 105*c^3*f^3) + 10*d*f*(8*A*d*f*(5*d*e + 3*c*f) - B*(35*d^2*e^2 + 22*
c*d*e*f + 15*c^2*f^2))) + 4*b*d*f*(8*b*d*f*(6*b*c*C*e + 3*a*C*d*e + a*c*C*f
 - 10*A*b*d*f) - (7*b*d*e + 5*b*c*f - 4*a*d*f)*(4*a*C*d*f + b*(9*C*d*e + 7*
c*C*f - 10*B*d*f)))*x))/(960*b*d^4*f^4) + ((d*e - c*f)*(16*a^2*d^2*f^2*(2*d
*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)) + 4*a
*b*d*f*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3) + 8*d*f
*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))) - b^2*(C*(6
3*d^4*e^4 + 28*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 12*c^3*d*e*f^3 + 7*c^4*f^
4) + 2*d*f*(8*A*d*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) - B*(35*d^3*e^3 + 15*
c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[e + f*x])])/(128*d^(9/2)*f^(11/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(a+bx)^2\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}dx = \frac{C(a+bx)^3(c+dx)^{3/2}\sqrt{e+fx}}{5bdf} + \frac{\int \frac{(a+bx)^2\sqrt{c+dx}\left(-\frac{1}{2}b(6bcCe+3aCde+acC}{}}{}$$

$$= -\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{40bd^2f^2}$$

$$= -\frac{(4aCdf+b(9Cde+7cCf-10Bdf))(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{40bd^2f^2}$$

$$= -\frac{\left(16a^2d^2f^2\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right.\right.\right.}{}$$

$$= -\frac{\left(16a^2d^2f^2\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right.\right.\right.}{}$$

$$= -\frac{\left(16a^2d^2f^2\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right.\right.\right.}{}$$

$$= -\frac{\left(16a^2d^2f^2\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right.\right.\right.}{}$$

<u>Mathematica</u> [B] time = 6.70, size = 3220, normalized size = 3.12

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] ((-(b*e) + a*f)^2*(d*e - c*f)^2*(C*e^2 - B*e*f + A*f^2)*Sqrt[d/((d^2*e)/(d*
e - c*f) - (c*d*f)/(d*e - c*f))]*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)
)^2*Sqrt[(d*(e + f*x))/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*(
(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]*((2*d*f*(c + d*x))/((d*e - c*f
)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c
+ d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e
)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e
- c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e
)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(2*d^3*f^6*Sqrt[c + d*x]*Sqrt[e +
 f*x]) + (2*b^2*C*(d*e - c*f)^3*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c
+ d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(9/2)*((
3*(35/(64*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/
(d*e - c*f))))^4) + 35/(48*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e
- c*f) - (c*d*f)/(d*e - c*f))))^3) + 7/(8*(1 + (d*f*(c + d*x))/((d*e - c*f)
*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((
d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1)))/10 + (21*(d
*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x)
)/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sq
rt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f
]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[
(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e -
 c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))])))/(512*d^2*f^2*(c + d*
```

```
x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e
- c*f))))^4)))/(3*d^4*f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(
7/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*b*(d*e - c*f)^2*(-4*b*C*e + b*B*
f + 2*a*C*f)*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f
)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(7/2)*((3*(5/(8*(1 + (d*f*(
c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3) + 5
/(6*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
 c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d
*f)/(d*e - c*f))))^(-1)))/8 + (15*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d
*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (
c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sq
rt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d
*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*
f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d
*e - c*f)))]))))/(256*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*
((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3)))/(3*d^3*f^4*(d/((d^2*e)/(
d*e - c*f) - (c*d*f)/(d*e - c*f)))^(5/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) +
 (2*(d*e - c*f)*(6*b^2*C*e^2 - 3*b^2*B*e*f - 6*a*b*C*e*f + A*b^2*f^2 + 2*a*
b*B*f^2 + a^2*C*f^2)*(c + d*x)^(3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d
*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(5/2)*((3/(4*(1 + (
d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2
) + (1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e -
 c*f))))^(-1))/2 + (3*(d*e - c*f)^2*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c
*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e
- c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c
 + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])]
)/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*Sqrt[1 +
 (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))]
)))/(32*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e
 - c*f) - (c*d*f)/(d*e - c*f))))^2)))/(3*d^2*f^4*(d/((d^2*e)/(d*e - c*f) -
(c*d*f)/(d*e - c*f)))^(3/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*(-(b*e) +
 a*f)*(4*b*C*e^2 - 3*b*B*e*f - 2*a*C*e*f + 2*A*b*f^2 + a*B*f^2)*(c + d*x)^(
3/2)*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) -
 (c*d*f)/(d*e - c*f))))^(3/2)*(3/(4*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2
*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))) + (3*(d*e - c*f)^2*((d^2*e)/(d*e
- c*f) - (c*d*f)/(d*e - c*f))^2*((2*d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d
*e - c*f) - (c*d*f)/(d*e - c*f))) - (2*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*ArcSin
h[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f)
 - (c*d*f)/(d*e - c*f)])])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d
*f)/(d*e - c*f)]*Sqrt[1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f)
 - (c*d*f)/(d*e - c*f)))]))))/(16*d^2*f^2*(c + d*x)^2*(1 + (d*f*(c + d*x))/(
(d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))))))/(3*d*f^4*Sqrt[
d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*
f)])
```

IntegrateAlgebraic [B] time = 9.37, size = 2260, normalized size = 2.19

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x)^2*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f
*x],x]
```

```
[Out] (Sqrt[c - (d*e)/f + (d*(e + f*x))/f]*(2895*b^2*C*d^4*e^4*Sqrt[e + f*x] - 42
0*b^2*c*C*d^3*e^3*f*Sqrt[e + f*x] - 2790*b^2*B*d^4*e^3*f*Sqrt[e + f*x] - 55
80*a*b*C*d^4*e^3*f*Sqrt[e + f*x] - 270*b^2*c^2*C*d^2*e^2*f^2*Sqrt[e + f*x]
+ 450*b^2*B*c*d^3*e^2*f^2*Sqrt[e + f*x] + 900*a*b*c*C*d^3*e^2*f^2*Sqrt[e +
f*x] + 2640*A*b^2*d^4*e^2*f^2*Sqrt[e + f*x] + 5280*a*b*B*d^4*e^2*f^2*Sqrt[e
 + f*x] + 2640*a^2*C*d^4*e^2*f^2*Sqrt[e + f*x] - 180*b^2*c^3*C*d*e*f^3*Sqrt
[e + f*x] + 270*b^2*B*c^2*d^2*e*f^3*Sqrt[e + f*x] + 540*a*b*c^2*C*d^2*e*f^3
```

```
*Sqrt[e + f*x] - 480*A*b^2*c*d^3*e*f^3*Sqrt[e + f*x] - 960*a*b*B*c*d^3*e*f^
3*Sqrt[e + f*x] - 480*a^2*c*C*d^3*e*f^3*Sqrt[e + f*x] - 4800*a*A*b*d^4*e*f^
3*Sqrt[e + f*x] - 2400*a^2*B*d^4*e*f^3*Sqrt[e + f*x] - 105*b^2*c^4*C*f^4*Sq
rt[e + f*x] + 150*b^2*B*c^3*d*f^4*Sqrt[e + f*x] + 300*a*b*c^3*C*d*f^4*Sqrt[
e + f*x] - 240*A*b^2*c^2*d^2*f^4*Sqrt[e + f*x] - 480*a*b*B*c^2*d^2*f^4*Sqrt
[e + f*x] - 240*a^2*c^2*C*d^2*f^4*Sqrt[e + f*x] + 960*a*A*b*c*d^3*f^4*Sqrt[
e + f*x] + 480*a^2*B*c*d^3*f^4*Sqrt[e + f*x] + 1920*a^2*A*d^4*f^4*Sqrt[e +
f*x] - 4470*b^2*C*d^4*e^3*(e + f*x)^(3/2) + 370*b^2*c*C*d^3*e^2*f*(e + f*x)
^(3/2) + 3260*b^2*B*d^4*e^2*f*(e + f*x)^(3/2) + 6520*a*b*C*d^4*e^2*f*(e + f
*x)^(3/2) + 190*b^2*c^2*C*d^2*e*f^2*(e + f*x)^(3/2) - 280*b^2*B*c*d^3*e*f^2
*(e + f*x)^(3/2) - 560*a*b*c*C*d^3*e*f^2*(e + f*x)^(3/2) - 2080*A*b^2*d^4*e
*f^2*(e + f*x)^(3/2) - 4160*a*b*B*d^4*e*f^2*(e + f*x)^(3/2) - 2080*a^2*C*d^
4*e*f^2*(e + f*x)^(3/2) + 70*b^2*c^3*C*d*f^3*(e + f*x)^(3/2) - 100*b^2*B*c^
2*d^2*f^3*(e + f*x)^(3/2) - 200*a*b*c^2*C*d^2*f^3*(e + f*x)^(3/2) + 160*A*b
^2*c*d^3*f^3*(e + f*x)^(3/2) + 320*a*b*B*c*d^3*f^3*(e + f*x)^(3/2) + 160*a^
2*c*C*d^3*f^3*(e + f*x)^(3/2) + 1920*a*A*b*d^4*f^3*(e + f*x)^(3/2) + 960*a^
2*B*d^4*f^3*(e + f*x)^(3/2) + 4104*b^2*C*d^4*e^2*(e + f*x)^(5/2) - 208*b^2*
c*C*d^3*e*f*(e + f*x)^(5/2) - 2000*b^2*B*d^4*e*f*(e + f*x)^(5/2) - 4000*a*b
*C*d^4*e*f*(e + f*x)^(5/2) - 56*b^2*c^2*C*d^2*f^2*(e + f*x)^(5/2) + 80*b^2*
B*c*d^3*f^2*(e + f*x)^(5/2) + 160*a*b*c*C*d^3*f^2*(e + f*x)^(5/2) + 640*A*b
^2*d^4*f^2*(e + f*x)^(5/2) + 1280*a*b*B*d^4*f^2*(e + f*x)^(5/2) + 640*a^2*C
*d^4*f^2*(e + f*x)^(5/2) - 1968*b^2*C*d^4*e*(e + f*x)^(7/2) + 48*b^2*c*C*d^
3*f*(e + f*x)^(7/2) + 480*b^2*B*d^4*f*(e + f*x)^(7/2) + 960*a*b*C*d^4*f*(e
+ f*x)^(7/2) + 384*b^2*C*d^4*(e + f*x)^(9/2)))/(1920*d^4*f^5) + ((63*b^2*C*
d^5*e^5*Sqrt[d/f] - 35*b^2*c*C*d^4*e^4*Sqrt[d/f]*f - 70*b^2*B*d^5*e^4*Sqrt[
d/f]*f - 140*a*b*C*d^5*e^4*Sqrt[d/f]*f - 10*b^2*c^2*C*d^3*e^3*Sqrt[d/f]*f^2
 + 40*b^2*B*c*d^4*e^3*Sqrt[d/f]*f^2 + 80*a*b*c*C*d^4*e^3*Sqrt[d/f]*f^2 + 80
*A*b^2*d^5*e^3*Sqrt[d/f]*f^2 + 160*a*b*B*d^5*e^3*Sqrt[d/f]*f^2 + 80*a^2*C*d
^5*e^3*Sqrt[d/f]*f^2 - 6*b^2*c^3*C*d^2*e^2*Sqrt[d/f]*f^3 + 12*b^2*B*c^2*d^3
*e^2*Sqrt[d/f]*f^3 + 24*a*b*c^2*C*d^3*e^2*Sqrt[d/f]*f^3 - 48*A*b^2*c*d^4*e^
2*Sqrt[d/f]*f^3 - 96*a*b*B*c*d^4*e^2*Sqrt[d/f]*f^3 - 48*a^2*c*C*d^4*e^2*Sqr
t[d/f]*f^3 - 192*a*A*b*d^5*e^2*Sqrt[d/f]*f^3 - 96*a^2*B*d^5*e^2*Sqrt[d/f]*f
^3 - 5*b^2*c^4*C*d*e*Sqrt[d/f]*f^4 + 8*b^2*B*c^3*d^2*e*Sqrt[d/f]*f^4 + 16*a
*b*c^3*C*d^2*e*Sqrt[d/f]*f^4 - 16*A*b^2*c^2*d^3*e*Sqrt[d/f]*f^4 - 32*a*b*B*
c^2*d^3*e*Sqrt[d/f]*f^4 - 16*a^2*c^2*C*d^3*e*Sqrt[d/f]*f^4 + 128*a*A*b*c*d^
4*e*Sqrt[d/f]*f^4 + 64*a^2*B*c*d^4*e*Sqrt[d/f]*f^4 + 128*a^2*A*d^5*e*Sqrt[d
/f]*f^4 - 7*b^2*c^5*C*Sqrt[d/f]*f^5 + 10*b^2*B*c^4*d*Sqrt[d/f]*f^5 + 20*a*b
*c^4*C*d*Sqrt[d/f]*f^5 - 16*A*b^2*c^3*d^2*Sqrt[d/f]*f^5 - 32*a*b*B*c^3*d^2*
Sqrt[d/f]*f^5 - 16*a^2*c^3*C*d^2*Sqrt[d/f]*f^5 + 64*a*A*b*c^2*d^3*Sqrt[d/f]
*f^5 + 32*a^2*B*c^2*d^3*Sqrt[d/f]*f^5 - 128*a^2*A*c*d^4*Sqrt[d/f]*f^5)*Log[
-(Sqrt[d/f]*Sqrt[e + f*x]) + Sqrt[c - (d*e)/f + (d*(e + f*x))/f]])/(128*d^5
*f^5)
```

fricas [A] time = 13.80, size = 2176, normalized size = 2.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [-1/7680*(15*(63*C*b^2*d^5*e^5 - 35*(C*b^2*c*d^4 + 2*(2*C*a*b + B*b^2)*d^5)
*e^4*f - 10*(C*b^2*c^2*d^3 - 4*(2*C*a*b + B*b^2)*c*d^4 - 8*(C*a^2 + 2*B*a*b
 + A*b^2)*d^5)*e^3*f^2 - 6*(C*b^2*c^3*d^2 - 2*(2*C*a*b + B*b^2)*c^2*d^3 + 8
*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 16*(B*a^2 + 2*A*a*b)*d^5)*e^2*f^3 - (5*C
*b^2*c^4*d - 128*A*a^2*d^5 - 8*(2*C*a*b + B*b^2)*c^3*d^2 + 16*(C*a^2 + 2*B*
a*b + A*b^2)*c^2*d^3 - 64*(B*a^2 + 2*A*a*b)*c*d^4)*e*f^4 - (7*C*b^2*c^5 + 1
28*A*a^2*c*d^4 - 10*(2*C*a*b + B*b^2)*c^4*d + 16*(C*a^2 + 2*B*a*b + A*b^2)*
c^3*d^2 - 32*(B*a^2 + 2*A*a*b)*c^2*d^3)*f^5)*sqrt(d*f)*log(8*d^2*f^2*x^2 +
d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x
```

```
+ c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(384*C*b^2*d^5*f^5*x^4 +
945*C*b^2*d^5*e^4*f - 210*(C*b^2*c*d^4 + 5*(2*C*a*b + B*b^2)*d^5)*e^3*f^2 -
 2*(68*C*b^2*c^2*d^3 - 125*(2*C*a*b + B*b^2)*c*d^4 - 600*(C*a^2 + 2*B*a*b +
 A*b^2)*d^5)*e^2*f^3 - 10*(11*C*b^2*c^3*d^2 - 17*(2*C*a*b + B*b^2)*c^2*d^3
+ 32*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 144*(B*a^2 + 2*A*a*b)*d^5)*e*f^4 - 1
5*(7*C*b^2*c^4*d - 128*A*a^2*d^5 - 10*(2*C*a*b + B*b^2)*c^3*d^2 + 16*(C*a^2
 + 2*B*a*b + A*b^2)*c^2*d^3 - 32*(B*a^2 + 2*A*a*b)*c*d^4)*f^5 - 48*(9*C*b^2
*d^5*e*f^4 - (C*b^2*c*d^4 + 10*(2*C*a*b + B*b^2)*d^5)*f^5)*x^3 + 8*(63*C*b^
2*d^5*e^2*f^3 - 2*(4*C*b^2*c*d^4 + 35*(2*C*a*b + B*b^2)*d^5)*e*f^4 - (7*C*b
^2*c^2*d^3 - 10*(2*C*a*b + B*b^2)*c*d^4 - 80*(C*a^2 + 2*B*a*b + A*b^2)*d^5)
*f^5)*x^2 - 2*(315*C*b^2*d^5*e^3*f^2 - 7*(7*C*b^2*c*d^4 + 50*(2*C*a*b + B*b
^2)*d^5)*e^2*f^3 - (39*C*b^2*c^2*d^3 - 60*(2*C*a*b + B*b^2)*c*d^4 - 400*(C*
a^2 + 2*B*a*b + A*b^2)*d^5)*e*f^4 - 5*(7*C*b^2*c^3*d^2 - 10*(2*C*a*b + B*b^
2)*c^2*d^3 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 96*(B*a^2 + 2*A*a*b)*d^5)
*f^5)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^6), 1/3840*(15*(63*C*b^2*d^5*e
^5 - 35*(C*b^2*c*d^4 + 2*(2*C*a*b + B*b^2)*d^5)*e^4*f - 10*(C*b^2*c^2*d^3 -
 4*(2*C*a*b + B*b^2)*c*d^4 - 8*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*e^3*f^2 - 6*(
C*b^2*c^3*d^2 - 2*(2*C*a*b + B*b^2)*c^2*d^3 + 8*(C*a^2 + 2*B*a*b + A*b^2)*c
*d^4 + 16*(B*a^2 + 2*A*a*b)*d^5)*e^2*f^3 - (5*C*b^2*c^4*d - 128*A*a^2*d^5 -
 8*(2*C*a*b + B*b^2)*c^3*d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3 - 64*(B
*a^2 + 2*A*a*b)*c*d^4)*e*f^4 - (7*C*b^2*c^5 + 128*A*a^2*c*d^4 - 10*(2*C*a*b
 + B*b^2)*c^4*d + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^3*d^2 - 32*(B*a^2 + 2*A*a*
b)*c^2*d^3)*f^5)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqr
t(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) +
 2*(384*C*b^2*d^5*f^5*x^4 + 945*C*b^2*d^5*e^4*f - 210*(C*b^2*c*d^4 + 5*(2*C
*a*b + B*b^2)*d^5)*e^3*f^2 - 2*(68*C*b^2*c^2*d^3 - 125*(2*C*a*b + B*b^2)*c*
d^4 - 600*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*e^2*f^3 - 10*(11*C*b^2*c^3*d^2 - 1
7*(2*C*a*b + B*b^2)*c^2*d^3 + 32*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4 + 144*(B*a
^2 + 2*A*a*b)*d^5)*e*f^4 - 15*(7*C*b^2*c^4*d - 128*A*a^2*d^5 - 10*(2*C*a*b
+ B*b^2)*c^3*d^2 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^3 - 32*(B*a^2 + 2*A*a
*b)*c*d^4)*f^5 - 48*(9*C*b^2*d^5*e*f^4 - (C*b^2*c*d^4 + 10*(2*C*a*b + B*b^2
)*d^5)*f^5)*x^3 + 8*(63*C*b^2*d^5*e^2*f^3 - 2*(4*C*b^2*c*d^4 + 35*(2*C*a*b
+ B*b^2)*d^5)*e*f^4 - (7*C*b^2*c^2*d^3 - 10*(2*C*a*b + B*b^2)*c*d^4 - 80*(C
*a^2 + 2*B*a*b + A*b^2)*d^5)*f^5)*x^2 - 2*(315*C*b^2*d^5*e^3*f^2 - 7*(7*C*b
^2*c*d^4 + 50*(2*C*a*b + B*b^2)*d^5)*e^2*f^3 - (39*C*b^2*c^2*d^3 - 60*(2*C*
a*b + B*b^2)*c*d^4 - 400*(C*a^2 + 2*B*a*b + A*b^2)*d^5)*e*f^4 - 5*(7*C*b^2*
c^3*d^2 - 10*(2*C*a*b + B*b^2)*c^2*d^3 + 16*(C*a^2 + 2*B*a*b + A*b^2)*c*d^4
 + 96*(B*a^2 + 2*A*a*b)*d^5)*f^5)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^6)
]
```

giac [A] time = 2.76, size = 1505, normalized size = 1.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/1920*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(4*(d*x + c)*(6*(d*x + c)*(8
*(d*x + c)*C*b^2/(d^5*f) - (31*C*b^2*c*d^20*f^8 - 20*C*a*b*d^21*f^8 - 10*B*
b^2*d^21*f^8 + 9*C*b^2*d^21*f^7*e)/(d^25*f^9)) + (263*C*b^2*c^2*d^20*f^8 -
340*C*a*b*c*d^21*f^8 - 170*B*b^2*c*d^21*f^8 + 80*C*a^2*d^22*f^8 + 160*B*a*b
*d^22*f^8 + 80*A*b^2*d^22*f^8 + 154*C*b^2*c*d^21*f^7*e - 140*C*a*b*d^22*f^7
*e - 70*B*b^2*d^22*f^7*e + 63*C*b^2*d^22*f^6*e^2)/(d^25*f^9)) - 5*(121*C*b^
2*c^3*d^20*f^8 - 236*C*a*b*c^2*d^21*f^8 - 118*B*b^2*c^2*d^21*f^8 + 112*C*a^
2*c*d^22*f^8 + 224*B*a*b*c*d^22*f^8 + 112*A*b^2*c*d^22*f^8 - 96*B*a^2*d^23*
f^8 - 192*A*a*b*d^23*f^8 + 109*C*b^2*c^2*d^21*f^7*e - 200*C*a*b*c*d^22*f^7*
e - 100*B*b^2*c*d^22*f^7*e + 80*C*a^2*d^23*f^7*e + 160*B*a*b*d^23*f^7*e + 8
0*A*b^2*d^23*f^7*e + 91*C*b^2*c*d^22*f^6*e^2 - 140*C*a*b*d^23*f^6*e^2 - 70*
B*b^2*d^23*f^6*e^2 + 63*C*b^2*d^23*f^5*e^3)/(d^25*f^9))*(d*x + c) + 15*(7*C
```

```
*b^2*c^4*d^20*f^8 - 20*C*a*b*c^3*d^21*f^8 - 10*B*b^2*c^3*d^21*f^8 + 16*C*a^
2*c^2*d^22*f^8 + 32*B*a*b*c^2*d^22*f^8 + 16*A*b^2*c^2*d^22*f^8 - 32*B*a^2*c
*d^23*f^8 - 64*A*a*b*c*d^23*f^8 + 128*A*a^2*d^24*f^8 + 12*C*b^2*c^3*d^21*f^
7*e - 36*C*a*b*c^2*d^22*f^7*e - 18*B*b^2*c^2*d^22*f^7*e + 32*C*a^2*c*d^23*f
^7*e + 64*B*a*b*c*d^23*f^7*e + 32*A*b^2*c*d^23*f^7*e - 96*B*a^2*d^24*f^7*e
- 192*A*a*b*d^24*f^7*e + 18*C*b^2*c^2*d^22*f^6*e^2 - 60*C*a*b*c*d^23*f^6*e^
2 - 30*B*b^2*c*d^23*f^6*e^2 + 80*C*a^2*d^24*f^6*e^2 + 160*B*a*b*d^24*f^6*e^
2 + 80*A*b^2*d^24*f^6*e^2 + 28*C*b^2*c*d^23*f^5*e^3 - 140*C*a*b*d^24*f^5*e^
3 - 70*B*b^2*d^24*f^5*e^3 + 63*C*b^2*d^24*f^4*e^4)/(d^25*f^9))*sqrt(d*x + c
) - 15*(7*C*b^2*c^5*f^5 - 20*C*a*b*c^4*d*f^5 - 10*B*b^2*c^4*d*f^5 + 16*C*a^
2*c^3*d^2*f^5 + 32*B*a*b*c^3*d^2*f^5 + 16*A*b^2*c^3*d^2*f^5 - 32*B*a^2*c^2*
d^3*f^5 - 64*A*a*b*c^2*d^3*f^5 + 128*A*a^2*c*d^4*f^5 + 5*C*b^2*c^4*d*f^4*e
- 16*C*a*b*c^3*d^2*f^4*e - 8*B*b^2*c^3*d^2*f^4*e + 16*C*a^2*c^2*d^3*f^4*e +
 32*B*a*b*c^2*d^3*f^4*e + 16*A*b^2*c^2*d^3*f^4*e - 64*B*a^2*c*d^4*f^4*e - 1
28*A*a*b*c*d^4*f^4*e - 128*A*a^2*d^5*f^4*e + 6*C*b^2*c^3*d^2*f^3*e^2 - 24*C
*a*b*c^2*d^3*f^3*e^2 - 12*B*b^2*c^2*d^3*f^3*e^2 + 48*C*a^2*c*d^4*f^3*e^2 +
96*B*a*b*c*d^4*f^3*e^2 + 48*A*b^2*c*d^4*f^3*e^2 + 96*B*a^2*d^5*f^3*e^2 + 19
2*A*a*b*d^5*f^3*e^2 + 10*C*b^2*c^2*d^3*f^2*e^3 - 80*C*a*b*c*d^4*f^2*e^3 - 4
0*B*b^2*c*d^4*f^2*e^3 - 80*C*a^2*d^5*f^2*e^3 - 160*B*a*b*d^5*f^2*e^3 - 80*A
*b^2*d^5*f^2*e^3 + 35*C*b^2*c*d^4*f*e^4 + 140*C*a*b*d^5*f*e^4 + 70*B*b^2*d^
5*f*e^4 - 63*C*b^2*d^5*e^5)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x +
c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^4*f^5))*d/abs(d)
```

maple [B] time = 0.05, size = 3958, normalized size = 3.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] 1/3840*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(1440*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+
e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*d^5*e^2*f^3+1280*C*x^2*a^2*
d^4*f^4*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-2880*B*(d*f)^(1/2)*((d*x+c)*(f*
x+e))^(1/2)*a^2*d^4*e*f^3-2100*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*d^
4*e^3*f+2400*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*d^4*e^2*f^2+2880*A*l
n(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
a*b*d^5*e^2*f^3+720*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*b^2*c*d^4*e^2*f^3-960*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(
f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c*d^4*e*f^4-2400*B*ln(1
/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b
*d^5*e^3*f^2-600*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*b^2*c*d^4*e^3*f^2+720*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x
+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c*d^4*e^2*f^3+2100*C*ln(1/
2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*
d^5*e^4*f+525*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d
*e)/(d*f)^(1/2))*b^2*c*d^4*e^4*f+2400*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)
*b^2*d^4*e^2*f^2-960*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
)+c*f+d*e)/(d*f)^(1/2))*a*b*c^2*d^3*f^5+105*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^5*f^5-420*C*(d*f)^(1/2
)*((d*x+c)*(f*x+e))^(1/2)*b^2*c*d^3*e^3*f-5760*A*(d*f)^(1/2)*((d*x+c)*(f*x+
e))^(1/2)*a*b*d^4*e*f^3+4800*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*d^4*
e^2*f^2-4200*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*d^4*e^3*f-1200*C*ln(
1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*
b*c*d^4*e^3*f^2-360*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)
+c*f+d*e)/(d*f)^(1/2))*a*b*c^2*d^3*e^2*f^3+1440*B*ln(1/2*(2*d*f*x+2*((d*x+c
)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c*d^4*e^2*f^3-1920*A
*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2)
)*a*b*c*d^4*e*f^4-1200*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2)+c*f+d*e)/(d*f)^(1/2))*a^2*d^5*e^3*f^2-150*B*ln(1/2*(2*d*f*x+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^4*d*f^5+240*C*ln(1/2
```

```
*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c
^3*d^2*f^5+240*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+
d*e)/(d*f)^(1/2))*b^2*c^3*d^2*f^5-480*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c^2*d^3*f^5+3840*A*(d*f)^(1/2)
*((d*x+c)*(f*x+e))^(1/2)*a^2*d^4*f^4+1920*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x
+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c*d^4*f^5-1920*A*ln(1/2*(2
*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*d^5*
e*f^4-1200*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)
/(d*f)^(1/2))*b^2*d^5*e^3*f^2-210*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2
*c^4*f^4+1890*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*d^4*e^4-945*C*ln(1/
2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*
d^5*e^5+1050*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*
e)/(d*f)^(1/2))*b^2*d^5*e^4*f+768*C*x^4*b^2*d^4*f^4*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2)+960*B*x^3*b^2*d^4*f^4*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+1280
*A*x^2*b^2*d^4*f^4*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+960*B*(d*f)^(1/2)*((
d*x+c)*(f*x+e))^(1/2)*a^2*c*d^3*f^4+300*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/
2)*b^2*c^3*d*f^4-480*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*c^2*d^2*f^4+
1920*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a^2*d^4*f^4+480*B*ln(1/2*(2*d*
f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^3*d^2
*f^5-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(
d*f)^(1/2))*b^2*c^3*d^2*e*f^4-180*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^2*d^3*e^2*f^3+240*C*ln(1/2*(2*d*
f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a^2*c^2*d^3
*e*f^4-300*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)
/(d*f)^(1/2))*a*b*c^4*d*f^5+75*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^4*d*e*f^4+90*C*ln(1/2*(2*d*f*x+2*((
d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b^2*c^3*d^2*e^2*f^3
+150*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)
^(1/2))*b^2*c^2*d^3*e^3*f^2-480*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c
^2*d^2*f^4+240*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+
d*e)/(d*f)^(1/2))*b^2*c^2*d^3*e*f^4+680*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/
2)*a*b*c^2*d^2*e*f^3+640*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a*b*c*d^3*
f^4-3200*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a*b*d^4*e*f^3+1000*C*(d*f)
^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*c*d^3*e^2*f^2+320*C*x^2*a*b*c*d^3*f^4*((
d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-2240*C*x^2*a*b*d^4*e*f^3*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)-128*C*x^2*b^2*c*d^3*e*f^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)
^(1/2)-240*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*c*d^3*e*f^3-400*C*(d
*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a*b*c^2*d^2*f^4+2800*C*(d*f)^(1/2)*((d*
x+c)*(f*x+e))^(1/2)*x*a*b*d^4*e^2*f^2+156*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(
1/2)*x*b^2*c^2*d^2*e*f^3+196*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*c*
d^3*e^2*f^2-1280*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*c*d^3*e*f^3-200*
B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*c^2*d^2*f^4+1400*B*(d*f)^(1/2)*
((d*x+c)*(f*x+e))^(1/2)*x*b^2*d^4*e^2*f^2+3840*A*(d*f)^(1/2)*((d*x+c)*(f*x+
e))^(1/2)*x*a*b*d^4*f^4+320*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*c*d
^3*f^4-1600*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*d^4*e*f^3+1920*C*x^
3*a*b*d^4*f^4*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+96*C*x^3*b^2*c*d^3*f^4*((
d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-240*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))
^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^3*d^2*e*f^4+480*B*ln(1/2*(2*
d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*b*c^2*d
^3*e*f^4-864*C*x^3*b^2*d^4*e*f^3*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+2560*B
*x^2*a*b*d^4*f^4*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+160*B*x^2*b^2*c*d^3*f^
4*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)-1120*B*x^2*b^2*d^4*e*f^3*((d*x+c)*(f*
x+e))^(1/2)*(d*f)^(1/2)-112*C*x^2*b^2*c^2*d^2*f^4*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)+1008*C*x^2*b^2*d^4*e^2*f^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+3
20*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a^2*c*d^3*f^4-1600*C*(d*f)^(1/2)
*((d*x+c)*(f*x+e))^(1/2)*x*a^2*d^4*e*f^3+140*C*(d*f)^(1/2)*((d*x+c)*(f*x+e)
)^(1/2)*x*b^2*c^3*d*f^4-1260*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*d^
4*e^3*f+1920*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*c*d^3*f^4-640*A*(d*f
)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c*d^3*e*f^3-960*B*(d*f)^(1/2)*((d*x+c)*
```

```
(f*x+e))^(1/2)*a*b*c^2*d^2*f^4+340*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^
2*c^2*d^2*e*f^3+500*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c*d^3*e^2*f^2
-640*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*c*d^3*e*f^3+600*C*(d*f)^(1/2
)*((d*x+c)*(f*x+e))^(1/2)*a*b*c^3*d*f^4-220*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))
^(1/2)*b^2*c^3*d*e*f^3-272*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c^2*d^
2*e^2*f^2-480*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a*b*c*d^3*e*f^3)/((d*
x+c)*(f*x+e))^(1/2)/f^5/d^4/(d*f)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 details)Is c*f-d*e zero or nonzero?
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

3.48 $\int \frac{(a+bx)\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx$

Optimal. Leaf size=540

$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(24a^2Cd^2f^2+4bdfx(4aCdf+b(-8Bdf+5cCf+7Cde))+8abdf(-6Bdf+3cCf+5Cde\right.}{96bd^3f^3}$$

Rubi [A] time = 0.71, antiderivative size = 540, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1615, 147, 50, 63, 217, 206}

$$-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(24a^2Cd^2f^2+4bdfx(4aCdf+b(-8Bdf+5cCf+7Cde))+8abdf(-6Bdf+3cCf+5Cde)+b^2\left(8df(-6Adf+3Bcf+5Bde)-C\left(15c^2f^2+22cdef+35d^2e^2\right)\right)\right)}{96bd^3f^3}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(8adf\left(2df(-4Adf+Bcf+3Bde)-C\left(c^2f^2+2cdef+5d^2e^2\right)\right)+b\left(8df\left(2Adf(cf+3de)-B\left(c^2f^2+2cdef+5d^2e^2\right)\right)+C\left(9c^2def^2+5c^3f^3+15cd^2e^2f+35d^3e^3\right)\right)\right)}{64d^3f^4}+\frac{(de-cf)\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(8adf\left(2df(-4Adf+Bcf+3Bde)-C\left(c^2f^2+2cdef+5d^2e^2\right)\right)+b\left(8df\left(2Adf(cf+3de)-B\left(c^2f^2+2cdef+5d^2e^2\right)\right)+C\left(9c^2def^2+5c^3f^3+15cd^2e^2f+35d^3e^3\right)\right)\right)}{64d^{7/2}f^{9/2}}+\frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] -((8*a*d*f*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f +
c^2*f^2)) + b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3)
+ 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))))*Sqr
t[c + d*x]*Sqrt[e + f*x])/(64*d^3*f^4) + (C*(a + b*x)^2*(c + d*x)^(3/2)*Sqr
t[e + f*x])/(4*b*d*f) - ((c + d*x)^(3/2)*Sqrt[e + f*x]*(24*a^2*C*d^2*f^2 +
8*a*b*d*f*(5*C*d*e + 3*c*C*f - 6*B*d*f) + b^2*(8*d*f*(5*B*d*e + 3*B*c*f - 6
*A*d*f) - C*(35*d^2*e^2 + 22*c*d*e*f + 15*c^2*f^2)) + 4*b*d*f*(4*a*C*d*f +
b*(7*C*d*e + 5*c*C*f - 8*B*d*f))*x))/(96*b*d^3*f^3) + ((d*e - c*f)*(8*a*d*f
*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))
+ b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 + 5*c^3*f^3) + 8*d*f*(2
*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))))*ArcTanh[(Sqrt
[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(64*d^(7/2)*f^(9/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(a+bx)\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx = \frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf} + \int \frac{(a+bx)\sqrt{c+dx}\left(-\frac{1}{2}b(4bcCe+3aCde+acCf-\cdots\right.}{\cdots}$$

$$= \frac{C(a+bx)^2(c+dx)^{3/2}\sqrt{e+fx}}{4bdf} - \frac{(c+dx)^{3/2}\sqrt{e+fx}\left(24a^2Cd^2f^2+\cdots\right.}{\cdots}$$

$$= -\frac{\left(8adf\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right)\right)+\cdots\right.}{\cdots}$$

$$= -\frac{\left(8adf\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right)\right)+\cdots\right.}{\cdots}$$

$$= -\frac{\left(8adf\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right)\right)+\cdots\right.}{\cdots}$$

$$= -\frac{\left(8adf\left(2df(3Bde+Bcf-4Adf)-C\left(5d^2e^2+2cdef+c^2f^2\right)\right)+\cdots\right.}{\cdots}$$

Mathematica [A] time = 3.54, size = 478, normalized size = 0.89

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (d*Sqrt[f]*Sqrt[c + d*x]*(e + f*x)*(8*a*d*f*(6*d*f*(4*A*d*f + B*(-3*d*e + c
*f + 2*d*f*x)) + C*(-3*c^2*f^2 + 2*c*d*f*(-2*e + f*x) + d^2*(15*e^2 - 10*e*
f*x + 8*f^2*x^2))) + b*(C*(15*c^3*f^3 + c^2*d*f^2*(17*e - 10*f*x) + c*d^2*f
*(25*e^2 - 12*e*f*x + 8*f^2*x^2) + d^3*(-105*e^3 + 70*e^2*f*x - 56*e*f^2*x^
2 + 48*f^3*x^3)) + 8*d*f*(6*A*d*f*(-3*d*e + c*f + 2*d*f*x) + B*(-3*c^2*f^2
+ 2*c*d*f*(-2*e + f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f^2*x^2))))) + 3*(d*e -
 c*f)^(3/2)*(-8*a*d*f*(2*d*f*(-3*B*d*e - B*c*f + 4*A*d*f) + C*(5*d^2*e^2 +
2*c*d*e*f + c^2*f^2)) + b*(C*(35*d^3*e^3 + 15*c*d^2*e^2*f + 9*c^2*d*e*f^2 +
 5*c^3*f^3) + 8*d*f*(2*A*d*f*(3*d*e + c*f) - B*(5*d^2*e^2 + 2*c*d*e*f + c^2
*f^2))))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sq
rt[d*e - c*f]])/(192*d^4*f^(9/2)*Sqrt[e + f*x])
```

IntegrateAlgebraic [B] time = 3.39, size = 1096, normalized size = 2.03

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x)*Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x
],x]
```

```
[Out] (Sqrt[c - (d*e)/f + (d*(e + f*x))/f]*(-279*b*C*d^3*e^3*Sqrt[e + f*x] + 45*b
*c*C*d^2*e^2*f*Sqrt[e + f*x] + 264*b*B*d^3*e^2*f*Sqrt[e + f*x] + 264*a*C*d^
3*e^2*f*Sqrt[e + f*x] + 27*b*c^2*C*d*e*f^2*Sqrt[e + f*x] - 48*b*B*c*d^2*e*f
^2*Sqrt[e + f*x] - 48*a*c*C*d^2*e*f^2*Sqrt[e + f*x] - 240*A*b*d^3*e*f^2*Sqr
t[e + f*x] - 240*a*B*d^3*e*f^2*Sqrt[e + f*x] + 15*b*c^3*C*f^3*Sqrt[e + f*x]
 - 24*b*B*c^2*d*f^3*Sqrt[e + f*x] - 24*a*c^2*C*d*f^3*Sqrt[e + f*x] + 48*A*b
*c*d^2*f^3*Sqrt[e + f*x] + 48*a*B*c*d^2*f^3*Sqrt[e + f*x] + 192*a*A*d^3*f^3
*Sqrt[e + f*x] + 326*b*C*d^3*e^2*(e + f*x)^(3/2) - 28*b*c*C*d^2*e*f*(e + f*
x)^(3/2) - 208*b*B*d^3*e*f*(e + f*x)^(3/2) - 208*a*C*d^3*e*f*(e + f*x)^(3/2
) - 10*b*c^2*C*d*f^2*(e + f*x)^(3/2) + 16*b*B*c*d^2*f^2*(e + f*x)^(3/2) + 1
6*a*c*C*d^2*f^2*(e + f*x)^(3/2) + 96*A*b*d^3*f^2*(e + f*x)^(3/2) + 96*a*B*d
^3*f^2*(e + f*x)^(3/2) - 200*b*C*d^3*e*(e + f*x)^(5/2) + 8*b*c*C*d^2*f*(e +
 f*x)^(5/2) + 64*b*B*d^3*f*(e + f*x)^(5/2) + 64*a*C*d^3*f*(e + f*x)^(5/2) +
 48*b*C*d^3*(e + f*x)^(7/2)))/(192*d^3*f^4) + ((-35*b*C*d^4*e^4*Sqrt[d/f] +
 20*b*c*C*d^3*e^3*Sqrt[d/f]*f + 40*b*B*d^4*e^3*Sqrt[d/f]*f + 40*a*C*d^4*e^3
*Sqrt[d/f]*f + 6*b*c^2*C*d^2*e^2*Sqrt[d/f]*f^2 - 24*b*B*c*d^3*e^2*Sqrt[d/f]
*f^2 - 24*a*c*C*d^3*e^2*Sqrt[d/f]*f^2 - 48*A*b*d^4*e^2*Sqrt[d/f]*f^2 - 48*a
*B*d^4*e^2*Sqrt[d/f]*f^2 + 4*b*c^3*C*d*e*Sqrt[d/f]*f^3 - 8*b*B*c^2*d^2*e*Sq
rt[d/f]*f^3 - 8*a*c^2*C*d^2*e*Sqrt[d/f]*f^3 + 32*A*b*c*d^3*e*Sqrt[d/f]*f^3
+ 32*a*B*c*d^3*e*Sqrt[d/f]*f^3 + 64*a*A*d^4*e*Sqrt[d/f]*f^3 + 5*b*c^4*C*Sqr
t[d/f]*f^4 - 8*b*B*c^3*d*Sqrt[d/f]*f^4 - 8*a*c^3*C*d*Sqrt[d/f]*f^4 + 16*A*b
*c^2*d^2*Sqrt[d/f]*f^4 + 16*a*B*c^2*d^2*Sqrt[d/f]*f^4 - 64*a*A*c*d^3*Sqrt[d
/f]*f^4)*Log[-(Sqrt[d/f]*Sqrt[e + f*x]) + Sqrt[c - (d*e)/f + (d*(e + f*x))/
f]])/(64*d^4*f^4)
```

fricas [A] time = 3.76, size = 1114, normalized size = 2.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/768*(3*(35*C*b*d^4*e^4 - 20*(C*b*c*d^3 + 2*(C*a + B*b)*d^4)*e^3*f - 6*(C
*b*c^2*d^2 - 4*(C*a + B*b)*c*d^3 - 8*(B*a + A*b)*d^4)*e^2*f^2 - 4*(C*b*c^3*
d + 16*A*a*d^4 - 2*(C*a + B*b)*c^2*d^2 + 8*(B*a + A*b)*c*d^3)*e*f^3 - (5*C*
b*c^4 - 64*A*a*c*d^3 - 8*(C*a + B*b)*c^3*d + 16*(B*a + A*b)*c^2*d^2)*f^4)*s
qrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d
*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x)
+ 4*(48*C*b*d^4*f^4*x^3 - 105*C*b*d^4*e^3*f + 5*(5*C*b*c*d^3 + 24*(C*a + B*
```

```
b)*d^4)*e^2*f^2 + (17*C*b*c^2*d^2 - 32*(C*a + B*b)*c*d^3 - 144*(B*a + A*b)*
d^4)*e*f^3 + 3*(5*C*b*c^3*d + 64*A*a*d^4 - 8*(C*a + B*b)*c^2*d^2 + 16*(B*a
+ A*b)*c*d^3)*f^4 - 8*(7*C*b*d^4*e*f^3 - (C*b*c*d^3 + 8*(C*a + B*b)*d^4)*f^
4)*x^2 + 2*(35*C*b*d^4*e^2*f^2 - 2*(3*C*b*c*d^3 + 20*(C*a + B*b)*d^4)*e*f^3
 - (5*C*b*c^2*d^2 - 8*(C*a + B*b)*c*d^3 - 48*(B*a + A*b)*d^4)*f^4)*x)*sqrt(
d*x + c)*sqrt(f*x + e))/(d^4*f^5), -1/384*(3*(35*C*b*d^4*e^4 - 20*(C*b*c*d^
3 + 2*(C*a + B*b)*d^4)*e^3*f - 6*(C*b*c^2*d^2 - 4*(C*a + B*b)*c*d^3 - 8*(B*
a + A*b)*d^4)*e^2*f^2 - 4*(C*b*c^3*d + 16*A*a*d^4 - 2*(C*a + B*b)*c^2*d^2 +
 8*(B*a + A*b)*c*d^3)*e*f^3 - (5*C*b*c^4 - 64*A*a*c*d^3 - 8*(C*a + B*b)*c^3
*d + 16*(B*a + A*b)*c^2*d^2)*f^4)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*
f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f
 + c*d*f^2)*x)) - 2*(48*C*b*d^4*f^4*x^3 - 105*C*b*d^4*e^3*f + 5*(5*C*b*c*d^
3 + 24*(C*a + B*b)*d^4)*e^2*f^2 + (17*C*b*c^2*d^2 - 32*(C*a + B*b)*c*d^3 -
144*(B*a + A*b)*d^4)*e*f^3 + 3*(5*C*b*c^3*d + 64*A*a*d^4 - 8*(C*a + B*b)*c^
2*d^2 + 16*(B*a + A*b)*c*d^3)*f^4 - 8*(7*C*b*d^4*e*f^3 - (C*b*c*d^3 + 8*(C*
a + B*b)*d^4)*f^4)*x^2 + 2*(35*C*b*d^4*e^2*f^2 - 2*(3*C*b*c*d^3 + 20*(C*a +
 B*b)*d^4)*e*f^3 - (5*C*b*c^2*d^2 - 8*(C*a + B*b)*c*d^3 - 48*(B*a + A*b)*d^
4)*f^4)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^4*f^5)]
```

giac [**A**] time = 1.82, size = 736, normalized size = 1.36

$$\frac{\left(\sqrt{(dx+c)df-cdf+d^2e}\left(2\,(dx+c)\left(4\,(dx+c)\left(\frac{6\,(dx+c)Cb}{d^4f}-\frac{17\,Cbcd^{12}f^6-8\,Cad^{13}f^6-8\,Bbd^{13}f^6+7\,Cbd^{13}f^5e}{d^{16}f^7}\right)+\frac{59\,Cbc^2d^{12}f^6-56\,Cacd^{13}f^6-56\,Bbcd^{13}f^6+48\,Bad^{14}f^6+48\,Abd^{14}f^6+50\,Cbcd^{13}f^5e-40\,Cad^{14}f^5e-40\,Bbd^{14}f^5e+35\,Cbd^{14}f^4e^2}{d^{16}f^7}\right)-\frac{3\left(5\,Cbc^3d^{12}f^6-8\,Cac^2d^{13}f^6-8\,Bbc^2d^{13}f^6+16\,Bacd^{14}f^6+16\,Abcd^{14}f^6-64\,Aad^{15}f^6+9\,Cbc^2d^{13}f^5e-16\,Cacd^{14}f^5e-16\,Bbcd^{14}f^5e+48\,Bad^{15}f^5e+48\,Abd^{15}f^5e+15\,Cbcd^{14}f^4e^2-40\,Cad^{15}f^4e^2-40\,Bbd^{15}f^4e^2+35\,Cbd^{15}f^3e^3\right)}{d^{16}f^7}\right)\sqrt{dx+c}+\frac{3\left(5\,Cbc^4f^4-8\,Cac^3df^4-8\,Bbc^3df^4+16\,Bac^2d^2f^4+16\,Abc^2d^2f^4-64\,Aacd^3f^4+4\,Cbc^3df^3e-8\,Cac^2d^2f^3e-8\,Bbc^2d^2f^3e+32\,Bacd^3f^3e+32\,Abcd^3f^3e+64\,Aad^4f^3e+6\,Cbc^2d^2f^2e^2-24\,Cacd^3f^2e^2-24\,Bbcd^3f^2e^2-48\,Bad^4f^2e^2-48\,Abd^4f^2e^2+20\,Cbcd^3fe^3+40\,Cad^4fe^3+40\,Bbd^4fe^3-35\,Cbd^4e^4\right)\log\left(\left|-\sqrt{df}\sqrt{dx+c}+\sqrt{(dx+c)df-cdf+d^2e}\right|\right)}{\sqrt{df}\,d^3f^4}\right)d}{192\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/192*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*
x + c)*C*b/(d^4*f) - (17*C*b*c*d^12*f^6 - 8*C*a*d^13*f^6 - 8*B*b*d^13*f^6 +
 7*C*b*d^13*f^5*e)/(d^16*f^7)) + (59*C*b*c^2*d^12*f^6 - 56*C*a*c*d^13*f^6 -
 56*B*b*c*d^13*f^6 + 48*B*a*d^14*f^6 + 48*A*b*d^14*f^6 + 50*C*b*c*d^13*f^5*
e - 40*C*a*d^14*f^5*e - 40*B*b*d^14*f^5*e + 35*C*b*d^14*f^4*e^2)/(d^16*f^7)
) - 3*(5*C*b*c^3*d^12*f^6 - 8*C*a*c^2*d^13*f^6 - 8*B*b*c^2*d^13*f^6 + 16*B*
a*c*d^14*f^6 + 16*A*b*c*d^14*f^6 - 64*A*a*d^15*f^6 + 9*C*b*c^2*d^13*f^5*e -
 16*C*a*c*d^14*f^5*e - 16*B*b*c*d^14*f^5*e + 48*B*a*d^15*f^5*e + 48*A*b*d^1
5*f^5*e + 15*C*b*c*d^14*f^4*e^2 - 40*C*a*d^15*f^4*e^2 - 40*B*b*d^15*f^4*e^2
 + 35*C*b*d^15*f^3*e^3)/(d^16*f^7))*sqrt(d*x + c) + 3*(5*C*b*c^4*f^4 - 8*C*
a*c^3*d*f^4 - 8*B*b*c^3*d*f^4 + 16*B*a*c^2*d^2*f^4 + 16*A*b*c^2*d^2*f^4 - 6
4*A*a*c*d^3*f^4 + 4*C*b*c^3*d*f^3*e - 8*C*a*c^2*d^2*f^3*e - 8*B*b*c^2*d^2*f
^3*e + 32*B*a*c*d^3*f^3*e + 32*A*b*c*d^3*f^3*e + 64*A*a*d^4*f^3*e + 6*C*b*c
^2*d^2*f^2*e^2 - 24*C*a*c*d^3*f^2*e^2 - 24*B*b*c*d^3*f^2*e^2 - 48*B*a*d^4*f
^2*e^2 - 48*A*b*d^4*f^2*e^2 + 20*C*b*c*d^3*f*e^3 + 40*C*a*d^4*f*e^3 + 40*B*
b*d^4*f*e^3 - 35*C*b*d^4*e^4)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x
+ c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^3*f^4))*d/abs(d)
```

maple [**B**] time = 0.03, size = 2002, normalized size = 3.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] 1/384*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(192*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e)
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^3*f^4+105*C*ln(1/2*(2*d*f*x
+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^4*e^4-24*C
*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b*c*d^2*e*f^2-15*C*ln(1/2*(2*d*f*x+2
*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^4*f^4-96*B*l
n(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*
a*c*d^3*e*f^3+72*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*
f+d*e)/(d*f)^(1/2))*b*c*d^3*e^2*f^2+72*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e)
```

```
)^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c*d^3*e^2*f^2-60*C*ln(1/2*(2*d*
f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^3*e^3
*f-288*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^3*e*f^2-96*A*ln(1/2*(2*d*f
*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c*d^3*e*f^
3+24*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)
^(1/2))*b*c^2*d^2*e*f^3+24*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f
)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^2*d^2*e*f^3-12*C*ln(1/2*(2*d*f*x+2*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d*e*f^3-192*A*ln(
1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*
d^4*e*f^3+144*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d
*e)/(d*f)^(1/2))*a*d^4*e^2*f^2-120*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1
/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^4*e^3*f-120*C*ln(1/2*(2*d*f*x+2*(
(d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*d^4*e^3*f+384*A*
(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^3*f^3-210*C*(d*f)^(1/2)*((d*x+c)*(f
*x+e))^(1/2)*b*d^3*e^3-48*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)
^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^2*f^4+24*C*ln(1/2*(2*d*f*x+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*a*c^3*d*f^4+24*B*ln(1/2*(2
*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^3*d*
f^4-48*B*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+c*f+d*e)/(d*
f)^(1/2))*a*c^2*d^2*f^4+144*A*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*d^4*e^2*f^2+30*C*(d*f)^(1/2)*((d*x+c)*(f*x
+e))^(1/2)*b*c^3*f^3+192*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a*d^3*f^3+
96*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c*d^2*f^3+192*A*(d*f)^(1/2)*((d*
x+c)*(f*x+e))^(1/2)*x*b*d^3*f^3+240*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a
*d^3*e^2*f+96*C*x^3*b*d^3*f^3*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)+128*B*x^2
*b*d^3*f^3*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)+128*C*x^2*a*d^3*f^3*(d*f)^(1
/2)*((d*x+c)*(f*x+e))^(1/2)-18*C*ln(1/2*(2*d*f*x+2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)+c*f+d*e)/(d*f)^(1/2))*b*c^2*d^2*e^2*f^2-288*B*(d*f)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*a*d^3*e*f^2+240*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*
d^3*e^2*f-48*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c^2*d*f^3+96*B*(d*f)^(
1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c*d^2*f^3-48*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))
^(1/2)*b*c^2*d*f^3-64*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*c*d^2*e*f^2+3
4*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c^2*d*e*f^2+50*C*(d*f)^(1/2)*((d*
x+c)*(f*x+e))^(1/2)*b*c*d^2*e^2*f+32*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*
x*b*c*d^2*f^3-160*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b*d^3*e*f^2+32*C*
(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a*c*d^2*f^3-160*C*(d*f)^(1/2)*((d*x+c
)*(f*x+e))^(1/2)*x*a*d^3*e*f^2-20*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b
*c^2*d*f^3+140*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b*d^3*e^2*f-64*B*(d*
f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c*d^2*e*f^2+16*C*x^2*b*c*d^2*f^3*(d*f)^(
1/2)*((d*x+c)*(f*x+e))^(1/2)-112*C*x^2*b*d^3*e*f^2*(d*f)^(1/2)*((d*x+c)*(f*
x+e))^(1/2))/f^4/((d*x+c)*(f*x+e))^(1/2)/d^3/(d*f)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 details)Is c*f-d*e zero or nonzero?
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)*(c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
[Out] Timed out
```

3.49 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{\sqrt{e+fx}}\,dx$

Optimal. Leaf size=246

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(2df(4Adf-B(cf+3de))+C\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{8d^2f^3}-\frac{(de-cf)\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(2df(4A\right.}{}$$

Rubi [A] time = 0.23, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {951, 80, 50, 63, 217, 206}

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(2df(4Adf-B(cf+3de))+C\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{8d^2f^3}-\frac{(de-cf)\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)\left(2df(4Adf-B(cf+3de))+C\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{8d^{5/2}f^{7/2}}-\frac{(c+dx)^{3/2}\sqrt{e+fx}(-6Bdf+7cCf+5Cde)}{12d^2f^2}+\frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] ((C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + 2*d*f*(4*A*d*f - B*(3*d*e + c*f)))*
Sqrt[c + d*x]*Sqrt[e + f*x])/(8*d^2*f^3) - ((5*C*d*e + 7*c*C*f - 6*B*d*f)*(
c + d*x)^(3/2)*Sqrt[e + f*x])/(12*d^2*f^2) + (C*(c + d*x)^(5/2)*Sqrt[e + f*
x])/(3*d^2*f) - ((d*e - c*f)*(C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + 2*d*f*(
4*A*d*f - B*(3*d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e
 + f*x])])/(8*d^(5/2)*f^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{\sqrt{e+fx}}\,dx = \frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f} + \frac{\int \frac{\sqrt{c+dx}\left(\frac{1}{2}\left(-5cCde-c^2Cf+6Ad^2f\right)-\frac{1}{2}d(5Cde+7cCf-6Bdf)x\right)}{\sqrt{e+fx}}\,dx}{3d^2f}$$

$$= -\frac{(5Cde+7cCf-6Bdf)(c+dx)^{3/2}\sqrt{e+fx}}{12d^2f^2} + \frac{C(c+dx)^{5/2}\sqrt{e+fx}}{3d^2f} + \frac{(C(\ldots}{\ldots}$$

$$= \frac{\left(C\left(5d^2e^2+2cdef+c^2f^2\right)+2df(4Adf-B(3de+cf))\right)\sqrt{c+dx}\,\sqrt{e+fx}}{8d^2f^3} \ldots$$

$$= \frac{\left(C\left(5d^2e^2+2cdef+c^2f^2\right)+2df(4Adf-B(3de+cf))\right)\sqrt{c+dx}\,\sqrt{e+fx}}{8d^2f^3} \ldots$$

$$= \frac{\left(C\left(5d^2e^2+2cdef+c^2f^2\right)+2df(4Adf-B(3de+cf))\right)\sqrt{c+dx}\,\sqrt{e+fx}}{8d^2f^3} \ldots$$

$$= \frac{\left(C\left(5d^2e^2+2cdef+c^2f^2\right)+2df(4Adf-B(3de+cf))\right)\sqrt{c+dx}\,\sqrt{e+fx}}{8d^2f^3} \ldots$$

Mathematica [A] time = 1.07, size = 225, normalized size = 0.91

$$\frac{-d\sqrt{f}\sqrt{c+dx}\,(e+fx)\left(C\left(3c^2f^2-2cdf(fx-2e)+d^2\left(-15e^2+10efx-8f^2x^2\right)\right)-6df(4Adf+B(cf-3de+2dfx))\right)-3(de-cf)^{3/2}\sqrt{\frac{d(e+fx)}{de-cf}}\,\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)\left(2df(4Adf-B(cf+3de))+C\left(c^2f^2+2cdef+5d^2e^2\right)\right)}{24d^3f^{7/2}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (-(d*Sqrt[f]*Sqrt[c + d*x]*(e + f*x)*(-6*d*f*(4*A*d*f + B*(-3*d*e + c*f + 2
*d*f*x)) + C*(3*c^2*f^2 - 2*c*d*f*(-2*e + f*x) + d^2*(-15*e^2 + 10*e*f*x -
8*f^2*x^2)))) - 3*(d*e - c*f)^(3/2)*(C*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) +
2*d*f*(4*A*d*f - B*(3*d*e + c*f)))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*ArcSinh[
(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(24*d^3*f^(7/2)*Sqrt[e + f*x])
```

IntegrateAlgebraic [A] time = 1.12, size = 357, normalized size = 1.45

$$\frac{\sqrt{c+\frac{d(e+fx)}{f}-\frac{de}{f}}\left(24Ad^2f^2\sqrt{e+fx}+6Bcdf^2\sqrt{e+fx}+12Bd^2f(e+fx)^{3/2}-30Bd^2ef\sqrt{e+fx}-3c^2Cf^2\sqrt{e+fx}+2cCdf(e+fx)^{3/2}-6cCdef\sqrt{e+fx}+33Cd^2e^2\sqrt{e+fx}+8Cd^2(e+fx)^{5/2}-26Cd^2e(e+fx)^{3/2}\right)}{24d^2f^3}+\frac{\sqrt{\frac{d}{f}}\log\left(\sqrt{c+\frac{d(e+fx)}{f}-\frac{de}{f}}-\sqrt{\frac{d}{f}}\sqrt{e+fx}\right)\left(-8Acd^2f^3+8Ad^3ef^2+2Bc^2df^3+4Bcd^2ef^2-6Bd^3e^2f-c^3Cf^3-c^2Cdef^2-3cCd^2e^2f+5Cd^3e^3\right)}{8d^3f^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x]*(A + B*x + C*x^2))/Sqrt[e + f*x],x]
```

```
[Out] (Sqrt[c - (d*e)/f + (d*(e + f*x))/f]*(33*C*d^2*e^2*Sqrt[e + f*x] - 6*c*C*d*
e*f*Sqrt[e + f*x] - 30*B*d^2*e*f*Sqrt[e + f*x] - 3*c^2*C*f^2*Sqrt[e + f*x]
+ 6*B*c*d*f^2*Sqrt[e + f*x] + 24*A*d^2*f^2*Sqrt[e + f*x] - 26*C*d^2*e*(e +
f*x)^(3/2) + 2*c*C*d*f*(e + f*x)^(3/2) + 12*B*d^2*f*(e + f*x)^(3/2) + 8*C*d
^2*(e + f*x)^(5/2)))/(24*d^2*f^3) + (Sqrt[d/f]*(5*C*d^3*e^3 - 3*c*C*d^2*e^2
*f - 6*B*d^3*e^2*f - c^2*C*d*e*f^2 + 4*B*c*d^2*e*f^2 + 8*A*d^3*e*f^2 - c^3*
C*f^3 + 2*B*c^2*d*f^3 - 8*A*c*d^2*f^3)*Log[-(Sqrt[d/f]*Sqrt[e + f*x]) + Sqr
t[c - (d*e)/f + (d*(e + f*x))/f]])/(8*d^3*f^3)
```

fricas [A] time = 1.49, size = 576, normalized size = 2.34

$$\left[-\frac{3\left(5Cd^3e^3-3\left(Ccd^2+2Bd^3\right)e^2f-\left(Cc^2d-4Bcd^2-8Ad^3\right)ef^2-\left(Cc^3-2Bc^2d+8Acd^2\right)f^3\right)\sqrt{df}\log\left(8d^2f^2x^2+d^2e^2+6cdef+c^2f^2+4\left(2dfx+de+cf\right)\sqrt{df}\sqrt{dx+c}\sqrt{fx+e}+8\left(d^2ef+cdf^2\right)x\right)-4\left(8Cd^3f^3x^2+15Cd^3e^2f-2\left(2Ccd^2+9Bd^3\right)ef^2-3\left(Cc^2d-2Bcd^2-8Ad^3\right)f^3-2\left(5Cd^3ef^2-\left(Ccd^2+6Bd^3\right)f^3\right)x\right)\sqrt{dx+c}\sqrt{fx+e}}{96d^3f^4}, \frac{3\left(5Cd^3e^3-3\left(Ccd^2+2Bd^3\right)e^2f-\left(Cc^2d-4Bcd^2-8Ad^3\right)ef^2-\left(Cc^3-2Bc^2d+8Acd^2\right)f^3\right)\sqrt{-df}\arctan\left(\frac{\left(2dfx+de+cf\right)\sqrt{-df}\sqrt{dx+c}\sqrt{fx+e}}{2\left(d^2f^2x^2+cdef+\left(d^2ef+cdf^2\right)x\right)}\right)+2\left(8Cd^3f^3x^2+15Cd^3e^2f-2\left(2Ccd^2+9Bd^3\right)ef^2-3\left(Cc^2d-2Bcd^2-8Ad^3\right)f^3-2\left(5Cd^3ef^2-\left(Ccd^2+6Bd^3\right)f^3\right)x\right)\sqrt{dx+c}\sqrt{fx+e}}{48d^3f^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*C*d^3*e^3 - 3*(C*c*d^2 + 2*B*d^3)*e^2*f - (C*c^2*d - 4*B*c*d^2
 - 8*A*d^3)*e*f^2 - (C*c^3 - 2*B*c^2*d + 8*A*c*d^2)*f^3)*sqrt(d*f)*log(8*d^
2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*
f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(8*C*d^3*f^3*
x^2 + 15*C*d^3*e^2*f - 2*(2*C*c*d^2 + 9*B*d^3)*e*f^2 - 3*(C*c^2*d - 2*B*c*d
^2 - 8*A*d^3)*f^3 - 2*(5*C*d^3*e*f^2 - (C*c*d^2 + 6*B*d^3)*f^3)*x)*sqrt(d*x
 + c)*sqrt(f*x + e))/(d^3*f^4), 1/48*(3*(5*C*d^3*e^3 - 3*(C*c*d^2 + 2*B*d^3
)*e^2*f - (C*c^2*d - 4*B*c*d^2 - 8*A*d^3)*e*f^2 - (C*c^3 - 2*B*c^2*d + 8*A*
c*d^2)*f^3)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x
 + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(8
*C*d^3*f^3*x^2 + 15*C*d^3*e^2*f - 2*(2*C*c*d^2 + 9*B*d^3)*e*f^2 - 3*(C*c^2*
d - 2*B*c*d^2 - 8*A*d^3)*f^3 - 2*(5*C*d^3*e*f^2 - (C*c*d^2 + 6*B*d^3)*f^3)*
x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^3*f^4)]
```

giac [A] time = 1.35, size = 315, normalized size = 1.28

$$\frac{\left(\sqrt{(dx+c)df-cdf+d^2e}\,\sqrt{dx+c}\left(2\,(dx+c)\left(\frac{4\,(dx+c)C}{d^3f}-\frac{7\,Ccd^6f^4-6\,Bd^7f^4+5\,Cd^7f^3e}{d^9f^5}\right)+\frac{3\left(Cc^2d^6f^4-2\,Bcd^7f^4+8\,Ad^8f^4+2\,Ccd^7f^3e-6\,Bd^8f^3e+5\,Cd^8f^2e^2\right)}{d^9f^5}\right)-\frac{3\left(Cc^3f^3-2\,Bc^2df^3+8\,Acd^2f^3+Cc^2df^2e-4\,Bcd^2f^2e-8\,Ad^3f^2e+3\,Ccd^2fe^2+6\,Bd^3fe^2-5\,Cd^3e^3\right)\log\left(\left|-\sqrt{df}\,\sqrt{dx+c}+\sqrt{(dx+c)df-cdf+d^2e}\right|\right)}{\sqrt{df}\,d^2f^3}\right)d}{24\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*
x + c)*C/(d^3*f) - (7*C*c*d^6*f^4 - 6*B*d^7*f^4 + 5*C*d^7*f^3*e)/(d^9*f^5))
 + 3*(C*c^2*d^6*f^4 - 2*B*c*d^7*f^4 + 8*A*d^8*f^4 + 2*C*c*d^7*f^3*e - 6*B*d
^8*f^3*e + 5*C*d^8*f^2*e^2)/(d^9*f^5)) - 3*(C*c^3*f^3 - 2*B*c^2*d*f^3 + 8*A
*c*d^2*f^3 + C*c^2*d*f^2*e - 4*B*c*d^2*f^2*e - 8*A*d^3*f^2*e + 3*C*c*d^2*f*
e^2 + 6*B*d^3*f*e^2 - 5*C*d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt(
(d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^2*f^3))*d/abs(d)
```

maple [B] time = 0.02, size = 763, normalized size = 3.10

$$\frac{\sqrt{dx+c}\,\sqrt{fx+e}\left(24Acd^2f^3\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)-24Ad^3ef^2\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)-6Bc^2df^3\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)-12Bcd^2ef^2\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)+18Bd^3e^2f\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)+3Cc^3f^3\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)+3Cc^2def^2\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)+9Ccd^2e^2f\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)-15Cd^3e^3\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)+16\sqrt{(dx+c)(fx+e)}\,\sqrt{df}\,Cd^2f^2x^2+24\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,Bd^2f^2x+4\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,Ccdf^2x-20\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,Cd^2efx+48\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,Ad^2f^2+12\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,Bcdf^2-36\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,Bd^2ef-6\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,Cc^2f^2-8\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,Ccdef+30\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,Cd^2e^2\right)}{48\sqrt{(dx+c)(fx+e)}\,\sqrt{df}\,d^2f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] 1/48*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(16*C*x^2*d^2*f^2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2)+24*A*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2))/(d*f)^(1/2))*c*d^2*f^3-24*A*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e)
```

```
)^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*d^3*e*f^2-6*B*ln(1/2*(2*d*f*x+c*f+d*e+2*(
(d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*c^2*d*f^3-12*B*ln(1/2*(2*d
*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*c*d^2*e*f^
2+18*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)
^(1/2))*d^3*e^2*f+24*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*d^2*f^2+3*C*ln
(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*c
^3*f^3+3*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(
d*f)^(1/2))*c^2*d*e*f^2+9*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2))/(d*f)^(1/2))*c*d^2*e^2*f-15*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d
*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*d^3*e^3+4*C*(d*f)^(1/2)*((d*
x+c)*(f*x+e))^(1/2)*x*c*d*f^2-20*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*d^
2*e*f+48*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*f^2+12*B*(d*f)^(1/2)*((d
*x+c)*(f*x+e))^(1/2)*c*d*f^2-36*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*e
*f-6*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c^2*f^2-8*C*(d*f)^(1/2)*((d*x+c)
*(f*x+e))^(1/2)*c*d*e*f+30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d^2*e^2)/f
^3/((d*x+c)*(f*x+e))^(1/2)/d^2/(d*f)^(1/2)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 details)Is c*f-d*e zero or nonzero?
```

mupad [**B**] time = 90.55, size = 1832, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/(e + f*x)^(1/2),x)
```

```
[Out] ((((c + d*x)^(1/2) - c^(1/2))*(2*A*d^2*e + 2*A*c*d*f))/(f^3*((e + f*x)^(1/2
) - e^(1/2))) + ((2*A*c*f + 2*A*d*e)*((c + d*x)^(1/2) - c^(1/2))^3)/(f^2*((
e + f*x)^(1/2) - e^(1/2))^3) - (8*A*c^(1/2)*d*e^(1/2)*((c + d*x)^(1/2) - c^
(1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/((((c + d*x)^(1/2) - c^(1/2))
^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) - c^(1/2
))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) - c^(1/2))*((
C*c^3*d^3*f^3)/4 - (5*C*d^6*e^3)/4 + (C*c^2*d^4*e*f^2)/4 + (3*C*c*d^5*e^2*f
)/4))/(f^9*((e + f*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^5*((
33*C*d^4*e^3)/2 + (19*C*c^3*d*f^3)/2 + (275*C*c^2*d^2*e*f^2)/2 + (313*C*c*d
^3*e^2*f)/2))/(f^7*((e + f*x)^(1/2) - e^(1/2))^5) - (((c + d*x)^(1/2) - c^(
1/2))^7*((19*C*c^3*f^3)/2 + (33*C*d^3*e^3)/2 + (313*C*c*d^2*e^2*f)/2 + (275
*C*c^2*d*e*f^2)/2))/(f^6*((e + f*x)^(1/2) - e^(1/2))^7) - (((c + d*x)^(1/2)
 - c^(1/2))^3*((17*C*c^3*d^2*f^3)/12 - (85*C*d^5*e^3)/12 + (91*C*c^2*d^3*e*
f^2)/4 + (17*C*c*d^4*e^2*f)/4))/(f^8*((e + f*x)^(1/2) - e^(1/2))^3) + (((c
+ d*x)^(1/2) - c^(1/2))^11*((C*c^3*f^3)/4 - (5*C*d^3*e^3)/4 + (3*C*c*d^2*e^
2*f)/4 + (C*c^2*d*e*f^2)/4))/(d^2*f^4*((e + f*x)^(1/2) - e^(1/2))^11) - (((
c + d*x)^(1/2) - c^(1/2))^9*((17*C*c^3*f^3)/12 - (85*C*d^3*e^3)/12 + (17*C*
c*d^2*e^2*f)/4 + (91*C*c^2*d*e*f^2)/4))/(d*f^5*((e + f*x)^(1/2) - e^(1/2))^
9) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^8*(32*C*c^2*f + 96*C*c*d*
e))/(f^4*((e + f*x)^(1/2) - e^(1/2))^8) + (c^(1/2)*e^(1/2)*(96*C*c*d^3*e +
32*C*c^2*d^2*f)*((c + d*x)^(1/2) - c^(1/2))^4)/(f^6*((e + f*x)^(1/2) - e^(1
/2))^4) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^6*(128*C*d^3*e^2 + 6
4*C*c^2*d*f^2 + (704*C*c*d^2*e*f)/3))/(f^6*((e + f*x)^(1/2) - e^(1/2))^6))/
(((c + d*x)^(1/2) - c^(1/2))^12/((e + f*x)^(1/2) - e^(1/2))^12 + d^6/f^6 -
(6*d*((c + d*x)^(1/2) - c^(1/2))^10)/(f*((e + f*x)^(1/2) - e^(1/2))^10) - (
```

```
6*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/(f^5*((e + f*x)^(1/2) - e^(1/2))^2) +
(15*d^4*((c + d*x)^(1/2) - c^(1/2))^4)/(f^4*((e + f*x)^(1/2) - e^(1/2))^4)
- (20*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/(f^3*((e + f*x)^(1/2) - e^(1/2))^6
) + (15*d^2*((c + d*x)^(1/2) - c^(1/2))^8)/(f^2*((e + f*x)^(1/2) - e^(1/2))
^8)) + ((((c + d*x)^(1/2) - c^(1/2))*((B*c^2*d^2*f^2)/2 - (3*B*d^4*e^2)/2 +
 B*c*d^3*e*f))/(f^6*((e + f*x)^(1/2) - e^(1/2))) + (((c + d*x)^(1/2) - c^(1
/2))^3*((11*B*d^3*e^2)/2 + (7*B*c^2*d*f^2)/2 + 23*B*c*d^2*e*f))/(f^5*((e +
f*x)^(1/2) - e^(1/2))^3) + (((c + d*x)^(1/2) - c^(1/2))^5*((7*B*c^2*f^2)/2
+ (11*B*d^2*e^2)/2 + 23*B*c*d*e*f))/(f^4*((e + f*x)^(1/2) - e^(1/2))^5) + (
((c + d*x)^(1/2) - c^(1/2))^7*((B*c^2*f^2)/2 - (3*B*d^2*e^2)/2 + B*c*d*e*f)
)/(d*f^3*((e + f*x)^(1/2) - e^(1/2))^7) - (c^(1/2)*e^(1/2)*((c + d*x)^(1/2)
 - c^(1/2))^4*(32*B*d^2*e + 16*B*c*d*f))/(f^4*((e + f*x)^(1/2) - e^(1/2))^4
) - (8*B*c^(3/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^6)/(f^2*((e + f*x)^(1/
2) - e^(1/2))^6) - (8*B*c^(3/2)*d^2*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^2)/
(f^4*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^(1/2))^8/((e + f
*x)^(1/2) - e^(1/2))^8 + d^4/f^4 - (4*d*((c + d*x)^(1/2) - c^(1/2))^6)/(f*(
(e + f*x)^(1/2) - e^(1/2))^6) - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/(f^3*
((e + f*x)^(1/2) - e^(1/2))^2) + (6*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(f^2
*((e + f*x)^(1/2) - e^(1/2))^4)) + (2*A*atanh((f^(1/2)*((c + d*x)^(1/2) - c
^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(c*f - d*e))/(d^(1/2)*f^(3/
2)) + (C*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1
/2) - e^(1/2))))*(c*f - d*e)*(c^2*f^2 + 5*d^2*e^2 + 2*c*d*e*f))/(4*d^(5/2)*
f^(7/2)) - (B*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*
x)^(1/2) - e^(1/2))))*(c*f - d*e)*(c*f + 3*d*e))/(2*d^(3/2)*f^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

3.50 $\int \frac{\sqrt{c+dx}\,(A+Bx+Cx^2)}{(a+bx)\sqrt{e+fx}}\,dx$

Optimal. Leaf size=290

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2bdf(4Abdf-aC(cf+3de))+(2adf-bcf+bde)(4aCdf+b(-4Bdf+cCf+3Cde)))}{4b^3d^{3/2}f^{5/2}}$$

Rubi [A] time = 0.67, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1615, 154, 157, 63, 217, 206, 93, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2bdf(4Abdf-aC(cf+3de))+(2adf-bcf+bde)(4aCdf+b(-4Bdf+cCf+3Cde)))}{4b^3d^{3/2}f^{5/2}}-\frac{2\sqrt{bc-ad}\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)}{b^3\sqrt{be-af}}-\frac{\sqrt{c+dx}\sqrt{e+fx}(4aCdf+b(-4Bdf+cCf+3Cde))}{4b^2df^2}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)*Sqrt[e + f*x]),x]
```

```
[Out] -((4*a*C*d*f + b*(3*C*d*e + c*C*f - 4*B*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/
(4*b^2*d*f^2) + (C*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*b*d*f) + ((2*b*d*f*(4*
A*b*d*f - a*C*(3*d*e + c*f)) + (b*d*e - b*c*f + 2*a*d*f)*(4*a*C*d*f + b*(3*
C*d*e + c*C*f - 4*B*d*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e
+ f*x])])/(4*b^3*d^(3/2)*f^(5/2)) - (2*(A*b^2 - a*(b*B - a*C))*Sqrt[b*c - a
*d]*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])
])/(b^3*Sqrt[b*e - a*f])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)\sqrt{e+fx}}\,dx = \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf} + \frac{\int \frac{\sqrt{c+dx}\left(\frac{1}{2}b(4Abdf-aC(3de+cf))-\frac{1}{2}b(4aCdf+b(3Cde+cCf-4Bdf))x\right.}{(a+bx)\sqrt{e+fx}}}{2b^2df}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\,\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\,\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\,\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\,\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

$$= -\frac{(4aCdf+b(3Cde+cCf-4Bdf))\sqrt{c+dx}\,\sqrt{e+fx}}{4b^2df^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2bdf}$$

Mathematica [A] time = 3.45, size = 465, normalized size = 1.60

$$\frac{\frac{8\sqrt{de-cf}\left(a(aC-bB)+Ab^2\right)\sqrt{\frac{d(e+fx)}{de-cf}}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{f}\sqrt{e+fx}}-\frac{8\sqrt{ad-bc}\left(a(aC-bB)+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{af-be}}+\frac{4b\sqrt{e+fx}(aCf-bBf+bCe)\left(\sqrt{c+dx}(de-cf)\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)-\sqrt{f}(c+dx)\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}\right)}{f^{5/2}\sqrt{c+dx}\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}}+\frac{b^2C\sqrt{e+fx}\left(\sqrt{f}\sqrt{c+dx}(cf+d(e+2fx))-\frac{(de-cf)^{3/2}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{\frac{d(e+fx)}{de-cf}}}\right)}{df^{5/2}}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)*Sqrt[e + f*x]),x]
```

```
[Out] ((8*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e - c*
f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(Sqrt[f]*Sqrt[e + f*x
]) + (4*b*(b*C*e - b*B*f + a*C*f)*Sqrt[e + f*x]*(-(Sqrt[f]*Sqrt[d*e - c*f]*
(c + d*x)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (d*e - c*f)*Sqrt[c + d*x]*ArcS
inh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]]))/(f^(5/2)*Sqrt[d*e - c*f]*Sqr
t[c + d*x]*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (b^2*C*Sqrt[e + f*x]*(Sqrt[f]
*Sqrt[c + d*x]*(c*f + d*(e + 2*f*x)) - ((d*e - c*f)^(3/2)*ArcSinh[(Sqrt[f]*
Sqrt[c + d*x])/Sqrt[d*e - c*f]])/Sqrt[(d*(e + f*x))/(d*e - c*f)]))/(d*f^(5/
2)) - (8*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[-(b*c) + a*d]*ArcTanh[(Sqrt[-(b*e)
 + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])]])/Sqrt[-(b*e) + a
*f])/(4*b^3)
```

IntegrateAlgebraic [B] time = 34.33, size = 1375, normalized size = 4.74

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)*Sqrt[e + f*
x]),x]
```

```
[Out] (Sqrt[d/f]*Sqrt[e + f*x]*(-5*b*C*d*e + b*c*C*f + 4*b*B*d*f - 4*a*C*d*f + 2*
b*C*d*(e + f*x))*Sqrt[c - (d*e)/f + (d*(e + f*x))/f]*(d^2*e^2*f - 2*c*d*e*f
^2 + c^2*f^3 - 8*d^2*e*f*(e + f*x) + 8*c*d*f^2*(e + f*x) + 8*d^2*f*(e + f*x
)^2) + Sqrt[e + f*x]*(-5*b*C*d*e + b*c*C*f + 4*b*B*d*f - 4*a*C*d*f + 2*b*C*
d*(e + f*x))*(-4*d^3*e^2*Sqrt[e + f*x] + 8*c*d^2*e*f*Sqrt[e + f*x] - 4*c^2*
d*f^2*Sqrt[e + f*x] + 12*d^3*e*(e + f*x)^(3/2) - 12*c*d^2*f*(e + f*x)^(3/2)
 - 8*d^3*(e + f*x)^(5/2)))/(4*b^2*d*f^5*Sqrt[c - (d*e)/f + (d*(e + f*x))/f]
*((4*d^2*e*Sqrt[e + f*x])/f^2 - (4*c*d*Sqrt[e + f*x])/f - (8*d^2*(e + f*x)^
(3/2))/f^2) + 4*b^2*d*Sqrt[d/f]*f^5*(c^2 + (d^2*e^2)/f^2 - (2*c*d*e)/f - (8
*d^2*e*(e + f*x))/f^2 + (8*c*d*(e + f*x))/f + (8*d^2*(e + f*x)^2)/f^2)) + (
(2*A*Sqrt[d]*Sqrt[b*c - a*d])/(b*Sqrt[d/f]*Sqrt[f]*Sqrt[b*e - a*f]) - (2*a*
B*Sqrt[d]*Sqrt[b*c - a*d])/(b^2*Sqrt[d/f]*Sqrt[f]*Sqrt[b*e - a*f]) + (2*a^2
*C*Sqrt[d]*Sqrt[b*c - a*d])/(b^3*Sqrt[d/f]*Sqrt[f]*Sqrt[b*e - a*f]))*ArcTan
h[(-(b*d*e) + a*d*f + b*d*(e + f*x) - b*Sqrt[d/f]*f*Sqrt[e + f*x]*Sqrt[c -
(d*e)/f + (d*(e + f*x))/f])/(Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[f]*Sqrt[b*e - a*f
])] - (3*C*d^2*e^2*Log[-(Sqrt[d/f]*Sqrt[e + f*x]) + Sqrt[c - (d*e)/f + (d*(
e + f*x))/f]])/(4*b*(d/f)^(3/2)*f^4) + (c*C*d*e*Log[-(Sqrt[d/f]*Sqrt[e + f*
x]) + Sqrt[c - (d*e)/f + (d*(e + f*x))/f]])/(2*b*(d/f)^(3/2)*f^3) + (B*d^2*
e*Log[-(Sqrt[d/f]*Sqrt[e + f*x]) + Sqrt[c - (d*e)/f + (d*(e + f*x))/f]])/(b
*(d/f)^(3/2)*f^3) - (a*C*d^2*e*Log[-(Sqrt[d/f]*Sqrt[e + f*x]) + Sqrt[c - (d
*e)/f + (d*(e + f*x))/f]])/(b^2*(d/f)^(3/2)*f^3) + (c^2*C*Log[-(Sqrt[d/f]*S
qrt[e + f*x]) + Sqrt[c - (d*e)/f + (d*(e + f*x))/f]])/(4*b*(d/f)^(3/2)*f^2)
 - (B*c*d*Log[-(Sqrt[d/f]*Sqrt[e + f*x]) + Sqrt[c - (d*e)/f + (d*(e + f*x))
/f]])/(b*(d/f)^(3/2)*f^2) + (a*c*C*d*Log[-(Sqrt[d/f]*Sqrt[e + f*x]) + Sqrt[
c - (d*e)/f + (d*(e + f*x))/f]])/(b^2*(d/f)^(3/2)*f^2) - (2*A*d^2*Log[-(Sqr
t[d/f]*Sqrt[e + f*x]) + Sqrt[c - (d*e)/f + (d*(e + f*x))/f]])/(b*(d/f)^(3/2
)*f^2) + (2*a*B*d^2*Log[-(Sqrt[d/f]*Sqrt[e + f*x]) + Sqrt[c - (d*e)/f + (d*
(e + f*x))/f]])/(b^2*(d/f)^(3/2)*f^2) - (2*a^2*C*d^2*Log[-(Sqrt[d/f]*Sqrt[e
 + f*x]) + Sqrt[c - (d*e)/f + (d*(e + f*x))/f]])/(b^3*(d/f)^(3/2)*f^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 0.47
```

maple [B] time = 0.04, size = 1822, normalized size = 6.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x)
```

```
[Out] 1/8*(8*A*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*
f)^(1/2))*b^3*d^2*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+8*A*ln(
(-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^2*d^2*f^2*(d*f
)^(1/2)-8*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*b^3
*c*d*f^2*(d*f)^(1/2)-8*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*
(d*f)^(1/2))/(d*f)^(1/2))*a*b^2*d^2*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/
b^2)^(1/2)+4*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2
))/(d*f)^(1/2))*b^3*c*d*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-4
*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/
2))*b^3*d^2*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-8*B*ln((-2*a*
d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b*d^2*f^2*(d*f)^(1/2
)+8*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b
^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c*e)/(b*x+a))*a*b^2*c*d
*f^2*(d*f)^(1/2)+8*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f
)^(1/2))/(d*f)^(1/2))*a^2*b*d^2*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)
^(1/2)-4*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(
d*f)^(1/2))*a*b^2*c*d*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+4*C
*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2)
)*a*b^2*d^2*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-C*ln(1/2*(2*d
*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*b^3*c^2*f^
2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-2*C*ln(1/2*(2*d*f*x+c*f+d*e
+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*b^3*c*d*e*f*((a^2*d*f-
a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+3*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*b^3*d^2*e^2*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)+8*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*((a^2*d*f-a*b*
c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*f-a*d*e+2*b*c
*e)/(b*x+a))*a^3*d^2*f^2*(d*f)^(1/2)-8*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x+2*(
(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b-a*c*
f-a*d*e+2*b*c*e)/(b*x+a))*a^2*b*c*d*f^2*(d*f)^(1/2)+4*C*x*b^3*d*f*((d*x+c)*
```

```
(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+8*
B*b^3*d*f*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)-8*C*a*b^2*d*f*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*
f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*b^3*c*f*((d*x+c)*(f*x+e))^(1/2)*(
d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-6*C*b^3*d*e*((d*x+
c)*(f*x+e))^(1/2)*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
)*(f*x+e)^(1/2)*(d*x+c)^(1/2)/((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)
/(d*f)^(1/2)/d/f^2/b^4/((d*x+c)*(f*x+e))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)/(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for more details)Is ((-(2*a*d*f)/b^2)    +(c*f)/b    +(d*e)/b)    ^2    -
(4*d*f       *((a^2*d*f)/b^2        -(a*c*f)/b        -(a*d*e)/b        +c*
e))     /b^2 zero or nonzero?
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx} \left(A + Bx + Cx^{2}\right)}{(a + bx) \sqrt{e + fx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*(A + B*x + C*x**2)/((a + b*x)*sqrt(e + f*x)), x)
```

3.51 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^2\sqrt{e+fx}}\,dx$

Optimal. Leaf size=364

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe)\right)}{b^2f(bc-ad)(be-af)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(4a^3Cdf-a^2b(2B\right.}{}$$

Rubi [A] time = 1.10, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1613, 154, 157, 63, 217, 206, 93, 208}

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(2a^2Cdf-ab(Bdf+cCf+Cde)+b^2(Adf+cCe)\right)}{b^2f(bc-ad)(be-af)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(-a^2b(2Bdf+3cCf+5Cde)+4a^3Cdf+ab^2(Bcf+3Bde+4cCe)-b^3(-Acf+Ade+2Bce)\right)}{b^3\sqrt{bc-ad}(be-af)^{3/2}}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(4aCdf+b(-2Bdf-cCf+Cde))}{b^3\sqrt{d}f^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

```
[Out] ((2*a^2*C*d*f + b^2*(c*C*e + A*d*f) - a*b*(C*d*e + c*C*f + B*d*f))*Sqrt[c +
 d*x]*Sqrt[e + f*x])/(b^2*(b*c - a*d)*f*(b*e - a*f)) - ((A*b^2 - a*(b*B - a
*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*e - a*f)*(a + b*x)) -
 ((4*a*C*d*f + b*(C*d*e - c*C*f - 2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[e + f*x])])/(b^3*Sqrt[d]*f^(3/2)) + ((4*a^3*C*d*f - b^3*(2*B
*c*e + A*d*e - A*c*f) + a*b^2*(4*c*C*e + 3*B*d*e + B*c*f) - a^2*b*(5*C*d*e
+ 3*c*C*f + 2*B*d*f))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a
*d]*Sqrt[e + f*x])])/(b^3*Sqrt[b*c - a*d]*(b*e - a*f)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
```

```
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^2\sqrt{e+fx}}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} - \frac{\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+cf)+b^2(2Bce+Ade-Acf}{2b}\right.}{}}{}$$

$$= \frac{\left(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf)\right)\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} - \frac{\left(A\right.}{}$$

$$= \frac{\left(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf)\right)\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} - \frac{\left(A\right.}{}$$

$$= \frac{\left(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf)\right)\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} - \frac{\left(A\right.}{}$$

$$= \frac{\left(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf)\right)\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} - \frac{\left(A\right.}{}$$

$$= \frac{\left(2a^2Cdf+b^2(cCe+Adf)-ab(Cde+cCf+Bdf)\right)\sqrt{c+dx}\sqrt{e+fx}}{b^2(bc-ad)f(be-af)} - \frac{\left(A\right.}{}$$

Mathematica [A] time = 2.40, size = 417, normalized size = 1.15

$$\frac{-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\left(a(aC-bB)+Ab^2\right)}{(a+bx)(be-af)}+\frac{2b(cf-de)\left(a(aC-bB)+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{ad-bc}(af-be)^{3/2}}+\frac{4(bB-2aC)\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{f}\sqrt{e+fx}}-\frac{4(bB-2aC)\sqrt{ad-bc}\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{af-be}}+\frac{2bC\sqrt{e+fx}\left(\sqrt{f}\sqrt{c+dx}-\frac{\sqrt{de-cf}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{\frac{d(e+fx)}{de-cf}}}\right)}{f^{3/2}}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^2*Sqrt[e + f*x]),x]
```

```
[Out] ((-2*b*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*e - a*f)
*(a + b*x)) + (4*(b*B - 2*a*C)*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e - c*
f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(Sqrt[f]*Sqrt[e + f*x
]) + (2*b*C*Sqrt[e + f*x]*(Sqrt[f]*Sqrt[c + d*x] - (Sqrt[d*e - c*f]*ArcSinh
[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/Sqrt[(d*(e + f*x))/(d*e - c*f)])
)/f^(3/2) - (4*(b*B - 2*a*C)*Sqrt[-(b*c) + a*d]*ArcTanh[(Sqrt[-(b*e) + a*f]
*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/Sqrt[-(b*e) + a*f] + (
2*b*(A*b^2 + a*(-(b*B) + a*C))*(-(d*e) + c*f)*ArcTanh[(Sqrt[-(b*e) + a*f]*S
qrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b*c) + a*d]*(-(b
*e) + a*f)^(3/2)))/(2*b^3)
```

IntegrateAlgebraic [B] time = 169.66, size = 5591, normalized size = 15.36

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^2*Sqrt[e +
f*x]),x]
```

```
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [B] time = 10.82, size = 1388, normalized size = 3.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] (3*sqrt(d*f)*C*a^2*b*c*d^2*f - sqrt(d*f)*B*a*b^2*c*d^2*f - sqrt(d*f)*A*b^3*
c*d^2*f - 4*sqrt(d*f)*C*a^3*d^3*f + 2*sqrt(d*f)*B*a^2*b*d^3*f - 4*sqrt(d*f)
*C*a*b^2*c*d^2*e + 2*sqrt(d*f)*B*b^3*c*d^2*e + 5*sqrt(d*f)*C*a^2*b*d^3*e -
3*sqrt(d*f)*B*a*b^2*d^3*e + sqrt(d*f)*A*b^3*d^3*e)*arctan(-1/2*(b*c*d*f - 2
*a*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f
+ d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)
*d))/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*(a*b^3*f*
abs(d) - b^4*abs(d)*e)*d) + 2*(sqrt(d*f)*C*a^2*b*c^2*d^3*f^2 - sqrt(d*f)*B*
a*b^2*c^2*d^3*f^2 + sqrt(d*f)*A*b^3*c^2*d^3*f^2 - 2*sqrt(d*f)*C*a^2*b*c*d^4
*f*e + 2*sqrt(d*f)*B*a*b^2*c*d^4*f*e - 2*sqrt(d*f)*A*b^3*c*d^4*f*e - sqrt(d
*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2
*b*c*d^2*f + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*
```

```
f + d^2*e))^2*B*a*b^2*c*d^2*f - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^3*c*d^2*f + 2*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^3*d^3*f - 2*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*
b*d^3*f + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f
 + d^2*e))^2*A*a*b^2*d^3*f + sqrt(d*f)*C*a^2*b*d^5*e^2 - sqrt(d*f)*B*a*b^2*
d^5*e^2 + sqrt(d*f)*A*b^3*d^5*e^2 - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b*d^3*e + sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^2*d^3*e - sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^
3*d^3*e)/((b*c^2*d^2*f^2 - 2*b*c*d^3*f*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) -
sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*a*d^2*f + b*d^4*e^2 - 2*(sqrt(d*f)*s
qrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*b*d^2*e + (sqrt(d*f)*
sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*b)*(a*b^3*f*abs(d) -
 b^4*abs(d)*e)) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*C*abs(d
)/(b^2*d^2*f) - 1/2*(sqrt(d*f)*C*b*c*f - 4*sqrt(d*f)*C*a*d*f + 2*sqrt(d*f)*
B*b*d*f - sqrt(d*f)*C*b*d*e)*log((sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*
d*f - c*d*f + d^2*e))^2)/(b^3*f^2*abs(d))
```

maple [B] time = 0.05, size = 3670, normalized size = 10.08

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x)
```

```
[Out] -1/2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-2*A*b^4*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*
c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+4*C*ln((-2*a*d*f*x+b*c*
f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(
1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a^4*d*f^2*(d*f)^(1/2)+B*ln((-2*a*d
*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*
e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a^2*b^2*c*f^2*(d*f)^(1/2)
-2*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(
1/2))*a^2*b^2*d*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-3*C*ln((-
2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b
^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a^3*b*c*f^2*(d*f)^(1
/2)+4*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f
)^(1/2))*a^3*b*d*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-C*ln(1/2
*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a^2*b
^2*c*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-C*ln(1/2*(2*d*f*x+c*
f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a*b^3*d*e^2*((a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*B*a*b^3*f*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+2*C*x*b^4*e*((a
^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2)-4*C*a^2*b^2*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2)+2*C*a*b^3*e*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2
)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*
x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*b)/(b*x+a))*x*b^4*c*f^2*(d*f)^(1/2)-C*ln(1/2*(2*d*f*x+c*
f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*x*b^4*d*e^2*((a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a
*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)
*(f*x+e))^(1/2)*b)/(b*x+a))*a*b^3*c*f^2*(d*f)^(1/2)-2*B*ln((-2*a*d*f*x+b*c*
f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(
1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a^3*b*d*f^2*(d*f)^(1/2)-2*B*ln((-2
*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^
2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*x*b^4*c*e*f*(d*f)^(1/
2)+4*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)
^(1/2))*x*a^2*b^2*d*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-C*ln(
```

```
1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*x*
a*b^3*c*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+C*ln(1/2*(2*d*f*x
+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*x*b^4*c*e*f*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*
x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a*b^3*d*e*f*(d*f)^(1/2)+3*B*ln((-2*a*d*f*x+b
*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2
)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a^2*b^2*d*e*f*(d*f)^(1/2)-2*B*l
n((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d
*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a*b^3*c*e*f*(d*f
)^(1/2)+2*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/
(d*f)^(1/2))*a*b^3*d*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-5*C*
ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*
d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a^3*b*d*e*f*(d*
f)^(1/2)+4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a^
2*b^2*c*e*f*(d*f)^(1/2)-3*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/
2)*(d*f)^(1/2))/(d*f)^(1/2))*a^2*b^2*d*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*
e)/b^2)^(1/2)+C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2))/(d*f)^(1/2))*a*b^3*c*e*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-
2*C*x*a*b^3*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e
))^(1/2)*(d*f)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2
*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(
b*x+a))*x*b^4*d*e*f*(d*f)^(1/2)-2*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*
d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e
))^(1/2)*b)/(b*x+a))*x*a^2*b^2*d*f^2*(d*f)^(1/2)+B*ln((-2*a*d*f*x+b*c*f*x+b
*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*
((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*x*a*b^3*c*f^2*(d*f)^(1/2)-2*B*ln(1/2*(2
*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*x*a*b^3*
d*f^2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*B*ln(1/2*(2*d*f*x+c*f
+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*x*b^4*d*e*f*((a^2*
d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-
a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c
)*(f*x+e))^(1/2)*b)/(b*x+a))*x*a^3*b*d*f^2*(d*f)^(1/2)-3*C*ln((-2*a*d*f*x+b
*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2
)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*x*a^2*b^2*c*f^2*(d*f)^(1/2)+3*B
*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*x*a*b^3*d*e*f*
(d*f)^(1/2)-5*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*
d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))
*x*a^2*b^2*d*e*f*(d*f)^(1/2)+4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e
+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^
(1/2)*b)/(b*x+a))*x*a*b^3*c*e*f*(d*f)^(1/2)-3*C*ln(1/2*(2*d*f*x+c*f+d*e+2*(
(d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*x*a*b^3*d*e*f*((a^2*d*f-a*
b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2))/((d*x+c)*(f*x+e))^(1/2)/(a*f-b*e)/(b*x+a
)/(d*f)^(1/2)/((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)/f/b^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^2/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for more details)Is ((-(2*a*d*f)/b^2)    +(c*f)/b    +(d*e)/b)    ^2    -
```

```
(4*d*f      *((a^2*d*f)/b^2       -(a*c*f)/b       -(a*d*e)/b       +c*
e))     /b^2 zero or nonzero?
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^2),x)
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**2/(f*x+e)**(1/2),x)
[Out] Timed out
```

3.52 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^3\sqrt{e+fx}}\,dx$

Optimal. Leaf size=484

$$\frac{\sqrt{c+dx}\sqrt{e+fx}\left(4a^3Cdf-a^2bC(5cf+7de)+ab^2(-4Adf+Bcf+3Bde+8cCe)-b^3(-3Acf-Ade+4Bce)\right)}{4b^2(a+bx)(bc-ad)(be-af)^2}$$

Rubi [A] time = 1.56, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1613, 149, 157, 63, 217, 206, 93, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(3a^2b^2C\left(c^2f^2+10cdef+5d^2e^2\right)-4a^3bCdf(3cf+5de)+8a^4Cd^2f^2-ab^3\left(2cd\left(2Af^2-Bef+12Ce^2\right)+d^2e(3Be-4Af)+c^2f(8Ce-Bf)\right)-b^4\left(c^2\left(-\left(3Af^2-4Bef+8Ce^2\right)\right)-2cde(2Be-Af)+Ad^2e^2\right)\right)}{4b^3(bc-ad)^{3/2}(be-af)^{5/2}}+\frac{\sqrt{c+dx}\sqrt{e+fx}\left(-a^2bC(5cf+7de)+4a^3Cdf+ab^2(-4Adf+Bcf+3Bde+8cCe)-b^3(-3Acf-Ade+4Bce)\right)}{4b^2(a+bx)(bc-ad)(be-af)^2}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}+\frac{2C\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^3\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

```
[Out] ((4*a^3*C*d*f - a^2*b*C*(7*d*e + 5*c*f) - b^3*(4*B*c*e - A*d*e - 3*A*c*f) +
 a*b^2*(8*c*C*e + 3*B*d*e + B*c*f - 4*A*d*f))*Sqrt[c + d*x]*Sqrt[e + f*x])/
(4*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)) - ((A*b^2 - a*(b*B - a*C))*(c +
 d*x)^(3/2)*Sqrt[e + f*x])/(2*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (2*C
*Sqrt[d]*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^3*Sqr
t[f]) - ((8*a^4*C*d^2*f^2 - 4*a^3*b*C*d*f*(5*d*e + 3*c*f) + 3*a^2*b^2*C*(5*
d^2*e^2 + 10*c*d*e*f + c^2*f^2) - a*b^3*(d^2*e*(3*B*e - 4*A*f) + c^2*f*(8*C
*e - B*f) + 2*c*d*(12*C*e^2 - B*e*f + 2*A*f^2)) - b^4*(A*d^2*e^2 - 2*c*d*e*
(2*B*e - A*f) - c^2*(8*C*e^2 - 4*B*e*f + 3*A*f^2)))*ArcTanh[(Sqrt[b*e - a*f
]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(4*b^3*(b*c - a*d)^(3/2)
*(b*e - a*f)^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
```

```
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^3\sqrt{e+fx}}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+cf)+b^2(4Bce-Ade-3Ac}{2b}\right.}{(a+b}}{2(bc-}$$

$$= \frac{\left(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde\right.}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

$$= \frac{\left(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde\right.}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

$$= \frac{\left(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde\right.}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

$$= \frac{\left(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde\right.}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

$$= \frac{\left(4a^3Cdf-a^2bC(7de+5cf)-b^3(4Bce-Ade-3Acf)+ab^2(8cCe+3Bde\right.}{4b^2(bc-ad)(be-af)^2(a+bx)}$$

Mathematica [A] time = 5.67, size = 523, normalized size = 1.08

$$-\frac{\frac{2b^2(c+dx)^{3/2}\sqrt{e+fx}\left(a(aC-bB)+Ab^2\right)}{(a+bx)^2(bc-ad)(be-af)}+\frac{b\left(a(aC-bB)+Ab^2\right)(-4adf+3bcf+bde)\left(\sqrt{c+dx}\sqrt{e+fx}\sqrt{ad-bc}\sqrt{af-be}-(a+bx)(de-cf)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)\right)}{(a+bx)(ad-bc)^{3/2}(af-be)^{5/2}}+\frac{4b\sqrt{c+dx}\sqrt{e+fx}(bB-2aC)}{(a+bx)(be-af)}-\frac{4b(bB-2aC)(cf-de)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{ad-bc}(af-be)^{3/2}}+\frac{8C\sqrt{ad-bc}\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{af-be}}-\frac{8C\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{f}\sqrt{e+fx}}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^3*Sqrt[e + f*x]),x]
```

```
[Out] -1/4*((4*b*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*e - a*f)*(a + b*x
)) + (2*b^2*(A*b^2 + a*(-(b*B) + a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/((b*c
 - a*d)*(b*e - a*f)*(a + b*x)^2) - (8*C*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/
(d*e - c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(Sqrt[f]*Sqr
t[e + f*x]) + (8*C*Sqrt[-(b*c) + a*d]*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c +
d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/Sqrt[-(b*e) + a*f] - (4*b*(b*B -
 2*a*C)*(-(d*e) + c*f)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b
*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b*c) + a*d]*(-(b*e) + a*f)^(3/2)) + (b*
(A*b^2 + a*(-(b*B) + a*C))*(b*d*e + 3*b*c*f - 4*a*d*f)*(Sqrt[-(b*c) + a*d]*
Sqrt[-(b*e) + a*f]*Sqrt[c + d*x]*Sqrt[e + f*x] - (d*e - c*f)*(a + b*x)*ArcT
anh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])
)/((-(b*c) + a*d)^(3/2)*(-(b*e) + a*f)^(5/2)*(a + b*x)))/b^3
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^3*Sqrt[e +
f*x]),x]
```

```
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [B] time = 134.87, size = 8004, normalized size = 16.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] -1/4*(3*sqrt(d*f)*C*a^2*b^2*c^2*d^2*f^2 + sqrt(d*f)*B*a*b^3*c^2*d^2*f^2 + 3
*sqrt(d*f)*A*b^4*c^2*d^2*f^2 - 12*sqrt(d*f)*C*a^3*b*c*d^3*f^2 - 4*sqrt(d*f)
*A*a*b^3*c*d^3*f^2 + 8*sqrt(d*f)*C*a^4*d^4*f^2 - 8*sqrt(d*f)*C*a*b^3*c^2*d^
2*f*e - 4*sqrt(d*f)*B*b^4*c^2*d^2*f*e + 30*sqrt(d*f)*C*a^2*b^2*c*d^3*f*e +
2*sqrt(d*f)*B*a*b^3*c*d^3*f*e - 2*sqrt(d*f)*A*b^4*c*d^3*f*e - 20*sqrt(d*f)*
C*a^3*b*d^4*f*e + 4*sqrt(d*f)*A*a*b^3*d^4*f*e + 8*sqrt(d*f)*C*b^4*c^2*d^2*e
^2 - 24*sqrt(d*f)*C*a*b^3*c*d^3*e^2 + 4*sqrt(d*f)*B*b^4*c*d^3*e^2 + 15*sqrt
(d*f)*C*a^2*b^2*d^4*e^2 - 3*sqrt(d*f)*B*a*b^3*d^4*e^2 - sqrt(d*f)*A*b^4*d^4
*e^2)*arctan(-1/2*(b*c*d*f - 2*a*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c)
 - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^
2 - b^2*c*d*f*e + a*b*d^2*f*e)*d))/((a^2*b^4*c*f^2*abs(d) - a^3*b^3*d*f^2*a
```

```
bs(d) - 2*a*b^5*c*f*abs(d)*e + 2*a^2*b^4*d*f*abs(d)*e + b^6*c*abs(d)*e^2 -
a*b^5*d*abs(d)*e^2)*sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*
f*e)*d) - sqrt(d*f)*C*d*log((sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
 c*d*f + d^2*e))^2)/(b^3*f*abs(d)) - 1/2*(5*sqrt(d*f)*C*a^2*b^3*c^5*d^5*f^5
 - sqrt(d*f)*B*a*b^4*c^5*d^5*f^5 - 3*sqrt(d*f)*A*b^5*c^5*d^5*f^5 - 6*sqrt(d
*f)*C*a^3*b^2*c^4*d^6*f^5 + 2*sqrt(d*f)*B*a^2*b^3*c^4*d^6*f^5 + 2*sqrt(d*f)
*A*a*b^4*c^4*d^6*f^5 - 8*sqrt(d*f)*C*a*b^4*c^5*d^5*f^4*e + 4*sqrt(d*f)*B*b^
5*c^5*d^5*f^4*e - 11*sqrt(d*f)*C*a^2*b^3*c^4*d^6*f^4*e - sqrt(d*f)*B*a*b^4*
c^4*d^6*f^4*e + 13*sqrt(d*f)*A*b^5*c^4*d^6*f^4*e + 24*sqrt(d*f)*C*a^3*b^2*c
^3*d^7*f^4*e - 8*sqrt(d*f)*B*a^2*b^3*c^3*d^7*f^4*e - 8*sqrt(d*f)*A*a*b^4*c^
3*d^7*f^4*e - 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
c*d*f + d^2*e))^2*C*a^2*b^3*c^4*d^4*f^4 + 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
 c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^4*c^4*d^4*f^4 + 9*sqrt(d
*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^5
*c^4*d^4*f^4 + 44*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
 c*d*f + d^2*e))^2*C*a^3*b^2*c^3*d^5*f^4 - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^3*c^3*d^5*f^4 - 28*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A
*a*b^4*c^3*d^5*f^4 - 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)
*d*f - c*d*f + d^2*e))^2*C*a^4*b*c^2*d^6*f^4 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^3*b^2*c^2*d^6*f^4 + 1
6*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))
^2*A*a^2*b^3*c^2*d^6*f^4 + 32*sqrt(d*f)*C*a*b^4*c^4*d^6*f^3*e^2 - 16*sqrt(d
*f)*B*b^5*c^4*d^6*f^3*e^2 - 6*sqrt(d*f)*C*a^2*b^3*c^3*d^7*f^3*e^2 + 14*sqrt
(d*f)*B*a*b^4*c^3*d^7*f^3*e^2 - 22*sqrt(d*f)*A*b^5*c^3*d^7*f^3*e^2 - 36*sqr
t(d*f)*C*a^3*b^2*c^2*d^8*f^3*e^2 + 12*sqrt(d*f)*B*a^2*b^3*c^2*d^8*f^3*e^2 +
 12*sqrt(d*f)*A*a*b^4*c^2*d^8*f^3*e^2 + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a*b^4*c^4*d^4*f^3*e - 12*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*b
^5*c^4*d^4*f^3*e - 56*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d
*f - c*d*f + d^2*e))^2*C*a^2*b^3*c^3*d^5*f^3*e + 32*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^4*c^3*d^5*f^3*e
- 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e
))^2*A*b^5*c^3*d^5*f^3*e - 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*
x + c)*d*f - c*d*f + d^2*e))^2*C*a^3*b^2*c^2*d^6*f^3*e - 16*sqrt(d*f)*(sqrt
(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b^3*c^2*
d^6*f^3*e + 52*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*
d*f + d^2*e))^2*A*a*b^4*c^2*d^6*f^3*e + 64*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^4*b*c*d^7*f^3*e - 16*sqrt(d
*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^3
*b^2*c*d^7*f^3*e - 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d
*f - c*d*f + d^2*e))^2*A*a^2*b^3*c*d^7*f^3*e + 15*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^2*b^3*c^3*d^3*f^3 -
3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))
^4*B*a*b^4*c^3*d^3*f^3 - 9*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^4*A*b^5*c^3*d^3*f^3 - 58*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^3*b^2*c^2*d^4*f^3 +
 14*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e
))^4*B*a^2*b^3*c^2*d^4*f^3 + 30*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((
d*x + c)*d*f - c*d*f + d^2*e))^4*A*a*b^4*c^2*d^4*f^3 + 88*sqrt(d*f)*(sqrt(d
*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^4*b*c*d^5*f^
3 - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^
2*e))^4*B*a^3*b^2*c*d^5*f^3 - 40*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt(
(d*x + c)*d*f - c*d*f + d^2*e))^4*A*a^2*b^3*c*d^5*f^3 - 48*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^5*d^6*f^3 +
 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e
))^4*B*a^4*b*d^6*f^3 + 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
c)*d*f - c*d*f + d^2*e))^4*A*a^3*b^2*d^6*f^3 - 48*sqrt(d*f)*C*a*b^4*c^3*d^7
*f^2*e^3 + 24*sqrt(d*f)*B*b^5*c^3*d^7*f^2*e^3 + 34*sqrt(d*f)*C*a^2*b^3*c^2*
```

```
d^8*f^2*e^3 - 26*sqrt(d*f)*B*a*b^4*c^2*d^8*f^2*e^3 + 18*sqrt(d*f)*A*b^5*c^2
*d^8*f^2*e^3 + 24*sqrt(d*f)*C*a^3*b^2*c*d^9*f^2*e^3 - 8*sqrt(d*f)*B*a^2*b^3
*c*d^9*f^2*e^3 - 8*sqrt(d*f)*A*a*b^4*c*d^9*f^2*e^3 - 24*sqrt(d*f)*(sqrt(d*f
)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a*b^4*c^3*d^5*f^
2*e^2 + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f
+ d^2*e))^2*B*b^5*c^3*d^5*f^2*e^2 + 130*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b^3*c^2*d^6*f^2*e^2 - 58*sqr
t(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*
a*b^4*c^2*d^6*f^2*e^2 - 14*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^2*A*b^5*c^2*d^6*f^2*e^2 - 92*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^3*b^2*c*d^7*f^2
*e^2 + 56*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f +
 d^2*e))^2*B*a^2*b^3*c*d^7*f^2*e^2 - 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c)
- sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a*b^4*c*d^7*f^2*e^2 - 32*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^4*
b*d^8*f^2*e^2 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
 c*d*f + d^2*e))^2*B*a^3*b^2*d^8*f^2*e^2 + 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a^2*b^3*d^8*f^2*e^2 - 24*s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*
C*a*b^4*c^3*d^3*f^2*e + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
 c)*d*f - c*d*f + d^2*e))^4*B*b^5*c^3*d^3*f^2*e + 101*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^2*b^3*c^2*d^4*f^
2*e - 49*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f +
d^2*e))^4*B*a*b^4*c^2*d^4*f^2*e - 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*b^5*c^2*d^4*f^2*e - 188*sqrt(d*f)*(s
qrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^3*b^2*c
*d^5*f^2*e + 84*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c
*d*f + d^2*e))^4*B*a^2*b^3*c*d^5*f^2*e + 20*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
 c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*a*b^4*c*d^5*f^2*e + 120*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a
^4*b*d^6*f^2*e - 56*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^4*B*a^3*b^2*d^6*f^2*e - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*a^2*b^3*d^6*f^2*e - 5*sqrt(
d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*C*a^
2*b^3*c^2*d^2*f^2 + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^6*B*a*b^4*c^2*d^2*f^2 + 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*A*b^5*c^2*d^2*f^2 + 20*sqrt(d
*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*C*a^3
*b^2*c*d^3*f^2 - 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
- c*d*f + d^2*e))^6*B*a^2*b^3*c*d^3*f^2 - 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
 c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*A*a*b^4*c*d^3*f^2 - 16*sqrt(d*
f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*C*a^4*
b*d^4*f^2 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d
*f + d^2*e))^6*B*a^3*b^2*d^4*f^2 + 32*sqrt(d*f)*C*a*b^4*c^2*d^8*f*e^4 - 16*
sqrt(d*f)*B*b^5*c^2*d^8*f*e^4 - 31*sqrt(d*f)*C*a^2*b^3*c*d^9*f*e^4 + 19*sqr
t(d*f)*B*a*b^4*c*d^9*f*e^4 - 7*sqrt(d*f)*A*b^5*c*d^9*f*e^4 - 6*sqrt(d*f)*C*
a^3*b^2*d^10*f*e^4 + 2*sqrt(d*f)*B*a^2*b^3*d^10*f*e^4 + 2*sqrt(d*f)*A*a*b^4
*d^10*f*e^4 - 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
c*d*f + d^2*e))^2*C*a*b^4*c^2*d^6*f*e^3 + 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
+ c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*b^5*c^2*d^6*f*e^3 - 32*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a
^2*b^3*c*d^7*f*e^3 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*
d*f - c*d*f + d^2*e))^2*B*a*b^4*c*d^7*f*e^3 + 16*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^5*c*d^7*f*e^3 + 68*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C
*a^3*b^2*d^8*f*e^3 - 32*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)
*d*f - c*d*f + d^2*e))^2*B*a^2*b^3*d^8*f*e^3 - 4*sqrt(d*f)*(sqrt(d*f)*sqrt(
d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a*b^4*d^8*f*e^3 - 16*sq
rt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C
```

```
*a*b^4*c^2*d^4*f*e^2 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c
)*d*f - c*d*f + d^2*e))^4*B*b^5*c^2*d^4*f*e^2 + 97*sqrt(d*f)*(sqrt(d*f)*sqr
t(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^2*b^3*c*d^5*f*e^2 -
 45*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e
))^4*B*a*b^4*c*d^5*f*e^2 - 7*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x
 + c)*d*f - c*d*f + d^2*e))^4*A*b^5*c*d^5*f*e^2 - 90*sqrt(d*f)*(sqrt(d*f)*s
qrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a^3*b^2*d^6*f*e^2 +
 46*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e
))^4*B*a^2*b^3*d^6*f*e^2 - 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x
 + c)*d*f - c*d*f + d^2*e))^4*A*a*b^4*d^6*f*e^2 + 8*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*C*a*b^4*c^2*d^2*f*e -
4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))
^6*B*b^5*c^2*d^2*f*e - 34*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
c)*d*f - c*d*f + d^2*e))^6*C*a^2*b^3*c*d^3*f*e + 18*sqrt(d*f)*(sqrt(d*f)*sq
rt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*B*a*b^4*c*d^3*f*e - 2*
sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6
*A*b^5*c*d^3*f*e + 28*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d
*f - c*d*f + d^2*e))^6*C*a^3*b^2*d^4*f*e - 16*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x
 + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*B*a^2*b^3*d^4*f*e + 4*sqrt(d
*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*A*a*b
^4*d^4*f*e - 8*sqrt(d*f)*C*a*b^4*c*d^9*e^5 + 4*sqrt(d*f)*B*b^5*c*d^9*e^5 +
9*sqrt(d*f)*C*a^2*b^3*d^10*e^5 - 5*sqrt(d*f)*B*a*b^4*d^10*e^5 + sqrt(d*f)*A
*b^5*d^10*e^5 + 24*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
- c*d*f + d^2*e))^2*C*a*b^4*c*d^7*e^4 - 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x +
c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*b^5*c*d^7*e^4 - 27*sqrt(d*f)*
(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b^3
*d^8*e^4 + 15*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d
*f + d^2*e))^2*B*a*b^4*d^8*e^4 - 3*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^5*d^8*e^4 - 24*sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*C*a*b^4*c*d^5*e^3 +
 12*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e
))^4*B*b^5*c*d^5*e^3 + 27*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
c)*d*f - c*d*f + d^2*e))^4*C*a^2*b^3*d^6*e^3 - 15*sqrt(d*f)*(sqrt(d*f)*sqrt
(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*B*a*b^4*d^6*e^3 + 3*sqrt
(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*A*b
^5*d^6*e^3 + 8*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*
d*f + d^2*e))^6*C*a*b^4*c*d^3*e^2 - 4*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) -
sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*B*b^5*c*d^3*e^2 - 9*sqrt(d*f)*(sqrt(
d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^6*C*a^2*b^3*d^4*e
^2 + 5*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^
2*e))^6*B*a*b^4*d^4*e^2 - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x +
c)*d*f - c*d*f + d^2*e))^6*A*b^5*d^4*e^2)/((a^2*b^4*c*f^2*abs(d) - a^3*b^3*
d*f^2*abs(d) - 2*a*b^5*c*f*abs(d)*e + 2*a^2*b^4*d*f*abs(d)*e + b^6*c*abs(d)
*e^2 - a*b^5*d*abs(d)*e^2)*(b*c^2*d^2*f^2 - 2*b*c*d^3*f*e - 2*(sqrt(d*f)*sq
rt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*b*c*d*f + 4*(sqrt(d*f)
*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*a*d^2*f + b*d^4*e^2
 - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*b*d^
2*e + (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^4*b)^
2)
```

maple [B] time = 0.10, size = 9100, normalized size = 18.80

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^3/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for more details)Is ((-(2*a*d*f)/b^2)    +(c*f)/b    +(d*e)/b)    ^2    -
(4*d*f       *((a^2*d*f)/b^2       -(a*c*f)/b       -(a*d*e)/b       +c*
e))     /b^2 zero or nonzero?
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**3/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

3.53 $\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^4\sqrt{e+fx}}\,dx$

Optimal. Leaf size=685

$$-\frac{(de-cf)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(-\left(a^2\left(2df(-4Adf+Bcf+3Bde)-C\left(c^2f^2+2cdef+5d^2e^2\right)\right)\right)+ab\left(-2cc\right.\right.}{8(bc}$$

Rubi [A] time = 1.78, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1613, 149, 151, 12, 93, 208}

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(-a^2b^2\left(4df(-2Adf+Bcf+4Bde)-C\left(3c^2f^2+44cdef+33d^2e^2\right)\right)-2a^3bdf(-2Bdf+7cCf+13Cde)+8a^4Cd^2f^2-ab^3\left(2cd\left(13Af^2-14Bef+30Ce^2\right)+d^2e(3Be-10Af)+3c^2f(4Ce-Bf)\right)-b^4\left(-3c^2\left(5Af^2-6Bef+8Ce^2\right)-2cde(3Be-2Af)+3Ad^2e^2\right)\right)}{24b^2(a+bx)(bc-ad)^2(be-af)^3}-\frac{(de-cf)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(a^2\left(-\left(2df(-4Adf+Bcf+3Bde)-C\left(c^2f^2+2cdef+5d^2e^2\right)\right)\right)+ab\left(-2cd\left(6Af^2-7Bef+6Ce^2\right)+d^2e(Be-4Af)+c^2(-f)(4Ce-Bf)\right)+b^2\left(c^2\left(5Af^2-6Bef+8Ce^2\right)-2cde(Be-Af)+Ad^2e^2\right)\right)}{8(bc-ad)^{5/2}(be-af)^{7/2}}+\frac{\sqrt{c+dx}\sqrt{e+fx}\left(-a^2b(-2Bdf+7cCf+9Cde)+4a^3Cdf+ab^2(-8Adf+Bcf+3Bde+12cCe)-b^3(-5Acf-3Ade+6Bce)\right)}{12b^2(a+bx)^2(bc-ad)(be-af)^2}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

```
[Out] ((4*a^3*C*d*f - b^3*(6*B*c*e - 3*A*d*e - 5*A*c*f) + a*b^2*(12*c*C*e + 3*B*d
*e + B*c*f - 8*A*d*f) - a^2*b*(9*C*d*e + 7*c*C*f - 2*B*d*f))*Sqrt[c + d*x]*
Sqrt[e + f*x])/(12*b^2*(b*c - a*d)*(b*e - a*f)^2*(a + b*x)^2) - ((8*a^4*C*d
^2*f^2 - 2*a^3*b*d*f*(13*C*d*e + 7*c*C*f - 2*B*d*f) - b^4*(3*A*d^2*e^2 - 2*
c*d*e*(3*B*e - 2*A*f) - 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) - a*b^3*(d^2*e
*(3*B*e - 10*A*f) + 3*c^2*f*(4*C*e - B*f) + 2*c*d*(30*C*e^2 - 14*B*e*f + 13
*A*f^2)) - a^2*b^2*(4*d*f*(4*B*d*e + B*c*f - 2*A*d*f) - C*(33*d^2*e^2 + 44*
c*d*e*f + 3*c^2*f^2)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(24*b^2*(b*c - a*d)^2*(
b*e - a*f)^3*(a + b*x)) - ((A*b^2 - a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e +
 f*x])/(3*b*(b*c - a*d)*(b*e - a*f)*(a + b*x)^3) - ((d*e - c*f)*(b^2*(A*d^2
*e^2 - 2*c*d*e*(B*e - A*f) + c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2)) + a*b*(d^2*
e*(B*e - 4*A*f) - c^2*f*(4*C*e - B*f) - 2*c*d*(6*C*e^2 - 7*B*e*f + 6*A*f^2)
) - a^2*(2*d*f*(3*B*d*e + B*c*f - 4*A*d*f) - C*(5*d^2*e^2 + 2*c*d*e*f + c^2
*f^2)))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f
*x])])/(8*(b*c - a*d)^(5/2)*(b*e - a*f)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\left(A+Bx+Cx^2\right)}{(a+bx)^4\sqrt{e+fx}}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)(c+dx)^{3/2}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} - \frac{\int \frac{\sqrt{c+dx}\left(-\frac{a^2C(3de+cf)+b^2(6Bce-3Ade-5Acf)-}{2b}\right.}{}}{}$$

$$= \frac{\left(4a^3Cdf-b^3(6Bce-3Ade-5Acf)+ab^2(12cCe+3Bde+Bcf-8Adf)-a^2\right.}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$= \frac{\left(4a^3Cdf-b^3(6Bce-3Ade-5Acf)+ab^2(12cCe+3Bde+Bcf-8Adf)-a^2\right.}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$= \frac{\left(4a^3Cdf-b^3(6Bce-3Ade-5Acf)+ab^2(12cCe+3Bde+Bcf-8Adf)-a^2\right.}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$= \frac{\left(4a^3Cdf-b^3(6Bce-3Ade-5Acf)+ab^2(12cCe+3Bde+Bcf-8Adf)-a^2\right.}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

$$= \frac{\left(4a^3Cdf-b^3(6Bce-3Ade-5Acf)+ab^2(12cCe+3Bde+Bcf-8Adf)-a^2\right.}{12b^2(bc-ad)(be-af)^2(a+bx)^2}$$

Mathematica [A] time = 6.34, size = 729, normalized size = 1.06

$$-\frac{\left(a^2C-abB+Ab^2\right)\left(-\frac{3\left(8a^2d^2f^2-4abdf(3cf+de)+b^2\left(5c^2f^2+2cdef+d^2e^2\right)\right)\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{(a+bx)(af-be)}-\frac{(de-cf)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{ad-bc}(af-be)^{3/2}}\right)}{8(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(\frac{1}{2}b(-6adf+5bcf+3bde)-abdf\right)}{2(a+bx)^2(bc-ad)(be-af)}\right)}{3b^2(bc-ad)(be-af)}-\frac{(c+dx)^{3/2}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(a+bx)^3(bc-ad)(be-af)}+\frac{(bB-2aC)(-4adf+3bcf+bde)\left(\frac{\sqrt{c+dx}\sqrt{e+fx}}{(a+bx)(be-af)}+\frac{(de-cf)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{ad-bc}(af-be)^{3/2}}\right)}{4b^2(bc-ad)(be-af)}-\frac{C\sqrt{c+dx}\sqrt{e+fx}}{b^2(a+bx)(be-af)}-\frac{C(de-cf)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{b^2\sqrt{ad-bc}(af-be)^{3/2}}-\frac{(c+dx)^{3/2}\sqrt{e+fx}(bB-2aC)}{2b(a+bx)^2(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^4*Sqrt[e + f*x]),x]
```

```
[Out] -((C*Sqrt[c + d*x]*Sqrt[e + f*x])/(b^2*(b*e - a*f)*(a + b*x))) - ((A*b^2 -
a*(b*B - a*C))*(c + d*x)^(3/2)*Sqrt[e + f*x])/(3*b*(b*c - a*d)*(b*e - a*f)*
(a + b*x)^3) - ((b*B - 2*a*C)*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*b*(b*c - a*
d)*(b*e - a*f)*(a + b*x)^2) - (C*(d*e - c*f)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sq
rt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(b^2*Sqrt[-(b*c) + a*d]*(
-(b*e) + a*f)^(3/2)) + ((b*B - 2*a*C)*(b*d*e + 3*b*c*f - 4*a*d*f)*((Sqrt[c
+ d*x]*Sqrt[e + f*x])/((b*e - a*f)*(a + b*x)) + ((d*e - c*f)*ArcTanh[(Sqrt[
-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b
*c) + a*d]*(-(b*e) + a*f)^(3/2))))/(4*b^2*(b*c - a*d)*(b*e - a*f)) - ((A*b^
2 - a*b*B + a^2*C)*(-1/2*((-(a*b*d*f) + (b*(3*b*d*e + 5*b*c*f - 6*a*d*f))/2
)*(c + d*x)^(3/2)*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^2) - (3
*(8*a^2*d^2*f^2 - 4*a*b*d*f*(d*e + 3*c*f) + b^2*(d^2*e^2 + 2*c*d*e*f + 5*c^
2*f^2))*((Sqrt[c + d*x]*Sqrt[e + f*x])/((-(b*e) + a*f)*(a + b*x)) - ((d*e -
 c*f)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e
 + f*x])])/(Sqrt[-(b*c) + a*d]*(-(b*e) + a*f)^(3/2))))/(8*(b*c - a*d)*(b*e
- a*f))))/(3*b^2*(b*c - a*d)*(b*e - a*f))
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x]*(A + B*x + C*x^2))/((a + b*x)^4*Sqrt[e +
f*x]),x]
```

```
[Out] $Aborted
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [B] time = 0.16, size = 15990, normalized size = 23.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(d*x+c)^(1/2)/(b*x+a)^4/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for mo
re details)Is (a*d-b*c)    *(a*f-b*e)    positive, negative or zero?
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x)^(1/2)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(a + b*x)^4),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(d*x+c)**(1/2)/(b*x+a)**4/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

3.54 $\int \frac{(a+bx)^2\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$

Optimal. Leaf size=718

$$\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)\left(16a^2d^2f^2\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)-16abdf\left(2df\left(4Adf(c\right.\right.\right.$$

Rubi [A] time = 1.34, antiderivative size = 715, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1615, 153, 147, 63, 217, 206}

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] ((8*b*B*d*f - 2*a*C*d*f - 7*b*C*(d*e + c*f))*(a + b*x)^2*Sqrt[c + d*x]*Sqrt
[e + f*x])/(24*b*d^2*f^2) + (C*(a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*
b*d*f) - (Sqrt[c + d*x]*Sqrt[e + f*x]*(32*a^3*C*d^3*f^3 - 8*a^2*b*d^2*f^2*(
16*B*d*f - 11*C*(d*e + c*f)) - 16*a*b^2*d*f*(C*(15*d^2*e^2 + 14*c*d*e*f + 1
5*c^2*f^2) + 6*d*f*(4*A*d*f - 3*B*(d*e + c*f))) + b^3*(5*C*(21*d^3*e^3 + 19
*c*d^2*e^2*f + 19*c^2*d*e*f^2 + 21*c^3*f^3) + 8*d*f*(18*A*d*f*(d*e + c*f) -
 B*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2))) + 2*b*d*f*(6*b*d*f*(6*b*c*C*e +
 a*C*d*e + a*c*C*f - 8*A*b*d*f) - (4*a*d*f - 5*b*(d*e + c*f))*(8*b*B*d*f -
2*a*C*d*f - 7*b*C*(d*e + c*f)))*x))/(192*b*d^4*f^4) + ((16*a^2*d^2*f^2*(C*(
3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f))) - 16*
a*b*d*f*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3) + 2*d*f*
(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))) + b^2*(C*(35
*d^4*e^4 + 20*c*d^3*e^3*f + 18*c^2*d^2*e^2*f^2 + 20*c^3*d*e*f^3 + 35*c^4*f^
4) + 8*d*f*(2*A*d*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) - B*(5*d^3*e^3 + 3*
c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[e + f*x])])/(64*d^(9/2)*f^(9/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
```

```
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(a+bx)^2\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx = \frac{C(a+bx)^3\sqrt{c+dx}\,\sqrt{e+fx}}{4bdf} + \frac{\int \frac{(a+bx)^2\left(-\frac{1}{2}b(6bcCe+aCde+acCf-8Abdf)+\frac{1}{2}b(8bBdf-2a\cdots}{\sqrt{c+dx}\,\sqrt{e+fx}}}{4b^2df}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+bx)^3\sqrt{c\cdots}}{4b\cdots}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+bx)^3\sqrt{c\cdots}}{4b\cdots}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+bx)^3\sqrt{c\cdots}}{4b\cdots}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+bx)^3\sqrt{c\cdots}}{4b\cdots}$$

$$= \frac{(8bBdf-2aCdf-7bC(de+cf))(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{24bd^2f^2} + \frac{C(a+bx)^3\sqrt{c\cdots}}{4b\cdots}$$

Mathematica [B]   time = 6.49, size = 2195, normalized size = 3.06

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*(b*e - a*f)^2*Sqrt[d*e - c*f]*(C*e^2 - f*(B*e - A*f))*Sqrt[(d*(e + f*x))
/(d*e - c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]])/(d*f^(9/2)*
Sqrt[e + f*x]) + (2*b^2*C*(d*e - c*f)^3*Sqrt[c + d*x]*Sqrt[e + f*x]*(1 + (d
*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(9
/2)*((35/(16*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*
f)/(d*e - c*f))))^4) + 35/(24*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d
*e - c*f) - (c*d*f)/(d*e - c*f))))^3) + 7/(6*(1 + (d*f*(c + d*x))/((d*e - c
*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))
/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1))/8 + (35*S
qrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*ArcSinh[(Sqr
t[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*
d*f)/(d*e - c*f)])])/(128*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*(1 + (d*f*(c + d*x)
)/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(9/2))))/(d^4*
f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(7/2)*Sqrt[(d*(e + f*x)
)/(d*e - c*f)]) + (2*b*(d*e - c*f)^2*(-4*b*C*e + b*B*f + 2*a*C*f)*Sqrt[c +
d*x]*Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) -
 (c*d*f)/(d*e - c*f))))^(7/2)*((15/(8*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d
^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^3) + 5/(4*(1 + (d*f*(c + d*x))/(
(d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c
 + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(-1))/6
 + (5*Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)]*ArcSi
nh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f
) - (c*d*f)/(d*e - c*f)])])/(16*Sqrt[d]*Sqrt[f]*Sqrt[c + d*x]*(1 + (d*f*(c
+ d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f))))^(7/2))))
/(d^3*f^4*(d/((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))^(5/2)*Sqrt[(d*(e
+ f*x))/(d*e - c*f)]) + (2*(d*e - c*f)*(6*b^2*C*e^2 - 3*b^2*B*e*f - 6*a*b*C
*e*f + A*b^2*f^2 + 2*a*b*B*f^2 + a^2*C*f^2)*Sqrt[c + d*x]*Sqrt[e + f*x]*(1
+ (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)))
)^(5/2)*((3/(2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*
d*f)/(d*e - c*f))))^2) + (1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e -
c*f) - (c*d*f)/(d*e - c*f))))^(-1))/4 + (3*Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*
e - c*f) - (c*d*f)/(d*e - c*f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sq
rt[d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(8*Sqrt[d]
*Sqrt[f]*Sqrt[c + d*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*
f) - (c*d*f)/(d*e - c*f))))^(5/2))))/(d^2*f^4*(d/((d^2*e)/(d*e - c*f) - (c*
d*f)/(d*e - c*f)))^(3/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + (2*(-(b*e) + a*
f)*(4*b*C*e^2 - 3*b*B*e*f - 2*a*C*e*f + 2*A*b*f^2 + a*B*f^2)*Sqrt[c + d*x]*
Sqrt[e + f*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f) - (c*d
*f)/(d*e - c*f))))^(3/2)*(1/(2*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(
d*e - c*f) - (c*d*f)/(d*e - c*f))))) + (Sqrt[d*e - c*f]*Sqrt[(d^2*e)/(d*e -
 c*f) - (c*d*f)/(d*e - c*f)]*ArcSinh[(Sqrt[d]*Sqrt[f]*Sqrt[c + d*x])/(Sqrt[
d*e - c*f]*Sqrt[(d^2*e)/(d*e - c*f) - (c*d*f)/(d*e - c*f)])])/(2*Sqrt[d]*Sq
rt[f]*Sqrt[c + d*x]*(1 + (d*f*(c + d*x))/((d*e - c*f)*((d^2*e)/(d*e - c*f)
- (c*d*f)/(d*e - c*f))))^(3/2))))/(d*f^4*Sqrt[d/((d^2*e)/(d*e - c*f) - (c*d
*f)/(d*e - c*f))]*Sqrt[(d*(e + f*x))/(d*e - c*f)])
```

IntegrateAlgebraic [B]   time = 1.81, size = 2158, normalized size = 3.01

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x)^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e +
f*x]),x]
```

```
[Out] ((d*e - c*f)*Sqrt[e + f*x]*(279*b^2*C*d^3*e^3*f^3 + 219*b^2*c*C*d^2*e^2*f^4
 - 264*b^2*B*d^3*e^2*f^4 - 528*a*b*C*d^3*e^2*f^4 + 165*b^2*c^2*C*d*e*f^5 -
192*b^2*B*c*d^2*e*f^5 - 384*a*b*c*C*d^2*e*f^5 + 240*A*b^2*d^3*e*f^5 + 480*a
*b*B*d^3*e*f^5 + 240*a^2*C*d^3*e*f^5 + 105*b^2*c^3*C*f^6 - 120*b^2*B*c^2*d*
f^6 - 240*a*b*c^2*C*d*f^6 + 144*A*b^2*c*d^2*f^6 + 288*a*b*B*c*d^2*f^6 + 144
*a^2*c*C*d^2*f^6 - 384*a*A*b*d^3*f^6 - 192*a^2*B*d^3*f^6 - (511*b^2*C*d^4*e
^3*f^2*(e + f*x))/(c + d*x) - (803*b^2*c*C*d^3*e^2*f^3*(e + f*x))/(c + d*x)
 + (584*b^2*B*d^4*e^2*f^3*(e + f*x))/(c + d*x) + (1168*a*b*C*d^4*e^2*f^3*(e
 + f*x))/(c + d*x) - (605*b^2*c^2*C*d^2*e*f^4*(e + f*x))/(c + d*x) + (704*b
^2*B*c*d^3*e*f^4*(e + f*x))/(c + d*x) + (1408*a*b*c*C*d^3*e*f^4*(e + f*x))/
(c + d*x) - (624*A*b^2*d^4*e*f^4*(e + f*x))/(c + d*x) - (1248*a*b*B*d^4*e*f
^4*(e + f*x))/(c + d*x) - (624*a^2*C*d^4*e*f^4*(e + f*x))/(c + d*x) - (385*
b^2*c^3*C*d*f^5*(e + f*x))/(c + d*x) + (440*b^2*B*c^2*d^2*f^5*(e + f*x))/(c
 + d*x) + (880*a*b*c^2*C*d^2*f^5*(e + f*x))/(c + d*x) - (528*A*b^2*c*d^3*f^
5*(e + f*x))/(c + d*x) - (1056*a*b*B*c*d^3*f^5*(e + f*x))/(c + d*x) - (528*
a^2*c*C*d^3*f^5*(e + f*x))/(c + d*x) + (1152*a*A*b*d^4*f^5*(e + f*x))/(c +
d*x) + (576*a^2*B*d^4*f^5*(e + f*x))/(c + d*x) + (385*b^2*C*d^5*e^3*f*(e +
f*x)^2)/(c + d*x)^2 + (605*b^2*c*C*d^4*e^2*f^2*(e + f*x)^2)/(c + d*x)^2 - (
440*b^2*B*d^5*e^2*f^2*(e + f*x)^2)/(c + d*x)^2 - (880*a*b*C*d^5*e^2*f^2*(e
+ f*x)^2)/(c + d*x)^2 + (803*b^2*c^2*C*d^3*e*f^3*(e + f*x)^2)/(c + d*x)^2 -
 (704*b^2*B*c*d^4*e*f^3*(e + f*x)^2)/(c + d*x)^2 - (1408*a*b*c*C*d^4*e*f^3*
(e + f*x)^2)/(c + d*x)^2 + (528*A*b^2*d^5*e*f^3*(e + f*x)^2)/(c + d*x)^2 +
(1056*a*b*B*d^5*e*f^3*(e + f*x)^2)/(c + d*x)^2 + (528*a^2*C*d^5*e*f^3*(e +
f*x)^2)/(c + d*x)^2 + (511*b^2*c^3*C*d^2*f^4*(e + f*x)^2)/(c + d*x)^2 - (58
4*b^2*B*c^2*d^3*f^4*(e + f*x)^2)/(c + d*x)^2 - (1168*a*b*c^2*C*d^3*f^4*(e +
 f*x)^2)/(c + d*x)^2 + (624*A*b^2*c*d^4*f^4*(e + f*x)^2)/(c + d*x)^2 + (124
8*a*b*B*c*d^4*f^4*(e + f*x)^2)/(c + d*x)^2 + (624*a^2*c*C*d^4*f^4*(e + f*x)
^2)/(c + d*x)^2 - (1152*a*A*b*d^5*f^4*(e + f*x)^2)/(c + d*x)^2 - (576*a^2*B
*d^5*f^4*(e + f*x)^2)/(c + d*x)^2 - (105*b^2*C*d^6*e^3*(e + f*x)^3)/(c + d*
x)^3 - (165*b^2*c*C*d^5*e^2*f*(e + f*x)^3)/(c + d*x)^3 + (120*b^2*B*d^6*e^2
*f*(e + f*x)^3)/(c + d*x)^3 + (240*a*b*C*d^6*e^2*f*(e + f*x)^3)/(c + d*x)^3
 - (219*b^2*c^2*C*d^4*e*f^2*(e + f*x)^3)/(c + d*x)^3 + (192*b^2*B*c*d^5*e*f
^2*(e + f*x)^3)/(c + d*x)^3 + (384*a*b*c*C*d^5*e*f^2*(e + f*x)^3)/(c + d*x)
^3 - (144*A*b^2*d^6*e*f^2*(e + f*x)^3)/(c + d*x)^3 - (288*a*b*B*d^6*e*f^2*(
e + f*x)^3)/(c + d*x)^3 - (144*a^2*C*d^6*e*f^2*(e + f*x)^3)/(c + d*x)^3 - (
279*b^2*c^3*C*d^3*f^3*(e + f*x)^3)/(c + d*x)^3 + (264*b^2*B*c^2*d^4*f^3*(e
+ f*x)^3)/(c + d*x)^3 + (528*a*b*c^2*C*d^4*f^3*(e + f*x)^3)/(c + d*x)^3 - (
240*A*b^2*c*d^5*f^3*(e + f*x)^3)/(c + d*x)^3 - (480*a*b*B*c*d^5*f^3*(e + f*
x)^3)/(c + d*x)^3 - (240*a^2*c*C*d^5*f^3*(e + f*x)^3)/(c + d*x)^3 + (384*a*
A*b*d^6*f^3*(e + f*x)^3)/(c + d*x)^3 + (192*a^2*B*d^6*f^3*(e + f*x)^3)/(c +
 d*x)^3))/(192*d^4*f^4*Sqrt[c + d*x]*(-f + (d*(e + f*x))/(c + d*x))^4) + ((
35*b^2*C*d^4*e^4 + 20*b^2*c*C*d^3*e^3*f - 40*b^2*B*d^4*e^3*f - 80*a*b*C*d^4
*e^3*f + 18*b^2*c^2*C*d^2*e^2*f^2 - 24*b^2*B*c*d^3*e^2*f^2 - 48*a*b*c*C*d^3
*e^2*f^2 + 48*A*b^2*d^4*e^2*f^2 + 96*a*b*B*d^4*e^2*f^2 + 48*a^2*C*d^4*e^2*f
^2 + 20*b^2*c^3*C*d*e*f^3 - 24*b^2*B*c^2*d^2*e*f^3 - 48*a*b*c^2*C*d^2*e*f^3
 + 32*A*b^2*c*d^3*e*f^3 + 64*a*b*B*c*d^3*e*f^3 + 32*a^2*c*C*d^3*e*f^3 - 128
*a*A*b*d^4*e*f^3 - 64*a^2*B*d^4*e*f^3 + 35*b^2*c^4*C*f^4 - 40*b^2*B*c^3*d*f
^4 - 80*a*b*c^3*C*d*f^4 + 48*A*b^2*c^2*d^2*f^4 + 96*a*b*B*c^2*d^2*f^4 + 48*
a^2*c^2*C*d^2*f^4 - 128*a*A*b*c*d^3*f^4 - 64*a^2*B*c*d^3*f^4 + 128*a^2*A*d^
4*f^4)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(64*d^(9/2
)*f^(9/2))
```

fricas [A] time = 5.32, size = 1436, normalized size = 2.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] [1/768*(3*(35*C*b^2*d^4*e^4 + 20*(C*b^2*c*d^3 - 2*(2*C*a*b + B*b^2)*d^4)*e^
3*f + 6*(3*C*b^2*c^2*d^2 - 4*(2*C*a*b + B*b^2)*c*d^3 + 8*(C*a^2 + 2*B*a*b +
 A*b^2)*d^4)*e^2*f^2 + 4*(5*C*b^2*c^3*d - 6*(2*C*a*b + B*b^2)*c^2*d^2 + 8*(
C*a^2 + 2*B*a*b + A*b^2)*c*d^3 - 16*(B*a^2 + 2*A*a*b)*d^4)*e*f^3 + (35*C*b^
2*c^4 + 128*A*a^2*d^4 - 40*(2*C*a*b + B*b^2)*c^3*d + 48*(C*a^2 + 2*B*a*b +
A*b^2)*c^2*d^2 - 64*(B*a^2 + 2*A*a*b)*c*d^3)*f^4)*sqrt(d*f)*log(8*d^2*f^2*x
^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*f)*sqrt
(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) + 4*(48*C*b^2*d^4*f^4*x^
3 - 105*C*b^2*d^4*e^3*f - 5*(19*C*b^2*c*d^3 - 24*(2*C*a*b + B*b^2)*d^4)*e^2
*f^2 - (95*C*b^2*c^2*d^2 - 112*(2*C*a*b + B*b^2)*c*d^3 + 144*(C*a^2 + 2*B*a
*b + A*b^2)*d^4)*e*f^3 - 3*(35*C*b^2*c^3*d - 40*(2*C*a*b + B*b^2)*c^2*d^2 +
 48*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 - 64*(B*a^2 + 2*A*a*b)*d^4)*f^4 - 8*(7*
C*b^2*d^4*e*f^3 + (7*C*b^2*c*d^3 - 8*(2*C*a*b + B*b^2)*d^4)*f^4)*x^2 + 2*(3
5*C*b^2*d^4*e^2*f^2 + 2*(17*C*b^2*c*d^3 - 20*(2*C*a*b + B*b^2)*d^4)*e*f^3 +
 (35*C*b^2*c^2*d^2 - 40*(2*C*a*b + B*b^2)*c*d^3 + 48*(C*a^2 + 2*B*a*b + A*b
^2)*d^4)*f^4)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^5*f^5), -1/384*(3*(35*C*b^
2*d^4*e^4 + 20*(C*b^2*c*d^3 - 2*(2*C*a*b + B*b^2)*d^4)*e^3*f + 6*(3*C*b^2*c
^2*d^2 - 4*(2*C*a*b + B*b^2)*c*d^3 + 8*(C*a^2 + 2*B*a*b + A*b^2)*d^4)*e^2*f
^2 + 4*(5*C*b^2*c^3*d - 6*(2*C*a*b + B*b^2)*c^2*d^2 + 8*(C*a^2 + 2*B*a*b +
A*b^2)*c*d^3 - 16*(B*a^2 + 2*A*a*b)*d^4)*e*f^3 + (35*C*b^2*c^4 + 128*A*a^2*
d^4 - 40*(2*C*a*b + B*b^2)*c^3*d + 48*(C*a^2 + 2*B*a*b + A*b^2)*c^2*d^2 - 6
4*(B*a^2 + 2*A*a*b)*c*d^3)*f^4)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d*e + c*f)
*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f +
 c*d*f^2)*x)) - 2*(48*C*b^2*d^4*f^4*x^3 - 105*C*b^2*d^4*e^3*f - 5*(19*C*b^2
*c*d^3 - 24*(2*C*a*b + B*b^2)*d^4)*e^2*f^2 - (95*C*b^2*c^2*d^2 - 112*(2*C*a
*b + B*b^2)*c*d^3 + 144*(C*a^2 + 2*B*a*b + A*b^2)*d^4)*e*f^3 - 3*(35*C*b^2*
c^3*d - 40*(2*C*a*b + B*b^2)*c^2*d^2 + 48*(C*a^2 + 2*B*a*b + A*b^2)*c*d^3 -
 64*(B*a^2 + 2*A*a*b)*d^4)*f^4 - 8*(7*C*b^2*d^4*e*f^3 + (7*C*b^2*c*d^3 - 8*
(2*C*a*b + B*b^2)*d^4)*f^4)*x^2 + 2*(35*C*b^2*d^4*e^2*f^2 + 2*(17*C*b^2*c*d
^3 - 20*(2*C*a*b + B*b^2)*d^4)*e*f^3 + (35*C*b^2*c^2*d^2 - 40*(2*C*a*b + B*
b^2)*c*d^3 + 48*(C*a^2 + 2*B*a*b + A*b^2)*d^4)*f^4)*x)*sqrt(d*x + c)*sqrt(f
*x + e))/(d^5*f^5)]
```

giac [A] time = 2.51, size = 951, normalized size = 1.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/192*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*(2*(d*x + c)*(4*(d*x + c)*(6*(d*
x + c)*C*b^2/(d^5*f) - (25*C*b^2*c*d^19*f^6 - 16*C*a*b*d^20*f^6 - 8*B*b^2*d
^20*f^6 + 7*C*b^2*d^20*f^5*e)/(d^24*f^7)) + (163*C*b^2*c^2*d^19*f^6 - 208*C
*a*b*c*d^20*f^6 - 104*B*b^2*c*d^20*f^6 + 48*C*a^2*d^21*f^6 + 96*B*a*b*d^21*
f^6 + 48*A*b^2*d^21*f^6 + 90*C*b^2*c*d^20*f^5*e - 80*C*a*b*d^21*f^5*e - 40*
B*b^2*d^21*f^5*e + 35*C*b^2*d^21*f^4*e^2)/(d^24*f^7)) - 3*(93*C*b^2*c^3*d^1
9*f^6 - 176*C*a*b*c^2*d^20*f^6 - 88*B*b^2*c^2*d^20*f^6 + 80*C*a^2*c*d^21*f^
6 + 160*B*a*b*c*d^21*f^6 + 80*A*b^2*c*d^21*f^6 - 64*B*a^2*d^22*f^6 - 128*A*
a*b*d^22*f^6 + 73*C*b^2*c^2*d^20*f^5*e - 128*C*a*b*c*d^21*f^5*e - 64*B*b^2*
c*d^21*f^5*e + 48*C*a^2*d^22*f^5*e + 96*B*a*b*d^22*f^5*e + 48*A*b^2*d^22*f^
5*e + 55*C*b^2*c*d^21*f^4*e^2 - 80*C*a*b*d^22*f^4*e^2 - 40*B*b^2*d^22*f^4*e
^2 + 35*C*b^2*d^22*f^3*e^3)/(d^24*f^7))*sqrt(d*x + c) - 3*(35*C*b^2*c^4*f^4
 - 80*C*a*b*c^3*d*f^4 - 40*B*b^2*c^3*d*f^4 + 48*C*a^2*c^2*d^2*f^4 + 96*B*a*
b*c^2*d^2*f^4 + 48*A*b^2*c^2*d^2*f^4 - 64*B*a^2*c*d^3*f^4 - 128*A*a*b*c*d^3
*f^4 + 128*A*a^2*d^4*f^4 + 20*C*b^2*c^3*d*f^3*e - 48*C*a*b*c^2*d^2*f^3*e -
24*B*b^2*c^2*d^2*f^3*e + 32*C*a^2*c*d^3*f^3*e + 64*B*a*b*c*d^3*f^3*e + 32*A
*b^2*c*d^3*f^3*e - 64*B*a^2*d^4*f^3*e - 128*A*a*b*d^4*f^3*e + 18*C*b^2*c^2*
d^2*f^2*e^2 - 48*C*a*b*c*d^3*f^2*e^2 - 24*B*b^2*c*d^3*f^2*e^2 + 48*C*a^2*d^
4*f^2*e^2 + 96*B*a*b*d^4*f^2*e^2 + 48*A*b^2*d^4*f^2*e^2 + 20*C*b^2*c*d^3*f*
```

```
e^3 - 80*C*a*b*d^4*f*e^3 - 40*B*b^2*d^4*f*e^3 + 35*C*b^2*d^4*e^4)*log(abs(-
sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*
d^4*f^4))*d/abs(d)
```

maple [B] time = 0.05, size = 2528, normalized size = 3.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] 1/384*(144*A*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))
/(d*f)^(1/2))*b^2*d^4*e^2*f^2+192*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b
^2*d^3*f^3-384*A*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2))/(d*f)^(1/2))*a*b*c*d^3*f^4-384*A*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a*b*d^4*e*f^3+96*C*x^3*b^2*d^3*f^3*(
d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)+128*B*x^2*b^2*d^3*f^3*(d*f)^(1/2)*((d*x+
c)*(f*x+e))^(1/2)+96*A*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2))/(d*f)^(1/2))*b^2*c*d^3*e*f^3+60*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*
x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*b^2*c*d^3*e^3*f-72*B*ln(1/2*(
2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*b^2*c^2
*d^2*e*f^3-72*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/
2))/(d*f)^(1/2))*b^2*c*d^3*e^2*f^2+96*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*
(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a^2*c*d^3*e*f^3+60*C*ln(1/2*(2*d*f
*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*b^2*c^3*d*e*
f^3+54*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*
f)^(1/2))*b^2*c^2*d^2*e^2*f^2+192*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a
^2*d^3*f^3-240*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1
/2))/(d*f)^(1/2))*a*b*c^3*d*f^4-240*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a*b*d^4*e^3*f+768*A*(d*f)^(1/2)*((d*
x+c)*(f*x+e))^(1/2)*a*b*d^3*f^3-288*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b
^2*c*d^2*f^3-288*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*d^3*e*f^2+288*B*
ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))
*a*b*c^2*d^2*f^4+288*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2))/(d*f)^(1/2))*a*b*d^4*e^2*f^2+240*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))
^(1/2)*b^2*c^2*d*f^3+240*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*d^3*e^2*
f+105*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f
)^(1/2))*b^2*c^4*f^4+105*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2
)*(d*f)^(1/2))/(d*f)^(1/2))*b^2*d^4*e^4-192*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d
*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a^2*c*d^3*f^4-192*B*ln(1/2*(
2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a^2*d^4
*e*f^3+144*A*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))
/(d*f)^(1/2))*b^2*c^2*d^2*f^4-120*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x
+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*b^2*c^3*d*f^4-120*B*ln(1/2*(2*d*f*x+c*
f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*b^2*d^4*e^3*f+144
*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/
2))*a^2*c^2*d^2*f^4+144*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2))/(d*f)^(1/2))*a^2*d^4*e^2*f^2+384*B*(d*f)^(1/2)*((d*x+c)*(f*x+
e))^(1/2)*a^2*d^3*f^3-210*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c^3*f^3
-210*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*d^3*e^3+384*A*ln(1/2*(2*d*f*
x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a^2*d^4*f^4-2
88*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a^2*c*d^2*f^3-288*C*(d*f)^(1/2)*((
d*x+c)*(f*x+e))^(1/2)*a^2*d^3*e*f^2-190*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/
2)*b^2*c^2*d*e*f^2-190*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c*d^2*e^2*
f-160*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*d^3*e*f^2+140*C*(d*f)^(1/
2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*c^2*d*f^3-144*C*ln(1/2*(2*d*f*x+c*f+d*e+2*
((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a*b*c*d^3*e^2*f^2+192*B*l
n(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*
a*b*c*d^3*e*f^3-144*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2))/(d*f)^(1/2))*a*b*c^2*d^2*e*f^3-576*B*(d*f)^(1/2)*((d*x+c)*(f*x+e)
```

```
)^(1/2)*a*b*c*d^2*f^3-576*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*d^3*e*f
^2+224*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b^2*c*d^2*e*f^2+480*C*(d*f)^(1
/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*c^2*d*f^3+140*C*(d*f)^(1/2)*((d*x+c)*(f*x+e
))^(1/2)*x*b^2*d^3*e^2*f+256*C*x^2*a*b*d^3*f^3*(d*f)^(1/2)*((d*x+c)*(f*x+e)
)^(1/2)-112*C*x^2*b^2*c*d^2*f^3*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-112*C*x
^2*b^2*d^3*e*f^2*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)+384*B*(d*f)^(1/2)*((d*
x+c)*(f*x+e))^(1/2)*x*a*b*d^3*f^3-160*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)
*x*b^2*c*d^2*f^3+480*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*d^3*e^2*f+44
8*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*b*c*d^2*e*f^2-320*C*(d*f)^(1/2)*(
(d*x+c)*(f*x+e))^(1/2)*x*a*b*c*d^2*f^3-320*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^
(1/2)*x*a*b*d^3*e*f^2+136*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b^2*c*d^2
*e*f^2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(d*f)^(1/2)/f^4/d^4/((d*x+c)*(f*x+e))^(
1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 details)Is c*f-d*e zero or nonzero?
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^2*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

3.55 $\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$

Optimal. Leaf size=371

$$-\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\left(8a^2Cd^2f^2+2bdfx(2aCdf-b(6Bdf-5C(cf+de)))-6abdf(4Bdf-3C(cf+de))-\left(b^2\left(6\right.\right.\right.}{24bd^3f^3}$$

Rubi [A] time = 0.51, antiderivative size = 369, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1615, 147, 63, 217, 206}

$$-\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,\left(8a^2Cd^2f^2-2bdfx(-2aCdf+6bBdf-5bC(cf+de))-6abdf(4Bdf-3C(cf+de))+b^2\left(-\left(6df(4Adf-3B(cf+de))+C\left(15c^2f^2+14cdef+15d^2e^2\right)\right)\right)\right)}{24bd^3f^3}+\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)\left(2adf\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)-b\left(2df\left(4Adf(cf+de)-B\left(3c^2f^2+2cdef+3d^2e^2\right)\right)+C\left(3c^2def^2+5c^3f^3+3cd^2e^2f+5d^3e^3\right)\right)\right)}{8d^{7/2}f^{7/2}}+\frac{C(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}{3bdf}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (C*(a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*d*f) - (Sqrt[c + d*x]*Sqrt
[e + f*x]*(8*a^2*C*d^2*f^2 - 6*a*b*d*f*(4*B*d*f - 3*C*(d*e + c*f)) - b^2*(C
*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2) + 6*d*f*(4*A*d*f - 3*B*(d*e + c*f))
) - 2*b*d*f*(6*b*B*d*f - 2*a*C*d*f - 5*b*C*(d*e + c*f))*x))/(24*b*d^3*f^3)
+ ((2*a*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*
e + c*f))) - b*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3) +
 2*d*f*(4*A*d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))))*ArcT
anh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(8*d^(7/2)*f^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} + \frac{\int \frac{(a+bx)\left(-\frac{1}{2}b(4bcCe+aCde+acCf-6Abdf)+\frac{1}{2}b(6bBdf-2}{\sqrt{c+dx}\sqrt{e+fx}}}{3b^2df}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cd^2f^2-6abdf(4Bdf\right.}{}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cd^2f^2-6abdf(4Bdf\right.}{}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cd^2f^2-6abdf(4Bdf\right.}{}$$

$$= \frac{C(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}{3bdf} - \frac{\sqrt{c+dx}\sqrt{e+fx}\left(8a^2Cd^2f^2-6abdf(4Bdf\right.}{}$$

Mathematica [A] time = 1.96, size = 379, normalized size = 1.02

$$\frac{\sqrt{e+fx}\left(3\sqrt{de-cf}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)\left(b\left(2df\left(4Adf(cf+de)-B\left(3c^2f^2+2cdef+3d^2e^2\right)\right)+C\left(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3\right)\right)-2adf\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)\right)-\frac{d\sqrt{f}\sqrt{c+dx}(e+fx)\left(6adf(4Bdf+C(-3cf-3de+2dfx))+b\left(6df(4Adf+B(-3cf-3de+2dfx))+C\left(15c^2f^2+2cdf(7e-5fx)+d^2\left(15e^2-10efx+8f^2x^2\right)\right)\right)\right)}{\sqrt{\frac{d(e+fx)}{de-cf}}}\right)}{24d^3f^{7/2}(cf-de)\sqrt{\frac{d(e+fx)}{de-cf}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (Sqrt[e + f*x]*(-((d*Sqrt[f]*Sqrt[c + d*x]*(e + f*x)*(6*a*d*f*(4*B*d*f + C*
(-3*d*e - 3*c*f + 2*d*f*x)) + b*(6*d*f*(4*A*d*f + B*(-3*d*e - 3*c*f + 2*d*f
*x)) + C*(15*c^2*f^2 + 2*c*d*f*(7*e - 5*f*x) + d^2*(15*e^2 - 10*e*f*x + 8*f
^2*x^2)))))/Sqrt[(d*(e + f*x))/(d*e - c*f)]) + 3*Sqrt[d*e - c*f]*(-2*a*d*f*
(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f))) +
 b*(C*(5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3) + 2*d*f*(4*A*
d*f*(d*e + c*f) - B*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))))*ArcSinh[(Sqrt[f]
*Sqrt[c + d*x])/Sqrt[d*e - c*f]]))/(24*d^3*f^(7/2)*(-(d*e) + c*f)*Sqrt[(d*(
e + f*x))/(d*e - c*f)])
```

IntegrateAlgebraic [B] time = 0.90, size = 787, normalized size = 2.12

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x)*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[e + f*
x]),x]
```

```
[Out] ((d*e - c*f)*Sqrt[e + f*x]*(33*b*C*d^2*e^2*f^2 + 24*b*c*C*d*e*f^3 - 30*b*B*
d^2*e*f^3 - 30*a*C*d^2*e*f^3 + 15*b*c^2*C*f^4 - 18*b*B*c*d*f^4 - 18*a*c*C*d
*f^4 + 24*A*b*d^2*f^4 + 24*a*B*d^2*f^4 - (40*b*C*d^3*e^2*f*(e + f*x))/(c +
d*x) - (64*b*c*C*d^2*e*f^2*(e + f*x))/(c + d*x) + (48*b*B*d^3*e*f^2*(e + f*
x))/(c + d*x) + (48*a*C*d^3*e*f^2*(e + f*x))/(c + d*x) - (40*b*c^2*C*d*f^3*
(e + f*x))/(c + d*x) + (48*b*B*c*d^2*f^3*(e + f*x))/(c + d*x) + (48*a*c*C*d
^2*f^3*(e + f*x))/(c + d*x) - (48*A*b*d^3*f^3*(e + f*x))/(c + d*x) - (48*a*
B*d^3*f^3*(e + f*x))/(c + d*x) + (15*b*C*d^4*e^2*(e + f*x)^2)/(c + d*x)^2 +
 (24*b*c*C*d^3*e*f*(e + f*x)^2)/(c + d*x)^2 - (18*b*B*d^4*e*f*(e + f*x)^2)/
(c + d*x)^2 - (18*a*C*d^4*e*f*(e + f*x)^2)/(c + d*x)^2 + (33*b*c^2*C*d^2*f^
2*(e + f*x)^2)/(c + d*x)^2 - (30*b*B*c*d^3*f^2*(e + f*x)^2)/(c + d*x)^2 - (
30*a*c*C*d^3*f^2*(e + f*x)^2)/(c + d*x)^2 + (24*A*b*d^4*f^2*(e + f*x)^2)/(c
 + d*x)^2 + (24*a*B*d^4*f^2*(e + f*x)^2)/(c + d*x)^2))/(24*d^3*f^3*Sqrt[c +
 d*x]*(-f + (d*(e + f*x))/(c + d*x))^3) + ((-5*b*C*d^3*e^3 - 3*b*c*C*d^2*e^
2*f + 6*b*B*d^3*e^2*f + 6*a*C*d^3*e^2*f - 3*b*c^2*C*d*e*f^2 + 4*b*B*c*d^2*e
*f^2 + 4*a*c*C*d^2*e*f^2 - 8*A*b*d^3*e*f^2 - 8*a*B*d^3*e*f^2 - 5*b*c^3*C*f^
3 + 6*b*B*c^2*d*f^3 + 6*a*c^2*C*d*f^3 - 8*A*b*c*d^2*f^3 - 8*a*B*c*d^2*f^3 +
 16*a*A*d^3*f^3)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/
(8*d^(7/2)*f^(7/2))
```

fricas [A] time = 2.27, size = 720, normalized size = 1.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [-1/96*(3*(5*C*b*d^3*e^3 + 3*(C*b*c*d^2 - 2*(C*a + B*b)*d^3)*e^2*f + (3*C*b
*c^2*d - 4*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*e*f^2 + (5*C*b*c^3 - 16*A
*a*d^3 - 6*(C*a + B*b)*c^2*d + 8*(B*a + A*b)*c*d^2)*f^3)*sqrt(d*f)*log(8*d^
2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*d*f*x + d*e + c*f)*sqrt(d*
f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d*f^2)*x) - 4*(8*C*b*d^3*f^
3*x^2 + 15*C*b*d^3*e^2*f + 2*(7*C*b*c*d^2 - 9*(C*a + B*b)*d^3)*e*f^2 + 3*(5
*C*b*c^2*d - 6*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*f^3 - 2*(5*C*b*d^3*e*
f^2 + (5*C*b*c*d^2 - 6*(C*a + B*b)*d^3)*f^3)*x)*sqrt(d*x + c)*sqrt(f*x + e)
)/(d^4*f^4), 1/48*(3*(5*C*b*d^3*e^3 + 3*(C*b*c*d^2 - 2*(C*a + B*b)*d^3)*e^2
*f + (3*C*b*c^2*d - 4*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*e*f^2 + (5*C*b
*c^3 - 16*A*a*d^3 - 6*(C*a + B*b)*c^2*d + 8*(B*a + A*b)*c*d^2)*f^3)*sqrt(-d
*f)*arctan(1/2*(2*d*f*x + d*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)
/(d^2*f^2*x^2 + c*d*e*f + (d^2*e*f + c*d*f^2)*x)) + 2*(8*C*b*d^3*f^3*x^2 +
15*C*b*d^3*e^2*f + 2*(7*C*b*c*d^2 - 9*(C*a + B*b)*d^3)*e*f^2 + 3*(5*C*b*c^2
*d - 6*(C*a + B*b)*c*d^2 + 8*(B*a + A*b)*d^3)*f^3 - 2*(5*C*b*d^3*e*f^2 + (5
*C*b*c*d^2 - 6*(C*a + B*b)*d^3)*f^3)*x)*sqrt(d*x + c)*sqrt(f*x + e))/(d^4*f
^4)]
```

giac [A] time = 1.97, size = 447, normalized size = 1.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] 1/24*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*(4*(d*
x + c)*C*b/(d^4*f) - (13*C*b*c*d^11*f^4 - 6*C*a*d^12*f^4 - 6*B*b*d^12*f^4 +
 5*C*b*d^12*f^3*e)/(d^15*f^5)) + 3*(11*C*b*c^2*d^11*f^4 - 10*C*a*c*d^12*f^4
```

```
 - 10*B*b*c*d^12*f^4 + 8*B*a*d^13*f^4 + 8*A*b*d^13*f^4 + 8*C*b*c*d^12*f^3*e
 - 6*C*a*d^13*f^3*e - 6*B*b*d^13*f^3*e + 5*C*b*d^13*f^2*e^2)/(d^15*f^5)) +
3*(5*C*b*c^3*f^3 - 6*C*a*c^2*d*f^3 - 6*B*b*c^2*d*f^3 + 8*B*a*c*d^2*f^3 + 8*
A*b*c*d^2*f^3 - 16*A*a*d^3*f^3 + 3*C*b*c^2*d*f^2*e - 4*C*a*c*d^2*f^2*e - 4*
B*b*c*d^2*f^2*e + 8*B*a*d^3*f^2*e + 8*A*b*d^3*f^2*e + 3*C*b*c*d^2*f*e^2 - 6
*C*a*d^3*f*e^2 - 6*B*b*d^3*f*e^2 + 5*C*b*d^3*e^3)*log(abs(-sqrt(d*f)*sqrt(d
*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(d*f)*d^3*f^3))*d/abs(
d)
```

maple [B] time = 0.03, size = 1199, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] 1/48*(18*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(
d*f)^(1/2))*a*d^3*e^2*f+48*A*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^2*f^2+
48*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^2*f^2+30*C*(d*f)^(1/2)*((d*x+c
)*(f*x+e))^(1/2)*b*c^2*f^2+30*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*d^2*e
^2-24*A*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f
)^(1/2))*b*c*d^2*f^3-24*A*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)
*(d*f)^(1/2))/(d*f)^(1/2))*b*d^3*e*f^2-24*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x
+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a*c*d^2*f^3-24*B*ln(1/2*(2*d*f
*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a*d^3*e*f^2+
18*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(
1/2))*b*c^2*d*f^3+18*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d
*f)^(1/2))/(d*f)^(1/2))*b*d^3*e^2*f+18*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)
*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a*c^2*d*f^3+48*A*ln(1/2*(2*d*f*x+
c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a*d^3*f^3+16*C*
x^2*b*d^2*f^2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2)+12*B*ln(1/2*(2*d*f*x+c*f+
d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*b*c*d^2*e*f^2+12*C*
ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))
*a*c*d^2*e*f^2-9*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^
(1/2))/(d*f)^(1/2))*b*c^2*d*e*f^2-9*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f
*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*b*c*d^2*e^2*f+24*B*(d*f)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*x*b*d^2*f^2+24*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*a
*d^2*f^2-36*B*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c*d*f^2-36*B*(d*f)^(1/2
)*((d*x+c)*(f*x+e))^(1/2)*b*d^2*e*f-36*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2
)*a*c*d*f^2-36*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*a*d^2*e*f-15*C*ln(1/2*
(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*b*c^3*
f^3-15*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*
f)^(1/2))*b*d^3*e^3+28*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b*c*d*e*f-20*C
*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*b*d^2*e*f-20*C*(d*f)^(1/2)*((d*x+c)*
(f*x+e))^(1/2)*x*b*c*d*f^2)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/f^3/d^3/(d*f)^(1/2)
/((d*x+c)*(f*x+e))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 details)Is c*f-d*e zero or nonzero?
```

mupad [B] time = 105.19, size = 2621, normalized size = 7.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)*(A + B*x + C*x^2))/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] ((((c + d*x)^(1/2) - c^(1/2))*(2*A*b*c*f + 2*A*b*d*e))/(f^3*((e + f*x)^(1/2
) - e^(1/2))) + (((c + d*x)^(1/2) - c^(1/2))^3*(2*A*b*c*f + 2*A*b*d*e))/(d*
f^2*((e + f*x)^(1/2) - e^(1/2))^3) - (8*A*b*c^(1/2)*e^(1/2)*((c + d*x)^(1/2
) - c^(1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^
(1/2))^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) -
c^(1/2))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) - c^(1/
2))*((3*C*a*d^3*e^2)/2 + (3*C*a*c^2*d*f^2)/2 + C*a*c*d^2*e*f))/(f^6*((e + f
*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^3*((11*C*a*c^2*f^2)/2
+ (11*C*a*d^2*e^2)/2 + 25*C*a*c*d*e*f))/(f^5*((e + f*x)^(1/2) - e^(1/2))^3)
 + (((c + d*x)^(1/2) - c^(1/2))^7*((3*C*a*c^2*f^2)/2 + (3*C*a*d^2*e^2)/2 +
C*a*c*d*e*f))/(d^2*f^3*((e + f*x)^(1/2) - e^(1/2))^7) - (((c + d*x)^(1/2) -
 c^(1/2))^5*((11*C*a*c^2*f^2)/2 + (11*C*a*d^2*e^2)/2 + 25*C*a*c*d*e*f))/(d*
f^4*((e + f*x)^(1/2) - e^(1/2))^5) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^
(1/2))^4*(32*C*a*c*f + 32*C*a*d*e))/(f^4*((e + f*x)^(1/2) - e^(1/2))^4))/((
(c + d*x)^(1/2) - c^(1/2))^8/((e + f*x)^(1/2) - e^(1/2))^8 + d^4/f^4 - (4*d
*((c + d*x)^(1/2) - c^(1/2))^6)/(f*((e + f*x)^(1/2) - e^(1/2))^6) - (4*d^3*
((c + d*x)^(1/2) - c^(1/2))^2)/(f^3*((e + f*x)^(1/2) - e^(1/2))^2) + (6*d^2
*((c + d*x)^(1/2) - c^(1/2))^4)/(f^2*((e + f*x)^(1/2) - e^(1/2))^4)) - ((((
c + d*x)^(1/2) - c^(1/2))^3*((85*C*b*d^4*e^3)/12 + (85*C*b*c^3*d*f^3)/12 +
(17*C*b*c*d^3*e^2*f)/4 + (17*C*b*c^2*d^2*e*f^2)/4))/(f^8*((e + f*x)^(1/2) -
 e^(1/2))^3) - (((c + d*x)^(1/2) - c^(1/2))*((5*C*b*d^5*e^3)/4 + (5*C*b*c^3
*d^2*f^3)/4 + (3*C*b*c*d^4*e^2*f)/4 + (3*C*b*c^2*d^3*e*f^2)/4))/(f^9*((e +
f*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^5*((33*C*b*c^3*f^3)/2
 + (33*C*b*d^3*e^3)/2 + (327*C*b*c*d^2*e^2*f)/2 + (327*C*b*c^2*d*e*f^2)/2))
/(f^7*((e + f*x)^(1/2) - e^(1/2))^5) - (((c + d*x)^(1/2) - c^(1/2))^11*((5*
C*b*c^3*f^3)/4 + (5*C*b*d^3*e^3)/4 + (3*C*b*c*d^2*e^2*f)/4 + (3*C*b*c^2*d*e
*f^2)/4))/(d^3*f^4*((e + f*x)^(1/2) - e^(1/2))^11) + (((c + d*x)^(1/2) - c^
(1/2))^9*((85*C*b*c^3*f^3)/12 + (85*C*b*d^3*e^3)/12 + (17*C*b*c*d^2*e^2*f)/
4 + (17*C*b*c^2*d*e*f^2)/4))/(d^2*f^5*((e + f*x)^(1/2) - e^(1/2))^9) - (((c
 + d*x)^(1/2) - c^(1/2))^7*((33*C*b*c^3*f^3)/2 + (33*C*b*d^3*e^3)/2 + (327*
C*b*c*d^2*e^2*f)/2 + (327*C*b*c^2*d*e*f^2)/2))/(d*f^6*((e + f*x)^(1/2) - e^
(1/2))^7) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2))^6*(128*C*b*c^2*f^2
 + 128*C*b*d^2*e^2 + (896*C*b*c*d*e*f)/3))/(f^6*((e + f*x)^(1/2) - e^(1/2))
^6) + (64*C*b*c^(3/2)*e^(3/2)*((c + d*x)^(1/2) - c^(1/2))^8)/(f^4*((e + f*x
)^(1/2) - e^(1/2))^8) + (64*C*b*c^(3/2)*d^2*e^(3/2)*((c + d*x)^(1/2) - c^(1
/2))^4)/(f^6*((e + f*x)^(1/2) - e^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^1
2/((e + f*x)^(1/2) - e^(1/2))^12 + d^6/f^6 - (6*d*((c + d*x)^(1/2) - c^(1/2
))^10)/(f*((e + f*x)^(1/2) - e^(1/2))^10) - (6*d^5*((c + d*x)^(1/2) - c^(1/
2))^2)/(f^5*((e + f*x)^(1/2) - e^(1/2))^2) + (15*d^4*((c + d*x)^(1/2) - c^(
1/2))^4)/(f^4*((e + f*x)^(1/2) - e^(1/2))^4) - (20*d^3*((c + d*x)^(1/2) - c
^(1/2))^6)/(f^3*((e + f*x)^(1/2) - e^(1/2))^6) + (15*d^2*((c + d*x)^(1/2) -
 c^(1/2))^8)/(f^2*((e + f*x)^(1/2) - e^(1/2))^8)) - ((((c + d*x)^(1/2) - c^
(1/2))*((3*B*b*d^3*e^2)/2 + (3*B*b*c^2*d*f^2)/2 + B*b*c*d^2*e*f))/(f^6*((e
+ f*x)^(1/2) - e^(1/2))) - (((c + d*x)^(1/2) - c^(1/2))^3*((11*B*b*c^2*f^2)
/2 + (11*B*b*d^2*e^2)/2 + 25*B*b*c*d*e*f))/(f^5*((e + f*x)^(1/2) - e^(1/2))
^3) + (((c + d*x)^(1/2) - c^(1/2))^7*((3*B*b*c^2*f^2)/2 + (3*B*b*d^2*e^2)/2
 + B*b*c*d*e*f))/(d^2*f^3*((e + f*x)^(1/2) - e^(1/2))^7) - (((c + d*x)^(1/2
) - c^(1/2))^5*((11*B*b*c^2*f^2)/2 + (11*B*b*d^2*e^2)/2 + 25*B*b*c*d*e*f))/
(d*f^4*((e + f*x)^(1/2) - e^(1/2))^5) + (c^(1/2)*e^(1/2)*((c + d*x)^(1/2) -
 c^(1/2))^4*(32*B*b*c*f + 32*B*b*d*e))/(f^4*((e + f*x)^(1/2) - e^(1/2))^4))
/(((c + d*x)^(1/2) - c^(1/2))^8/((e + f*x)^(1/2) - e^(1/2))^8 + d^4/f^4 - (
4*d*((c + d*x)^(1/2) - c^(1/2))^6)/(f*((e + f*x)^(1/2) - e^(1/2))^6) - (4*d
^3*((c + d*x)^(1/2) - c^(1/2))^2)/(f^3*((e + f*x)^(1/2) - e^(1/2))^2) + (6*
d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(f^2*((e + f*x)^(1/2) - e^(1/2))^4)) + (
```

```
(((c + d*x)^(1/2) - c^(1/2))*(2*B*a*c*f + 2*B*a*d*e))/(f^3*((e + f*x)^(1/2)
 - e^(1/2))) + (((c + d*x)^(1/2) - c^(1/2))^3*(2*B*a*c*f + 2*B*a*d*e))/(d*f
^2*((e + f*x)^(1/2) - e^(1/2))^3) - (8*B*a*c^(1/2)*e^(1/2)*((c + d*x)^(1/2)
 - c^(1/2))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^(
1/2))^4/((e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) - c
^(1/2))^2)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - (4*A*a*atan((d*((e + f*x)^(
1/2) - e^(1/2)))/((-d*f)^(1/2)*((c + d*x)^(1/2) - c^(1/2)))))/(-d*f)^(1/2)
+ (B*b*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2
) - e^(1/2))))*(3*c^2*f^2 + 3*d^2*e^2 + 2*c*d*e*f))/(2*d^(5/2)*f^(5/2)) + (
C*a*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) -
 e^(1/2))))*(3*c^2*f^2 + 3*d^2*e^2 + 2*c*d*e*f))/(2*d^(5/2)*f^(5/2)) - (2*A
*b*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) -
e^(1/2))))*(c*f + d*e))/(d^(3/2)*f^(3/2)) - (2*B*a*atanh((f^(1/2)*((c + d*x
)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(c*f + d*e))/(d^
(3/2)*f^(3/2)) - (C*b*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*
((e + f*x)^(1/2) - e^(1/2))))*(c*f + d*e)*(5*c^2*f^2 + 5*d^2*e^2 - 2*c*d*e*
f))/(4*d^(7/2)*f^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((a + b*x)*(A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

3.56 $\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx$

Optimal. Leaf size=164

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{4d^{5/2}f^{5/2}}-\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(-4Bdf+5cCf+}{4d^2f^2}$$

Rubi [A] time = 0.15, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {951, 80, 63, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)}{4d^{5/2}f^{5/2}}-\frac{\sqrt{c+dx}\,\sqrt{e+fx}\,(-4Bdf+5cCf+3Cde)}{4d^2f^2}+\frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -((3*C*d*e + 5*c*C*f - 4*B*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*d^2*f^2) +
(C*(c + d*x)^(3/2)*Sqrt[e + f*x])/(2*d^2*f) + ((C*(3*d^2*e^2 + 2*c*d*e*f +
3*c^2*f^2) + 4*d*f*(2*A*d*f - B*(d*e + c*f)))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x
])/(Sqrt[d]*Sqrt[e + f*x])])/(4*d^(5/2)*f^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 951

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(c^p*(d + e*x)^(m + 2*p)*(f + g*x
)^(n + 1))/(g*e^(2*p)*(m + n + 2*p + 1)), x] + Dist[1/(g*e^(2*p)*(m + n + 2
*p + 1)), Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(2
*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)*
(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e
```

```
*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGt
Q[p, 0] && NeQ[m + n + 2*p + 1, 0] && (IntegerQ[n] ||  !IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{\int \frac{\frac{1}{2}\left(-3cCde-c^2Cf+4Ad^2f\right)-\frac{1}{2}d(3Cde+5cCf-4Bdf)x}{\sqrt{c+dx}\sqrt{e+fx}}\,dx}{2d^2f}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{\left(C\left(3d^2e^2+2\right.\right.}{}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{\left(C\left(3d^2e^2+2\right.\right.}{}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{\left(C\left(3d^2e^2+2\right.\right.}{}$$

$$= -\frac{(3Cde+5cCf-4Bdf)\sqrt{c+dx}\sqrt{e+fx}}{4d^2f^2} + \frac{C(c+dx)^{3/2}\sqrt{e+fx}}{2d^2f} + \frac{\left(C\left(3d^2e^2+2\right.\right.}{}$$

Mathematica [A] time = 0.79, size = 173, normalized size = 1.05

$$\frac{\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)+d\sqrt{f}\sqrt{c+dx}(e+fx)(4Bdf+C(-3cf-3de+2dfx))}{4d^3f^{5/2}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (d*Sqrt[f]*Sqrt[c + d*x]*(e + f*x)*(4*B*d*f + C*(-3*d*e - 3*c*f + 2*d*f*x))
 + Sqrt[d*e - c*f]*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f
- B*(d*e + c*f)))*Sqrt[(d*(e + f*x))/(d*e - c*f)]*ArcSinh[(Sqrt[f]*Sqrt[c +
 d*x])/Sqrt[d*e - c*f]])/(4*d^3*f^(5/2)*Sqrt[e + f*x])
```

IntegrateAlgebraic [A] time = 0.38, size = 229, normalized size = 1.40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\sqrt{c+dx}}\right)\left(8Ad^2f^2-4Bcdf^2-4Bd^2ef+3c^2Cf^2+2cCdef+3Cd^2e^2\right)}{4d^{5/2}f^{5/2}}+\frac{\sqrt{e+fx}\,(de-cf)\left(\frac{4Bd^2f(e+fx)}{c+dx}-4Bdf^2-\frac{3Cd^2e(e+fx)}{c+dx}-\frac{5cCdf(e+fx)}{c+dx}+3cCf^2+5Cdef\right)}{4d^2f^2\sqrt{c+dx}\left(\frac{d(e+fx)}{c+dx}-f\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] ((d*e - c*f)*Sqrt[e + f*x]*(5*C*d*e*f + 3*c*C*f^2 - 4*B*d*f^2 - (3*C*d^2*e*
(e + f*x))/(c + d*x) - (5*c*C*d*f*(e + f*x))/(c + d*x) + (4*B*d^2*f*(e + f*
x))/(c + d*x)))/(4*d^2*f^2*Sqrt[c + d*x]*(-f + (d*(e + f*x))/(c + d*x))^2)
+ ((3*C*d^2*e^2 + 2*c*C*d*e*f - 4*B*d^2*e*f + 3*c^2*C*f^2 - 4*B*c*d*f^2 + 8
*A*d^2*f^2)*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])])/(4*d^
(5/2)*f^(5/2))
```

fricas [A] time = 1.57, size = 380, normalized size = 2.32

$$\left[\frac{\left(3Cd^2e^2+2\left(Ccd-2Bd^2\right)ef+\left(3Cc^2-4Bcd+8Ad^2\right)f^2\right)\sqrt{df}\log\left(8d^2f^2x^2+d^2e^2+6cdef+c^2f^2+4\left(2dfx+de+cf\right)\sqrt{df}\sqrt{dx+c}\sqrt{fx+e}+8\left(d^2ef+cdf^2\right)x\right)+4\left(2Cd^2f^2x-3Cd^2ef-\left(3Ccd-4Bd^2\right)f^2\right)\sqrt{dx+c}\sqrt{fx+e}}{16d^3f^3}, -\frac{\left(3Cd^2e^2+2\left(Ccd-2Bd^2\right)ef+\left(3Cc^2-4Bcd+8Ad^2\right)f^2\right)\sqrt{-df}\arctan\left(\frac{\left(2dfx+de+cf\right)\sqrt{-df}\sqrt{dx+c}\sqrt{fx+e}}{2\left(d^2f^2x^2+cdef+\left(d^2ef+cdf^2\right)x\right)}\right)-2\left(2Cd^2f^2x-3Cd^2ef-\left(3Ccd-4Bd^2\right)f^2\right)\sqrt{dx+c}\sqrt{fx+e}}{8d^3f^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*C*d^2*e^2 + 2*(C*c*d - 2*B*d^2)*e*f + (3*C*c^2 - 4*B*c*d + 8*A*d^
2)*f^2)*sqrt(d*f)*log(8*d^2*f^2*x^2 + d^2*e^2 + 6*c*d*e*f + c^2*f^2 + 4*(2*
d*f*x + d*e + c*f)*sqrt(d*f)*sqrt(d*x + c)*sqrt(f*x + e) + 8*(d^2*e*f + c*d
*f^2)*x) + 4*(2*C*d^2*f^2*x - 3*C*d^2*e*f - (3*C*c*d - 4*B*d^2)*f^2)*sqrt(d
*x + c)*sqrt(f*x + e))/(d^3*f^3), -1/8*((3*C*d^2*e^2 + 2*(C*c*d - 2*B*d^2)*
e*f + (3*C*c^2 - 4*B*c*d + 8*A*d^2)*f^2)*sqrt(-d*f)*arctan(1/2*(2*d*f*x + d
*e + c*f)*sqrt(-d*f)*sqrt(d*x + c)*sqrt(f*x + e)/(d^2*f^2*x^2 + c*d*e*f + (
d^2*e*f + c*d*f^2)*x)) - 2*(2*C*d^2*f^2*x - 3*C*d^2*e*f - (3*C*c*d - 4*B*d^
2)*f^2)*sqrt(d*x + c)*sqrt(f*x + e))/(d^3*f^3)]
```

giac [A] time = 1.22, size = 194, normalized size = 1.18

$$\frac{\left(\sqrt{(dx+c)df-cdf+d^2e}\,\sqrt{dx+c}\left(\frac{2(dx+c)C}{d^3f}-\frac{5Ccd^5f^2-4Bd^6f^2+3Cd^6fe}{d^8f^3}\right)-\frac{\left(3Cc^2f^2-4Bcdf^2+8Ad^2f^2+2Ccdfe-4Bd^2fe+3Cd^2e^2\right)\log\left(\left|-\sqrt{df}\,\sqrt{dx+c}+\sqrt{(dx+c)df-cdf+d^2e}\right|\right)}{\sqrt{df}\,d^2f^2}\right)d}{4\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt((d*x + c)*d*f - c*d*f + d^2*e)*sqrt(d*x + c)*(2*(d*x + c)*C/(d^3*
f) - (5*C*c*d^5*f^2 - 4*B*d^6*f^2 + 3*C*d^6*f*e)/(d^8*f^3)) - (3*C*c^2*f^2
- 4*B*c*d*f^2 + 8*A*d^2*f^2 + 2*C*c*d*f*e - 4*B*d^2*f*e + 3*C*d^2*e^2)*log(
abs(-sqrt(d*f)*sqrt(d*x + c) + sqrt((d*x + c)*d*f - c*d*f + d^2*e)))/(sqrt(
d*f)*d^2*f^2))*d/abs(d)
```

maple [B] time = 0.02, size = 425, normalized size = 2.59

$$\frac{\left(8Ad^2f^2\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)-4Bcdf^2\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)-4Bd^2ef\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)+3Cc^2f^2\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)+2Ccdef\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)+3Cd^2e^2\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\,\sqrt{df}}{2\sqrt{df}}\right)+4\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,Cdfx+8\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,Bdf-6\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,Ccf-6\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,Cde\right)\sqrt{dx+c}\,\sqrt{fx+e}}{8\sqrt{df}\,\sqrt{(dx+c)(fx+e)}\,d^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] 1/8*(8*A*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*
f)^(1/2))*d^2*f^2-4*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*
f)^(1/2))/(d*f)^(1/2))*c*d*f^2-4*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+
e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*d^2*e*f+3*C*ln(1/2*(2*d*f*x+c*f+d*e+2*(
(d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*c^2*f^2+2*C*ln(1/2*(2*d*f*
x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*c*d*e*f+3*C*l
n(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*
d^2*e^2+4*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*x*d*f+8*B*(d*f)^(1/2)*((d*x
+c)*(f*x+e))^(1/2)*d*f-6*C*(d*f)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*c*f-6*C*(d*f
)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*d*e)*(d*x+c)^(1/2)*(f*x+e)^(1/2)/(d*f)^(1/2
)/f^2/d^2/((d*x+c)*(f*x+e))^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*f-d*e>0)', see `assume?` for more
 details)Is c*f-d*e zero or nonzero?
```

mupad [B] time = 25.89, size = 833, normalized size = 5.08

$$\frac{\frac{\left(2\,B\,c\,f+2\,B\,d\,e\right)\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{f^3\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)}+\frac{\left(2\,B\,c\,f+2\,B\,d\,e\right)\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^3}{d\,f^2\,{\left(\sqrt{e+f\,x}-\sqrt{e}\right)}^3}-\frac{8\,B\,\sqrt{c}\,\sqrt{e}\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{f^2\,{\left(\sqrt{e+f\,x}-\sqrt{e}\right)}^2}}{\frac{{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^4}{{\left(\sqrt{e+f\,x}-\sqrt{e}\right)}^4}+\frac{d^2}{f^2}-\frac{2\,d\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{f\,{\left(\sqrt{e+f\,x}-\sqrt{e}\right)}^2}}-\frac{\frac{\left(\sqrt{c+d\,x}-\sqrt{c}\right)\,\left(\frac{3\,C\,c^2\,d\,f^2}{2}+C\,c\,d^2\,e\,f+\frac{3\,C\,d^3\,e^2}{2}\right)}{f^6\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)}-\frac{{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^3\,\left(\frac{11\,C\,c^2\,f^2}{2}+25\,C\,c\,d\,e\,f+\frac{11\,C\,d^2\,e^2}{2}\right)}{f^5\,{\left(\sqrt{e+f\,x}-\sqrt{e}\right)}^3}+\frac{{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^7\,\left(\frac{3\,C\,c^2\,f^2}{2}+C\,c\,d\,e\,f+\frac{3\,C\,d^2\,e^2}{2}\right)}{d^2\,f^3\,{\left(\sqrt{e+f\,x}-\sqrt{e}\right)}^7}-\frac{{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^5\,\left(\frac{11\,C\,c^2\,f^2}{2}+25\,C\,c\,d\,e\,f+\frac{11\,C\,d^2\,e^2}{2}\right)}{d\,f^4\,{\left(\sqrt{e+f\,x}-\sqrt{e}\right)}^5}+\frac{\sqrt{c}\,\sqrt{e}\,\left(32\,C\,c\,f+32\,C\,d\,e\right)\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^4}{f^4\,{\left(\sqrt{e+f\,x}-\sqrt{e}\right)}^4}}{\frac{{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^8}{{\left(\sqrt{e+f\,x}-\sqrt{e}\right)}^8}+\frac{d^4}{f^4}-\frac{4\,d\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^6}{f\,{\left(\sqrt{e+f\,x}-\sqrt{e}\right)}^6}-\frac{4\,d^3\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^2}{f^3\,{\left(\sqrt{e+f\,x}-\sqrt{e}\right)}^2}+\frac{6\,d^2\,{\left(\sqrt{c+d\,x}-\sqrt{c}\right)}^4}{f^2\,{\left(\sqrt{e+f\,x}-\sqrt{e}\right)}^4}}-\frac{4\,A\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)}{\sqrt{-d\,f}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-d\,f}}-\frac{2\,B\,\mathrm{atanh}\left(\frac{\sqrt{f}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{d}\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)}\right)\,\left(c\,f+d\,e\right)}{d^{3/2}\,f^{3/2}}+\frac{C\,\mathrm{atanh}\left(\frac{\sqrt{f}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{d}\,\left(\sqrt{e+f\,x}-\sqrt{e}\right)}\right)\,\left(3\,c^2\,f^2+2\,c\,d\,e\,f+3\,d^2\,e^2\right)}{2\,d^{5/2}\,f^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] (((2*B*c*f + 2*B*d*e)*((c + d*x)^(1/2) - c^(1/2)))/(f^3*((e + f*x)^(1/2) -
e^(1/2))) + ((2*B*c*f + 2*B*d*e)*((c + d*x)^(1/2) - c^(1/2))^3)/(d*f^2*((e
+ f*x)^(1/2) - e^(1/2))^3) - (8*B*c^(1/2)*e^(1/2)*((c + d*x)^(1/2) - c^(1/2
))^2)/(f^2*((e + f*x)^(1/2) - e^(1/2))^2))/(((c + d*x)^(1/2) - c^(1/2))^4/(
(e + f*x)^(1/2) - e^(1/2))^4 + d^2/f^2 - (2*d*((c + d*x)^(1/2) - c^(1/2))^2
)/(f*((e + f*x)^(1/2) - e^(1/2))^2)) - ((((c + d*x)^(1/2) - c^(1/2))*((3*C*
d^3*e^2)/2 + (3*C*c^2*d*f^2)/2 + C*c*d^2*e*f))/(f^6*((e + f*x)^(1/2) - e^(1
/2))) - (((c + d*x)^(1/2) - c^(1/2))^3*((11*C*c^2*f^2)/2 + (11*C*d^2*e^2)/2
 + 25*C*c*d*e*f))/(f^5*((e + f*x)^(1/2) - e^(1/2))^3) + (((c + d*x)^(1/2) -
 c^(1/2))^7*((3*C*c^2*f^2)/2 + (3*C*d^2*e^2)/2 + C*c*d*e*f))/(d^2*f^3*((e +
 f*x)^(1/2) - e^(1/2))^7) - (((c + d*x)^(1/2) - c^(1/2))^5*((11*C*c^2*f^2)/
2 + (11*C*d^2*e^2)/2 + 25*C*c*d*e*f))/(d*f^4*((e + f*x)^(1/2) - e^(1/2))^5)
 + (c^(1/2)*e^(1/2)*(32*C*c*f + 32*C*d*e)*((c + d*x)^(1/2) - c^(1/2))^4)/(f
^4*((e + f*x)^(1/2) - e^(1/2))^4))/(((c + d*x)^(1/2) - c^(1/2))^8/((e + f*x
)^(1/2) - e^(1/2))^8 + d^4/f^4 - (4*d*((c + d*x)^(1/2) - c^(1/2))^6)/(f*((e
 + f*x)^(1/2) - e^(1/2))^6) - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/(f^3*((
e + f*x)^(1/2) - e^(1/2))^2) + (6*d^2*((c + d*x)^(1/2) - c^(1/2))^4)/(f^2*(
(e + f*x)^(1/2) - e^(1/2))^4)) - (4*A*atan((d*((e + f*x)^(1/2) - e^(1/2)))/
((-d*f)^(1/2)*((c + d*x)^(1/2) - c^(1/2)))))/(-d*f)^(1/2) - (2*B*atanh((f^(
1/2)*((c + d*x)^(1/2) - c^(1/2)))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(c
*f + d*e))/(d^(3/2)*f^(3/2)) + (C*atanh((f^(1/2)*((c + d*x)^(1/2) - c^(1/2)
))/(d^(1/2)*((e + f*x)^(1/2) - e^(1/2))))*(3*c^2*f^2 + 3*d^2*e^2 + 2*c*d*e*
f))/(2*d^(5/2)*f^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{\sqrt{c + dx}\,\sqrt{e + fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/(sqrt(c + d*x)*sqrt(e + f*x)), x)
```

3.57 $\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=188

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)}{b^2\sqrt{bc-ad}\sqrt{be-af}}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2aCdf+b(-2Bdf+cCf+Cde))}{b^2d^{3/2}f^{3/2}}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

Rubi [A] time = 0.34, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1615, 157, 63, 217, 206, 93, 208}

$$-\frac{2\left(Ab^2-a(bB-aC)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)}{b^2\sqrt{bc-ad}\sqrt{be-af}}-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)(2aCdf+b(-2Bdf+cCf+Cde))}{b^2d^{3/2}f^{3/2}}+\frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (C*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*d*f) - ((2*a*C*d*f + b*(C*d*e + c*C*f -
2*B*d*f))*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e + f*x])])/(b^2*d^
(3/2)*f^(3/2)) - (2*(A*b^2 - a*(b*B - a*C))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c
 + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b^2*Sqrt[b*c - a*d]*Sqrt[b*e -
a*f])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx = \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} + \frac{\int \frac{\frac{1}{2}b(2Abdf-aC(de+cf))-\frac{1}{2}b(2aCdf+b(Cde+cCf-2Bdf))x}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx}{b^2df}$$

$$= \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} + \left(A-\frac{a(bB-aC)}{b^2}\right)\int \frac{1}{(a+bx)\sqrt{c+dx}\sqrt{e+fx}}\,dx +$$

$$= \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} + \left(2\left(A-\frac{a(bB-aC)}{b^2}\right)\right)\text{Subst}\left(\int \frac{1}{-bc+ad-(-be+a$$

$$= \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} - \frac{2\left(A-\frac{a(bB-aC)}{b^2}\right)\tanh^{-1}\left(\frac{\sqrt{be-af}\sqrt{c+dx}}{\sqrt{bc-ad}\sqrt{e+fx}}\right)}{\sqrt{bc-ad}\sqrt{be-af}} + \frac{(-2aCdf -$$

$$= \frac{C\sqrt{c+dx}\sqrt{e+fx}}{bdf} - \frac{(2aCdf+b(Cde+cCf-2Bdf))\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^2d^{3/2}f^{3/2}}$$

Mathematica [A] time = 0.94, size = 304, normalized size = 1.62

$$\frac{2\left(\frac{\left(a(aC-bB)+Ab^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{ad-bc}\sqrt{af-be}} - \frac{\sqrt{e+fx}(aCf-bBf+bCe)\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{f^{3/2}\sqrt{de-cf}\sqrt{\frac{d(e+fx)}{de-cf}}} + \frac{bC\sqrt{e+fx}\left(\sqrt{f}\sqrt{c+dx}\sqrt{\frac{d(e+fx)}{de-cf}}+\sqrt{de-cf}\sinh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{de-cf}}\right)\right)}{2df^{3/2}\sqrt{\frac{d(e+fx)}{de-cf}}}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*(-(((b*C*e - b*B*f + a*C*f)*Sqrt[e + f*x]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x]
)/Sqrt[d*e - c*f]])/(f^(3/2)*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)
])) + (b*C*Sqrt[e + f*x]*(Sqrt[f]*Sqrt[c + d*x]*Sqrt[(d*(e + f*x))/(d*e - c
*f)] + Sqrt[d*e - c*f]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x])/Sqrt[d*e - c*f]]))/(
2*d*f^(3/2)*Sqrt[(d*(e + f*x))/(d*e - c*f)]) + ((A*b^2 + a*(-(b*B) + a*C))*
ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x
])])/(Sqrt[-(b*c) + a*d]*Sqrt[-(b*e) + a*f])))/b^2
```

IntegrateAlgebraic [A] time = 0.62, size = 227, normalized size = 1.21

$$-\frac{2\left(a^2C-abB+Ab^2\right)\tan^{-1}\left(\frac{\sqrt{e+fx}\sqrt{bc-ad}\sqrt{af-be}}{\sqrt{c+dx}\,(be-af)}\right)}{b^2\sqrt{bc-ad}\sqrt{af-be}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{e+fx}}{\sqrt{f}\sqrt{c+dx}}\right)(-2aCdf+2bBdf-bcCf-bCde)}{b^2d^{3/2}f^{3/2}}-\frac{C\sqrt{e+fx}\,(cf-de)}{bdf\sqrt{c+dx}\left(\frac{d(e+fx)}{c+dx}-f\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/((a + b*x)*Sqrt[c + d*x]*Sqrt[e + f*x]
),x]
```

```
[Out] -((C*(-(d*e) + c*f)*Sqrt[e + f*x])/(b*d*f*Sqrt[c + d*x]*(-f + (d*(e + f*x))
/(c + d*x)))) - (2*(A*b^2 - a*b*B + a^2*C)*ArcTan[(Sqrt[b*c - a*d]*Sqrt[-(b
*e) + a*f]*Sqrt[e + f*x])/((b*e - a*f)*Sqrt[c + d*x])]])/(b^2*Sqrt[b*c - a*d
]*Sqrt[-(b*e) + a*f]) + ((-(b*C*d*e) - b*c*C*f + 2*b*B*d*f - 2*a*C*d*f)*Arc
Tanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])]])/(b^2*d^(3/2)*f^(3/2)
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.03, size = 746, normalized size = 3.97

$$-\frac{\left(2\sqrt{df}\,Ab^2df\ln\left(\frac{-2adfx+bcfx+bdex-acf-ade+2bce+2\sqrt{\frac{a^2df-abcf-abde+b^2ce}{b^2}}\sqrt{(dx+c)(fx+e)}\,b}{bx+a}\right)-2\sqrt{df}\,Babdf\ln\left(\frac{-2adfx+bcfx+bdex-acf-ade+2bce+2\sqrt{\frac{a^2df-abcf-abde+b^2ce}{b^2}}\sqrt{(dx+c)(fx+e)}\,b}{bx+a}\right)-2\sqrt{\frac{a^2df-abcf-abde+b^2ce}{b^2}}\,Bb^2df\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\sqrt{df}}{2\sqrt{df}}\right)+2\sqrt{df}\,Ca^2df\ln\left(\frac{-2adfx+bcfx+bdex-acf-ade+2bce+2\sqrt{\frac{a^2df-abcf-abde+b^2ce}{b^2}}\sqrt{(dx+c)(fx+e)}\,b}{bx+a}\right)+2\sqrt{\frac{a^2df-abcf-abde+b^2ce}{b^2}}\,Cabdf\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\sqrt{df}}{2\sqrt{df}}\right)+\sqrt{\frac{a^2df-abcf-abde+b^2ce}{b^2}}\,Cb^2cf\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\sqrt{df}}{2\sqrt{df}}\right)+\sqrt{\frac{a^2df-abcf-abde+b^2ce}{b^2}}\,Cb^2de\ln\left(\frac{2dfx+cf+de+2\sqrt{(dx+c)(fx+e)}\sqrt{df}}{2\sqrt{df}}\right)-2\sqrt{(dx+c)(fx+e)}\sqrt{df}\sqrt{\frac{a^2df-abcf-abde+b^2ce}{b^2}}\,Cb^2\right)\sqrt{fx+e}\,\sqrt{dx+c}}{2\sqrt{(dx+c)(fx+e)}\sqrt{df}\sqrt{\frac{a^2df-abcf-abde+b^2ce}{b^2}}\,b^3df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] -1/2*(2*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*
b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*b^2*d
*f*(d*f)^(1/2)-2*B*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^
(1/2))/(d*f)^(1/2))*b^2*d*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-2
*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a
*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a*b*d*f*(d*f
)^(1/2)+2*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/
(d*f)^(1/2))*a*b*d*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+C*ln(1/2
*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*b^2*c
*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+C*ln(1/2*(2*d*f*x+c*f+d*e+
2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*b^2*d*e*((a^2*d*f-a*b*c
*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d
*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e)
)^(1/2)*b)/(b*x+a))*a^2*d*f*(d*f)^(1/2)-2*C*b^2*((d*x+c)*(f*x+e))^(1/2)*(d*
```

```
f)^(1/2)*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2))*(f*x+e)^(1/2)*(d*x+
c)^(1/2)/((d*x+c)*(f*x+e))^(1/2)/d/(d*f)^(1/2)/b^3/((a^2*d*f-a*b*c*f-a*b*d*
e+b^2*c*e)/b^2)^(1/2)/f
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for more details)Is ((-(2*a*d*f)/b^2)    +(c*f)/b    +(d*e)/b)    ^2    -
(4*d*f       *((a^2*d*f)/b^2       -(a*c*f)/b       -(a*d*e)/b       +c*
e))     /b^2 zero or nonzero?
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)*(c + d*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx + Cx^2}{(a + bx)\sqrt{c + dx}\sqrt{e + fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral((A + B*x + C*x**2)/((a + b*x)*sqrt(c + d*x)*sqrt(e + f*x)), x)
```

3.58 $\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=254

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(2a^3Cdf-3a^2bC(cf+de)+ab^2(-2Adf+Bcf+Bde+4cCe)-b^3(-Acf-Ade+2Bce)\right)}{b^2(bc-ad)^{3/2}(be-af)^{3/2}}$$

Rubi [A] time = 0.64, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {1613, 157, 63, 217, 206, 93, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(-3a^2bC(cf+de)+2a^3Cdf+ab^2(-2Adf+Bcf+Bde+4cCe)-b^3(-Acf-Ade+2Bce)\right)}{b^2(bc-ad)^{3/2}(be-af)^{3/2}}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{b(a+bx)(bc-ad)(be-af)}+\frac{2C\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{d}\sqrt{e+fx}}\right)}{b^2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -(((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*e
 - a*f)*(a + b*x))) + (2*C*ArcTanh[(Sqrt[f]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[e
+ f*x])])/(b^2*Sqrt[d]*Sqrt[f]) + ((2*a^3*C*d*f - 3*a^2*b*C*(d*e + c*f) - b
^3*(2*B*c*e - A*d*e - A*c*f) + a*b^2*(4*c*C*e + B*d*e + B*c*f - 2*A*d*f))*A
rcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/(b
^2*(b*c - a*d)^(3/2)*(b*e - a*f)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^2\sqrt{c+dx}\,\sqrt{e+fx}}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} - \frac{\int \frac{-\frac{a^2C(de+cf)+b^2(2Bce-Ade-Acf)-ab(2cCe+B}{2b}}{(a+bx)\sqrt{c+dx}\,\cdots}}{(bc-ad)(be}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{C\int \frac{1}{\sqrt{c+dx}\,\sqrt{e+fx}}\,dx}{b^2} - \frac{\left(2a^3Cdf\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{(2C)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{e-\frac{cf}{d}+\frac{fx^2}{d}}}\,dx, x,\right.}{b^2d}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{\left(2a^3Cdf-3a^2bC(de+cf)-b^3\right.}{}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\,\sqrt{e+fx}}{b(bc-ad)(be-af)(a+bx)} + \frac{2C\tanh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{e+fx}}\right)}{b^2\sqrt{d}\,\sqrt{f}} + \frac{\left(2a^3C\right.}{}$$

Mathematica [A]   time = 1.86, size = 325, normalized size = 1.28

$$\frac{-\frac{b\sqrt{c+dx}\,\sqrt{e+fx}\left(a(aC-bB)+Ab^2\right)}{(a+bx)(bc-ad)(be-af)}-\frac{\left(a(aC-bB)+Ab^2\right)(-2adf+bcf+bde)\tanh^{-1}\left(\frac{\sqrt{c+dx}\,\sqrt{af-be}}{\sqrt{e+fx}\,\sqrt{ad-bc}}\right)}{(ad-bc)^{3/2}(af-be)^{3/2}}+\frac{2(bB-2aC)\tanh^{-1}\left(\frac{\sqrt{c+dx}\,\sqrt{af-be}}{\sqrt{e+fx}\,\sqrt{ad-bc}}\right)}{\sqrt{ad-bc}\,\sqrt{af-be}}+\frac{2C\sqrt{e+fx}\sinh^{-1}\left(\frac{\sqrt{f}\,\sqrt{c+dx}}{\sqrt{de-cf}}\right)}{\sqrt{f}\,\sqrt{de-cf}\,\sqrt{\frac{d(e+fx)}{de-cf}}}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] (-((b*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)*
(b*e - a*f)*(a + b*x))) + (2*C*Sqrt[e + f*x]*ArcSinh[(Sqrt[f]*Sqrt[c + d*x]
)/Sqrt[d*e - c*f]])/(Sqrt[f]*Sqrt[d*e - c*f]*Sqrt[(d*(e + f*x))/(d*e - c*f)
]) + (2*(b*B - 2*a*C)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*
c) + a*d]*Sqrt[e + f*x])]])/(Sqrt[-(b*c) + a*d]*Sqrt[-(b*e) + a*f]) - ((A*b^
2 + a*(-(b*B) + a*C))*(b*d*e + b*c*f - 2*a*d*f)*ArcTanh[(Sqrt[-(b*e) + a*f]
*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])]])/((-(b*c) + a*d)^(3/2)*
(-(b*e) + a*f)^(3/2)))/b^2
```

IntegrateAlgebraic [A] time = 0.94, size = 330, normalized size = 1.30

$$-\frac{\sqrt{e+fx}\,(cf-de)\left(a^2C-abB+Ab^2\right)}{b\sqrt{c+dx}\,(bc-ad)(be-af)\left(-\frac{ad(e+fx)}{c+dx}+af+\frac{bc(e+fx)}{c+dx}-be\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e+fx}\,\sqrt{bc-ad}\,\sqrt{af-be}}{\sqrt{c+dx}\,(be-af)}\right)\left(2a^3Cdf-3a^2bcCf-3a^2bCde-2aAb^2df+ab^2Bcf+ab^2Bde+4ab^2cCe+Ab^3cf+Ab^3de-2b^3Bce\right)}{b^2(bc-ad)^{3/2}(be-af)\sqrt{af-be}}+\frac{2C\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{e+fx}}{\sqrt{f}\,\sqrt{c+dx}}\right)}{b^2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/((a + b*x)^2*Sqrt[c + d*x]*Sqrt[e + f*
x]),x]
```

```
[Out] -(((A*b^2 - a*b*B + a^2*C)*(-(d*e) + c*f)*Sqrt[e + f*x])/(b*(b*c - a*d)*(b*
e - a*f)*Sqrt[c + d*x]*(-(b*e) + a*f + (b*c*(e + f*x))/(c + d*x) - (a*d*(e
+ f*x))/(c + d*x)))) + ((-2*b^3*B*c*e + 4*a*b^2*c*C*e + A*b^3*d*e + a*b^2*B
*d*e - 3*a^2*b*C*d*e + A*b^3*c*f + a*b^2*B*c*f - 3*a^2*b*c*C*f - 2*a*A*b^2*
d*f + 2*a^3*C*d*f)*ArcTan[(Sqrt[b*c - a*d]*Sqrt[-(b*e) + a*f]*Sqrt[e + f*x]
)/((b*e - a*f)*Sqrt[c + d*x])]])/(b^2*(b*c - a*d)^(3/2)*(b*e - a*f)*Sqrt[-(b
*e) + a*f]) + (2*C*ArcTanh[(Sqrt[d]*Sqrt[e + f*x])/(Sqrt[f]*Sqrt[c + d*x])]
)/(b^2*Sqrt[d]*Sqrt[f])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [B] time = 9.37, size = 1356, normalized size = 5.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] (3*sqrt(d*f)*C*a^2*b*c*d^2*f - sqrt(d*f)*B*a*b^2*c*d^2*f - sqrt(d*f)*A*b^3*
c*d^2*f - 2*sqrt(d*f)*C*a^3*d^3*f + 2*sqrt(d*f)*A*a*b^2*d^3*f - 4*sqrt(d*f)
*C*a*b^2*c*d^2*e + 2*sqrt(d*f)*B*b^3*c*d^2*e + 3*sqrt(d*f)*C*a^2*b*d^3*e -
sqrt(d*f)*B*a*b^2*d^3*e - sqrt(d*f)*A*b^3*d^3*e)*arctan(-1/2*(b*c*d*f - 2*a
*d^2*f + b*d^2*e - (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f +
d^2*e))^2*b)/(sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d
))/((a*b^3*c*f*abs(d) - a^2*b^2*d*f*abs(d) - b^4*c*abs(d)*e + a*b^3*d*abs(d
)*e)*sqrt(a*b*c*d*f^2 - a^2*d^2*f^2 - b^2*c*d*f*e + a*b*d^2*f*e)*d) + 2*(sq
rt(d*f)*C*a^2*b*c^2*d^3*f^2 - sqrt(d*f)*B*a*b^2*c^2*d^3*f^2 + sqrt(d*f)*A*b
^3*c^2*d^3*f^2 - 2*sqrt(d*f)*C*a^2*b*c*d^4*f*e + 2*sqrt(d*f)*B*a*b^2*c*d^4*
f*e - 2*sqrt(d*f)*A*b^3*c*d^4*f*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sq
rt((d*x + c)*d*f - c*d*f + d^2*e))^2*C*a^2*b*c*d^2*f + sqrt(d*f)*(sqrt(d*f)
*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a*b^2*c*d^2*f - s
qrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*
A*b^3*c*d^2*f + 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f -
 c*d*f + d^2*e))^2*C*a^3*d^3*f - 2*sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^2*B*a^2*b*d^3*f + 2*sqrt(d*f)*(sqrt(d*f)*
sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*a*b^2*d^3*f + sqrt
(d*f)*C*a^2*b*d^5*e^2 - sqrt(d*f)*B*a*b^2*d^5*e^2 + sqrt(d*f)*A*b^3*d^5*e^2
 - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e)
)^2*C*a^2*b*d^3*e + sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^2*B*a*b^2*d^3*e - sqrt(d*f)*(sqrt(d*f)*sqrt(d*x + c) - s
qrt((d*x + c)*d*f - c*d*f + d^2*e))^2*A*b^3*d^3*e)/((b*c^2*d^2*f^2 - 2*b*c*
d^3*f*e - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*e))
```

```
^2*b*c*d*f + 4*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f - c*d*f + d^2*
e))^2*a*d^2*f + b*d^4*e^2 - 2*(sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*f
 - c*d*f + d^2*e))^2*b*d^2*e + (sqrt(d*f)*sqrt(d*x + c) - sqrt((d*x + c)*d*
f - c*d*f + d^2*e))^4*b)*(a*b^3*c*f*abs(d) - a^2*b^2*d*f*abs(d) - b^4*c*abs
(d)*e + a*b^3*d*abs(d)*e)) - sqrt(d*f)*C*log((sqrt(d*f)*sqrt(d*x + c) - sqr
t((d*x + c)*d*f - c*d*f + d^2*e))^2)/(b^2*f*abs(d))
```

maple [B] time = 0.06, size = 2973, normalized size = 11.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] -1/2*(d*x+c)^(1/2)*(f*x+e)^(1/2)*(-2*B*a*b^3*(d*f)^(1/2)*((a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)+2*A*b^4*(d*f)^(1/2)*((a
^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-B*ln((-2
*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^
2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*x*a*b^3*d*e*(d*f)^(1/
2)-2*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)
^(1/2))*x*a^2*b^2*d*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*ln(
1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*x*
a*b^3*c*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*ln(1/2*(2*d*f*x
+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*x*a*b^3*d*e*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-2*C*ln((-2*a*d*f*x+b*c*f*x+b*d*
e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d
*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*x*a^3*b*d*f*(d*f)^(1/2)+3*C*ln((-2*a*d*f*x
+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b
^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*x*a^2*b^2*c*f*(d*f)^(1/2)+3*C
*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*x*a^2*b^2*d*e*
(d*f)^(1/2)-4*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*
d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))
*x*a*b^3*c*e*(d*f)^(1/2)+2*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b
*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2
)*b)/(b*x+a))*x*a*b^3*d*f*(d*f)^(1/2)-B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*
f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f
*x+e))^(1/2)*b)/(b*x+a))*x*a*b^3*c*f*(d*f)^(1/2)+2*C*a^2*b^2*(d*f)^(1/2)*((
a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)-2*C*ln(
(-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e
+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a^4*d*f*(d*f)^(1/2
)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-
a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*x*b^4*c*f*(
d*f)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*x*
b^4*d*e*(d*f)^(1/2)+2*B*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+
2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/
(b*x+a))*x*b^4*c*e*(d*f)^(1/2)-2*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+
e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*x*b^4*c*e*((a^2*d*f-a*b*c*f-a*b*d*e+b^2
*c*e)/b^2)^(1/2)+2*A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*(
(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*
x+a))*a^2*b^2*d*f*(d*f)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+
2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(
1/2)*b)/(b*x+a))*a*b^3*c*f*(d*f)^(1/2)-A*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c
*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(
f*x+e))^(1/2)*b)/(b*x+a))*a*b^3*d*e*(d*f)^(1/2)-B*ln((-2*a*d*f*x+b*c*f*x+b*
d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*(
(d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a^2*b^2*c*f*(d*f)^(1/2)-B*ln((-2*a*d*f*x
+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b
^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a^2*b^2*d*e*(d*f)^(1/2)+2*B*l
```

```
n((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d
*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a*b^3*c*e*(d*f)^
(1/2)-2*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d
*f)^(1/2))*a^3*b*d*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*ln(1
/2*(2*d*f*x+c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a^2
*b^2*c*f*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)+2*C*ln(1/2*(2*d*f*x+
c*f+d*e+2*((d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a^2*b^2*d*e*((a
^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)-2*C*ln(1/2*(2*d*f*x+c*f+d*e+2*((
d*x+c)*(f*x+e))^(1/2)*(d*f)^(1/2))/(d*f)^(1/2))*a*b^3*c*e*((a^2*d*f-a*b*c*f
-a*b*d*e+b^2*c*e)/b^2)^(1/2)+3*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-a*c*f-a*d*e
+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c)*(f*x+e))^
(1/2)*b)/(b*x+a))*a^3*b*c*f*(d*f)^(1/2)+3*C*ln((-2*a*d*f*x+b*c*f*x+b*d*e*x-
a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1/2)*((d*x+c
)*(f*x+e))^(1/2)*b)/(b*x+a))*a^3*b*d*e*(d*f)^(1/2)-4*C*ln((-2*a*d*f*x+b*c*f
*x+b*d*e*x-a*c*f-a*d*e+2*b*c*e+2*((a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/b^2)^(1
/2)*((d*x+c)*(f*x+e))^(1/2)*b)/(b*x+a))*a^2*b^2*c*e*(d*f)^(1/2))/((d*x+c)*(
f*x+e))^(1/2)/(a*d-b*c)/(a*f-b*e)/(b*x+a)/(d*f)^(1/2)/((a^2*d*f-a*b*c*f-a*b
*d*e+b^2*c*e)/b^2)^(1/2)/b^3
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^2/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((-(2*a*d*f)/b^2)>0)', see `assume?
` for more details)Is ((-(2*a*d*f)/b^2)    +(c*f)/b    +(d*e)/b)    ^2    -
(4*d*f       *((a^2*d*f)/b^2       -(a*c*f)/b       -(a*d*e)/b       +c*
e))     /b^2 zero or nonzero?
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^2*(c + d*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**2/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

3.59 $\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=424

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(a^2\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)+ab\left(-2cd\left(4Af^2-7Bef\right.\right.\right.}{4(bc-ad)^{5}}$$

Rubi [A] time = 0.97, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1613, 151, 12, 93, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{be-af}}{\sqrt{e+fx}\sqrt{bc-ad}}\right)\left(a^2\left(4df(2Adf-B(cf+de))+C\left(3c^2f^2+2cdef+3d^2e^2\right)\right)+ab\left(-2cd\left(4Af^2-7Bef+4Ce^2\right)+d^2e(Be-8Af)+c^2(-f)(8Ce-Bf)\right)+b^2\left(c^2\left(3Af^2-4Bef+8Ce^2\right)-2cde(2Be-Af)+3Ad^2e^2\right)\right)}{4(bc-ad)^{5/2}(be-af)^{5/2}}+\frac{\sqrt{c+dx}\sqrt{e+fx}\left(a^2b(2Bdf-5C(cf+de))+2a^3Cdf+ab^2(-6Adf+Bcf+Bde+8cCe)-b^3(4Bce-3A(cf+de))\right)}{4b(a+bx)(bc-ad)^2(be-af)^2}-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{2b(a+bx)^2(bc-ad)(be-af)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(2*b*(b*c - a*d)*(b*
e - a*f)*(a + b*x)^2) + ((2*a^3*C*d*f + a*b^2*(8*c*C*e + B*d*e + B*c*f - 6*
A*d*f) - b^3*(4*B*c*e - 3*A*(d*e + c*f)) + a^2*b*(2*B*d*f - 5*C*(d*e + c*f)
))*Sqrt[c + d*x]*Sqrt[e + f*x])/(4*b*(b*c - a*d)^2*(b*e - a*f)^2*(a + b*x))
 - ((b^2*(3*A*d^2*e^2 - 2*c*d*e*(2*B*e - A*f) + c^2*(8*C*e^2 - 4*B*e*f + 3*
A*f^2)) + a*b*(d^2*e*(B*e - 8*A*f) - c^2*f*(8*C*e - B*f) - 2*c*d*(4*C*e^2 -
 7*B*e*f + 4*A*f^2)) + a^2*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(
2*A*d*f - B*(d*e + c*f))))*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*
c - a*d]*Sqrt[e + f*x])])/(4*(b*c - a*d)^(5/2)*(b*e - a*f)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^3\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} - \frac{\int \frac{-\frac{a^2C(de+cf)-ab(4cCe+Bde+Bcf-4Adf)+b^2(4Bce-\cdots}{2b}}{(a+\cdots}}{2(\cdots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{\left(2a^3Cdf+ab^2(8cCe+Bde+Bcf\cdots\right.}{\cdots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{\left(2a^3Cdf+ab^2(8cCe+Bde+Bcf\cdots\right.}{\cdots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{\left(2a^3Cdf+ab^2(8cCe+Bde+Bcf\cdots\right.}{\cdots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{2b(bc-ad)(be-af)(a+bx)^2} + \frac{\left(2a^3Cdf+ab^2(8cCe+Bde+Bcf\cdots\right.}{\cdots}$$

Mathematica [A] time = 2.09, size = 512, normalized size = 1.21

$$\frac{\frac{\left(a(aC-bB)+Ab^2\right)\left(\frac{\left(8a^2d^2f^2-8abdf(cf+de)+b^2\left(3c^2f^2+2cdef+3d^2e^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{(ad-bc)^{3/2}(af-be)^{3/2}}+\frac{3b\sqrt{c+dx}\sqrt{e+fx}(-2adf+bcf+bde)}{(a+bx)(bc-ad)(be-af)}\right)}{(bc-ad)(be-af)}-\frac{2b\sqrt{c+dx}\sqrt{e+fx}\left(a(aC-bB)+Ab^2\right)}{(a+bx)^2(bc-ad)(be-af)}-\frac{4b\sqrt{c+dx}\sqrt{e+fx}(bB-2aC)}{(a+bx)(bc-ad)(be-af)}-\frac{4(bB-2aC)(-2adf+bcf+bde)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{(ad-bc)^{3/2}(af-be)^{3/2}}+\frac{8C\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{\sqrt{ad-bc}\sqrt{af-be}}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] ((-2*b*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)
*(b*e - a*f)*(a + b*x)^2) - (4*b*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e + f*x])
/((b*c - a*d)*(b*e - a*f)*(a + b*x)) + (8*C*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqr
t[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/(Sqrt[-(b*c) + a*d]*Sqrt[-
(b*e) + a*f]) - (4*(b*B - 2*a*C)*(b*d*e + b*c*f - 2*a*d*f)*ArcTanh[(Sqrt[-(
b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e + f*x])])/((-(b*c) +
a*d)^(3/2)*(-(b*e) + a*f)^(3/2)) + ((A*b^2 + a*(-(b*B) + a*C))*((3*b*(b*d*e
 + b*c*f - 2*a*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(
a + b*x)) + ((8*a^2*d^2*f^2 - 8*a*b*d*f*(d*e + c*f) + b^2*(3*d^2*e^2 + 2*c*
d*e*f + 3*c^2*f^2))*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c)
 + a*d]*Sqrt[e + f*x])])/((-(b*c) + a*d)^(3/2)*(-(b*e) + a*f)^(3/2))))/((b*
c - a*d)*(b*e - a*f)))/(4*b^2)
```

IntegrateAlgebraic [B] time = 1.86, size = 911, normalized size = 2.15

[illegible]

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/((a + b*x)^3*Sqrt[c + d*x]*Sqrt[e + f*
x]),x]
```

```
[Out] -1/4*((-(d*e) + c*f)*Sqrt[e + f*x]*(-4*b^3*B*c*e^2 + 8*a*b^2*c*C*e^2 + 3*A*
b^3*d*e^2 + a*b^2*B*d*e^2 - 5*a^2*b*C*d*e^2 + 5*A*b^3*c*e*f + 3*a*b^2*B*c*e
*f - 11*a^2*b*c*C*e*f - 11*a*A*b^2*d*e*f + 3*a^2*b*B*d*e*f + 5*a^3*C*d*e*f
- 5*a*A*b^2*c*f^2 + a^2*b*B*c*f^2 + 3*a^3*c*C*f^2 + 8*a^2*A*b*d*f^2 - 4*a^3
*B*d*f^2 + (4*b^3*B*c^2*e*(e + f*x))/(c + d*x) - (8*a*b^2*c^2*C*e*(e + f*x)
)/(c + d*x) - (5*A*b^3*c*d*e*(e + f*x))/(c + d*x) - (3*a*b^2*B*c*d*e*(e + f
*x))/(c + d*x) + (11*a^2*b*c*C*d*e*(e + f*x))/(c + d*x) + (5*a*A*b^2*d^2*e*
(e + f*x))/(c + d*x) - (a^2*b*B*d^2*e*(e + f*x))/(c + d*x) - (3*a^3*C*d^2*e
*(e + f*x))/(c + d*x) - (3*A*b^3*c^2*f*(e + f*x))/(c + d*x) - (a*b^2*B*c^2*
f*(e + f*x))/(c + d*x) + (5*a^2*b*c^2*C*f*(e + f*x))/(c + d*x) + (11*a*A*b^
2*c*d*f*(e + f*x))/(c + d*x) - (3*a^2*b*B*c*d*f*(e + f*x))/(c + d*x) - (5*a
^3*c*C*d*f*(e + f*x))/(c + d*x) - (8*a^2*A*b*d^2*f*(e + f*x))/(c + d*x) + (
4*a^3*B*d^2*f*(e + f*x))/(c + d*x)))/((b*c - a*d)^2*(b*e - a*f)^2*Sqrt[c +
d*x]*(-(b*e) + a*f + (b*c*(e + f*x))/(c + d*x) - (a*d*(e + f*x))/(c + d*x))
^2) + ((-8*b^2*c^2*C*e^2 + 4*b^2*B*c*d*e^2 + 8*a*b*c*C*d*e^2 - 3*A*b^2*d^2*
e^2 - a*b*B*d^2*e^2 - 3*a^2*C*d^2*e^2 + 4*b^2*B*c^2*e*f + 8*a*b*c^2*C*e*f -
 2*A*b^2*c*d*e*f - 14*a*b*B*c*d*e*f - 2*a^2*c*C*d*e*f + 8*a*A*b*d^2*e*f + 4
*a^2*B*d^2*e*f - 3*A*b^2*c^2*f^2 - a*b*B*c^2*f^2 - 3*a^2*c^2*C*f^2 + 8*a*A*
b*c*d*f^2 + 4*a^2*B*c*d*f^2 - 8*a^2*A*d^2*f^2)*ArcTan[(Sqrt[b*c - a*d]*Sqrt
[-(b*e) + a*f]*Sqrt[e + f*x])/((b*e - a*f)*Sqrt[c + d*x])]])/(4*(b*c - a*d)^
(5/2)*(b*e - a*f)^2*Sqrt[-(b*e) + a*f])
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

maple [**B**] time = 0.13, size = 7119, normalized size = 16.79

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^3/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for mo
re details)Is (a*d-b*c)    *(a*f-b*e)    positive, negative or zero?
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^3*(c + d*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**3/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

3.60 $\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx$

Optimal. Leaf size=826

$$-\frac{\sqrt{c+dx}\sqrt{e+fx}\left(Ab^2-a(bB-aC)\right)}{3b(bc-ad)(be-af)(a+bx)^3}+\frac{\left(-2df\left(C\left(3d^2e^2+2cdfe+3c^2f^2\right)+4df(2Adf-B(de+cf))\right)a^3+\right.}{}$$

Rubi [A] time = 2.43, antiderivative size = 826, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {1613, 151, 12, 93, 208}

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/((a + b*x)^4*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -((A*b^2 - a*(b*B - a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/(3*b*(b*c - a*d)*(b*
e - a*f)*(a + b*x)^3) + ((2*a^3*C*d*f + a*b^2*(12*c*C*e + B*d*e + B*c*f - 1
0*A*d*f) - b^3*(6*B*c*e - 5*A*(d*e + c*f)) + a^2*b*(4*B*d*f - 7*C*(d*e + c*
f)))*Sqrt[c + d*x]*Sqrt[e + f*x])/(12*b*(b*c - a*d)^2*(b*e - a*f)^2*(a + b*
x)^2) + ((4*a^4*C*d^2*f^2 + 8*a^3*b*d*f*(B*d*f - 2*C*(d*e + c*f)) - b^4*(15
*A*d^2*e^2 - 2*c*d*e*(9*B*e - 7*A*f) + 3*c^2*(8*C*e^2 - 6*B*e*f + 5*A*f^2))
 - a*b^3*(d^2*e*(3*B*e - 44*A*f) - 3*c^2*f*(4*C*e - B*f) - 2*c*d*(6*C*e^2 -
 29*B*e*f + 22*A*f^2)) - a^2*b^2*(C*(3*d^2*e^2 - 34*c*d*e*f + 3*c^2*f^2) +
2*d*f*(22*A*d*f - 5*B*(d*e + c*f))))*Sqrt[c + d*x]*Sqrt[e + f*x])/(24*b*(b*
c - a*d)^3*(b*e - a*f)^3*(a + b*x)) + ((b^3*(5*A*d^3*e^3 - 3*c*d^2*e^2*(2*B
*e - A*f) + c^2*d*e*(8*C*e^2 - 4*B*e*f + 3*A*f^2) + c^3*f*(8*C*e^2 - 6*B*e*
f + 5*A*f^2)) + a*b^2*(d^3*e^2*(B*e - 18*A*f) - c^3*f^2*(4*C*e - B*f) - c*d
^2*e*(4*C*e^2 - 23*B*e*f + 12*A*f^2) - c^2*d*f*(40*C*e^2 - 23*B*e*f + 18*A*
f^2)) - 2*a^3*d*f*(C*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2) + 4*d*f*(2*A*d*f -
 B*(d*e + c*f))) + a^2*b*(C*(d^3*e^3 + 23*c*d^2*e^2*f + 23*c^2*d*e*f^2 + c^
3*f^3) + 4*d*f*(6*A*d*f*(d*e + c*f) - B*(d^2*e^2 + 10*c*d*e*f + c^2*f^2))))
*ArcTanh[(Sqrt[b*e - a*f]*Sqrt[c + d*x])/(Sqrt[b*c - a*d]*Sqrt[e + f*x])])/
(8*(b*c - a*d)^(7/2)*(b*e - a*f)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1613

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && ILtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{(a+bx)^4\sqrt{c+dx}\sqrt{e+fx}}\,dx = -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} - \frac{\int \frac{-\frac{a^2C(de+cf)-ab(6cCe+Bde+Bcf-6Adf)+b^2(6Bce-}{2b}}{\ldots}}{3(\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{\left(2a^3Cdf+ab^2(12cCe+Bde+Bcf\ldots\right.}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{\left(2a^3Cdf+ab^2(12cCe+Bde+Bcf\ldots\right.}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{\left(2a^3Cdf+ab^2(12cCe+Bde+Bcf\ldots\right.}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{\left(2a^3Cdf+ab^2(12cCe+Bde+Bcf\ldots\right.}{\ldots}$$

$$= -\frac{\left(Ab^2-a(bB-aC)\right)\sqrt{c+dx}\sqrt{e+fx}}{3b(bc-ad)(be-af)(a+bx)^3} + \frac{\left(2a^3Cdf+ab^2(12cCe+Bde+Bcf\ldots\right.}{\ldots}$$

Mathematica [A] time = 6.11, size = 794, normalized size = 0.96

$$-\frac{\frac{\left(a(aC-bB)+Ab^2\right)\left(\frac{b\sqrt{c+dx}\sqrt{e+fx}\left(44a^2d^2f^2-44abdf(cf+de)+b^2\left(15c^2f^2+14cdef+15d^2e^2\right)\right)}{(a+bx)(bc-ad)(be-af)}+\frac{3\left(8a^2d^2f^2-8abdf(cf+de)+b^2\left(5c^2f^2-2cdef+5d^2e^2\right)\right)(-2adf+bcf+bde)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{(ad-bc)^{3/2}(af-be)^{3/2}}-\frac{10b\sqrt{c+dx}\sqrt{e+fx}(-2adf+bcf+bde)}{(a+bx)^2}\right)}{2(bc-ad)^2(be-af)^2}-\frac{3(bB-2aC)\left(\frac{\left(8a^2d^2f^2-8abdf(cf+de)+b^2\left(3c^2f^2+2cdef+3d^2e^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{(ad-bc)^{3/2}(af-be)^{3/2}}+\frac{3b\sqrt{c+dx}\sqrt{e+fx}(-2adf+bcf+bde)}{(a+bx)(bc-ad)(be-af)}\right)}{(bc-ad)(be-af)}+\frac{4b\sqrt{c+dx}\sqrt{e+fx}\left(a(aC-bB)+Ab^2\right)}{(a+bx)^3(bc-ad)(be-af)}+\frac{6b\sqrt{c+dx}\sqrt{e+fx}(bB-2aC)}{(a+bx)^2(bc-ad)(be-af)}+\frac{12bC\sqrt{c+dx}\sqrt{e+fx}}{(a+bx)(bc-ad)(be-af)}+\frac{12C(-2adf+bcf+bde)\tanh^{-1}\left(\frac{\sqrt{c+dx}\sqrt{af-be}}{\sqrt{e+fx}\sqrt{ad-bc}}\right)}{(ad-bc)^{3/2}(af-be)^{3/2}}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/((a + b*x)^4*Sqrt[c + d*x]*Sqrt[e + f*x]),x]
```

```
[Out] -1/12*((4*b*(A*b^2 + a*(-(b*B) + a*C))*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c -
 a*d)*(b*e - a*f)*(a + b*x)^3) + (6*b*(b*B - 2*a*C)*Sqrt[c + d*x]*Sqrt[e +
f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)^2) + (12*b*C*Sqrt[c + d*x]*Sqrt[e
+ f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)) + (12*C*(b*d*e + b*c*f - 2*a*d*
f)*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqrt[-(b*c) + a*d]*Sqrt[e +
```

```
f*x])])/((-(b*c) + a*d)^(3/2)*(-(b*e) + a*f)^(3/2)) - (3*(b*B - 2*a*C)*((3*
b*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)*(b*e
- a*f)*(a + b*x)) + ((8*a^2*d^2*f^2 - 8*a*b*d*f*(d*e + c*f) + b^2*(3*d^2*e^
2 + 2*c*d*e*f + 3*c^2*f^2))*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(Sqr
t[-(b*c) + a*d]*Sqrt[e + f*x])]])/((-(b*c) + a*d)^(3/2)*(-(b*e) + a*f)^(3/2)
)))/((b*c - a*d)*(b*e - a*f)) + ((A*b^2 + a*(-(b*B) + a*C))*((-10*b*(b*d*e
+ b*c*f - 2*a*d*f)*Sqrt[c + d*x]*Sqrt[e + f*x])/(a + b*x)^2 + (b*(44*a^2*d^
2*f^2 - 44*a*b*d*f*(d*e + c*f) + b^2*(15*d^2*e^2 + 14*c*d*e*f + 15*c^2*f^2)
)*Sqrt[c + d*x]*Sqrt[e + f*x])/((b*c - a*d)*(b*e - a*f)*(a + b*x)) + (3*(b*
d*e + b*c*f - 2*a*d*f)*(8*a^2*d^2*f^2 - 8*a*b*d*f*(d*e + c*f) + b^2*(5*d^2*
e^2 - 2*c*d*e*f + 5*c^2*f^2))*ArcTanh[(Sqrt[-(b*e) + a*f]*Sqrt[c + d*x])/(S
qrt[-(b*c) + a*d]*Sqrt[e + f*x])]])/((-(b*c) + a*d)^(3/2)*(-(b*e) + a*f)^(3/
2))))/(2*(b*c - a*d)^2*(b*e - a*f)^2))/b^2
```

IntegrateAlgebraic [B] time = 5.66, size = 3507, normalized size = 4.25

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(A + B*x + C*x^2)/((a + b*x)^4*Sqrt[c + d*x]*Sqrt[e + f*
x]),x]
```

```
[Out] -1/24*((-(d*e) + c*f)*Sqrt[e + f*x]*(24*b^5*c^2*C*e^4 - 18*b^5*B*c*d*e^4 -
12*a*b^4*c*C*d*e^4 + 15*A*b^5*d^2*e^4 + 3*a*b^4*B*d^2*e^4 + 3*a^2*b^3*C*d^2
*e^4 - 30*b^5*B*c^2*e^3*f - 36*a*b^4*c^2*C*e^3*f + 24*A*b^5*c*d*e^3*f + 108
*a*b^4*B*c*d*e^3*f - 48*a^2*b^3*c*C*d*e^3*f - 84*a*A*b^4*d^2*e^3*f - 18*a^2
*b^3*B*d^2*e^3*f + 24*a^3*b^2*C*d^2*e^3*f + 33*A*b^5*c^2*e^2*f^2 + 57*a*b^4
*B*c^2*e^2*f^2 - 3*a^2*b^3*c^2*C*e^2*f^2 - 138*a*A*b^4*c*d*e^2*f^2 - 150*a^
2*b^3*B*c*d*e^2*f^2 + 150*a^3*b^2*c*C*d*e^2*f^2 + 195*a^2*A*b^3*d^2*e^2*f^2
 + 3*a^3*b^2*B*d^2*e^2*f^2 - 57*a^4*b*C*d^2*e^2*f^2 - 66*a*A*b^4*c^2*e*f^3
- 24*a^2*b^3*B*c^2*e*f^3 + 18*a^3*b^2*c^2*C*e*f^3 + 204*a^2*A*b^3*c*d*e*f^3
 + 48*a^3*b^2*B*c*d*e*f^3 - 108*a^4*b*c*C*d*e*f^3 - 198*a^3*A*b^2*d^2*e*f^3
 + 36*a^4*b*B*d^2*e*f^3 + 30*a^5*C*d^2*e*f^3 + 33*a^2*A*b^3*c^2*f^4 - 3*a^3
*b^2*B*c^2*f^4 - 3*a^4*b*c^2*C*f^4 - 90*a^3*A*b^2*c*d*f^4 + 12*a^4*b*B*c*d*
f^4 + 18*a^5*c*C*d*f^4 + 72*a^4*A*b*d^2*f^4 - 24*a^5*B*d^2*f^4 - (48*b^5*c^
3*C*e^3*(e + f*x))/(c + d*x) + (48*b^5*B*c^2*d*e^3*(e + f*x))/(c + d*x) + (
48*a*b^4*c^2*C*d*e^3*(e + f*x))/(c + d*x) - (40*A*b^5*c*d^2*e^3*(e + f*x))/
(c + d*x) - (56*a*b^4*B*c*d^2*e^3*(e + f*x))/(c + d*x) + (8*a^2*b^3*c*C*d^2
*e^3*(e + f*x))/(c + d*x) + (40*a*A*b^4*d^3*e^3*(e + f*x))/(c + d*x) + (8*a
^2*b^3*B*d^3*e^3*(e + f*x))/(c + d*x) - (8*a^3*b^2*C*d^3*e^3*(e + f*x))/(c
+ d*x) + (48*b^5*B*c^3*e^2*f*(e + f*x))/(c + d*x) + (48*a*b^4*c^3*C*e^2*f*(
e + f*x))/(c + d*x) - (64*A*b^5*c^2*d*e^2*f*(e + f*x))/(c + d*x) - (224*a*b
^4*B*c^2*d*e^2*f*(e + f*x))/(c + d*x) + (80*a^2*b^3*c^2*C*d*e^2*f*(e + f*x)
)/(c + d*x) + (248*a*A*b^4*c*d^2*e^2*f*(e + f*x))/(c + d*x) + (184*a^2*b^3*
B*c*d^2*e^2*f*(e + f*x))/(c + d*x) - (184*a^3*b^2*c*C*d^2*e^2*f*(e + f*x))/
(c + d*x) - (184*a^2*A*b^3*d^3*e^2*f*(e + f*x))/(c + d*x) - (8*a^3*b^2*B*d^
3*e^2*f*(e + f*x))/(c + d*x) + (56*a^4*b*C*d^3*e^2*f*(e + f*x))/(c + d*x) -
 (40*A*b^5*c^3*e*f^2*(e + f*x))/(c + d*x) - (56*a*b^4*B*c^3*e*f^2*(e + f*x)
)/(c + d*x) + (8*a^2*b^3*c^3*C*e*f^2*(e + f*x))/(c + d*x) + (248*a*A*b^4*c^
2*d*e*f^2*(e + f*x))/(c + d*x) + (184*a^2*b^3*B*c^2*d*e*f^2*(e + f*x))/(c +
 d*x) - (184*a^3*b^2*c^2*C*d*e*f^2*(e + f*x))/(c + d*x) - (496*a^2*A*b^3*c*
d^2*e*f^2*(e + f*x))/(c + d*x) - (80*a^3*b^2*B*c*d^2*e*f^2*(e + f*x))/(c +
d*x) + (224*a^4*b*c*C*d^2*e*f^2*(e + f*x))/(c + d*x) + (288*a^3*A*b^2*d^3*e
*f^2*(e + f*x))/(c + d*x) - (48*a^4*b*B*d^3*e*f^2*(e + f*x))/(c + d*x) - (4
8*a^5*C*d^3*e*f^2*(e + f*x))/(c + d*x) + (40*a*A*b^4*c^3*f^3*(e + f*x))/(c
+ d*x) + (8*a^2*b^3*B*c^3*f^3*(e + f*x))/(c + d*x) - (8*a^3*b^2*c^3*C*f^3*(
e + f*x))/(c + d*x) - (184*a^2*A*b^3*c^2*d*f^3*(e + f*x))/(c + d*x) - (8*a^
3*b^2*B*c^2*d*f^3*(e + f*x))/(c + d*x) + (56*a^4*b*c^2*C*d*f^3*(e + f*x))/(
c + d*x) + (288*a^3*A*b^2*c*d^2*f^3*(e + f*x))/(c + d*x) - (48*a^4*b*B*c*d^
2*f^3*(e + f*x))/(c + d*x) - (48*a^5*c*C*d^2*f^3*(e + f*x))/(c + d*x) - (14
```

```
4*a^4*A*b*d^3*f^3*(e + f*x))/(c + d*x) + (48*a^5*B*d^3*f^3*(e + f*x))/(c +
d*x) + (24*b^5*c^4*C*e^2*(e + f*x)^2)/(c + d*x)^2 - (30*b^5*B*c^3*d*e^2*(e
+ f*x)^2)/(c + d*x)^2 - (36*a*b^4*c^3*C*d*e^2*(e + f*x)^2)/(c + d*x)^2 + (3
3*A*b^5*c^2*d^2*e^2*(e + f*x)^2)/(c + d*x)^2 + (57*a*b^4*B*c^2*d^2*e^2*(e +
 f*x)^2)/(c + d*x)^2 - (3*a^2*b^3*c^2*C*d^2*e^2*(e + f*x)^2)/(c + d*x)^2 -
(66*a*A*b^4*c*d^3*e^2*(e + f*x)^2)/(c + d*x)^2 - (24*a^2*b^3*B*c*d^3*e^2*(e
 + f*x)^2)/(c + d*x)^2 + (18*a^3*b^2*c*C*d^3*e^2*(e + f*x)^2)/(c + d*x)^2 +
 (33*a^2*A*b^3*d^4*e^2*(e + f*x)^2)/(c + d*x)^2 - (3*a^3*b^2*B*d^4*e^2*(e +
 f*x)^2)/(c + d*x)^2 - (3*a^4*b*C*d^4*e^2*(e + f*x)^2)/(c + d*x)^2 - (18*b^
5*B*c^4*e*f*(e + f*x)^2)/(c + d*x)^2 - (12*a*b^4*c^4*C*e*f*(e + f*x)^2)/(c
+ d*x)^2 + (24*A*b^5*c^3*d*e*f*(e + f*x)^2)/(c + d*x)^2 + (108*a*b^4*B*c^3*
d*e*f*(e + f*x)^2)/(c + d*x)^2 - (48*a^2*b^3*c^3*C*d*e*f*(e + f*x)^2)/(c +
d*x)^2 - (138*a*A*b^4*c^2*d^2*e*f*(e + f*x)^2)/(c + d*x)^2 - (150*a^2*b^3*B
*c^2*d^2*e*f*(e + f*x)^2)/(c + d*x)^2 + (150*a^3*b^2*c^2*C*d^2*e*f*(e + f*x
)^2)/(c + d*x)^2 + (204*a^2*A*b^3*c*d^3*e*f*(e + f*x)^2)/(c + d*x)^2 + (48*
a^3*b^2*B*c*d^3*e*f*(e + f*x)^2)/(c + d*x)^2 - (108*a^4*b*c*C*d^3*e*f*(e +
f*x)^2)/(c + d*x)^2 - (90*a^3*A*b^2*d^4*e*f*(e + f*x)^2)/(c + d*x)^2 + (12*
a^4*b*B*d^4*e*f*(e + f*x)^2)/(c + d*x)^2 + (18*a^5*C*d^4*e*f*(e + f*x)^2)/(
c + d*x)^2 + (15*A*b^5*c^4*f^2*(e + f*x)^2)/(c + d*x)^2 + (3*a*b^4*B*c^4*f^
2*(e + f*x)^2)/(c + d*x)^2 + (3*a^2*b^3*c^4*C*f^2*(e + f*x)^2)/(c + d*x)^2
- (84*a*A*b^4*c^3*d*f^2*(e + f*x)^2)/(c + d*x)^2 - (18*a^2*b^3*B*c^3*d*f^2*
(e + f*x)^2)/(c + d*x)^2 + (24*a^3*b^2*c^3*C*d*f^2*(e + f*x)^2)/(c + d*x)^2
 + (195*a^2*A*b^3*c^2*d^2*f^2*(e + f*x)^2)/(c + d*x)^2 + (3*a^3*b^2*B*c^2*d
^2*f^2*(e + f*x)^2)/(c + d*x)^2 - (57*a^4*b*c^2*C*d^2*f^2*(e + f*x)^2)/(c +
 d*x)^2 - (198*a^3*A*b^2*c*d^3*f^2*(e + f*x)^2)/(c + d*x)^2 + (36*a^4*b*B*c
*d^3*f^2*(e + f*x)^2)/(c + d*x)^2 + (30*a^5*c*C*d^3*f^2*(e + f*x)^2)/(c + d
*x)^2 + (72*a^4*A*b*d^4*f^2*(e + f*x)^2)/(c + d*x)^2 - (24*a^5*B*d^4*f^2*(e
 + f*x)^2)/(c + d*x)^2))/((b*c - a*d)^3*(b*e - a*f)^3*Sqrt[c + d*x]*(-(b*e)
 + a*f + (b*c*(e + f*x))/(c + d*x) - (a*d*(e + f*x))/(c + d*x))^3) + ((8*b^
3*c^2*C*d*e^3 - 6*b^3*B*c*d^2*e^3 - 4*a*b^2*c*C*d^2*e^3 + 5*A*b^3*d^3*e^3 +
 a*b^2*B*d^3*e^3 + a^2*b*C*d^3*e^3 + 8*b^3*c^3*C*e^2*f - 4*b^3*B*c^2*d*e^2*
f - 40*a*b^2*c^2*C*d*e^2*f + 3*A*b^3*c*d^2*e^2*f + 23*a*b^2*B*c*d^2*e^2*f +
 23*a^2*b*c*C*d^2*e^2*f - 18*a*A*b^2*d^3*e^2*f - 4*a^2*b*B*d^3*e^2*f - 6*a^
3*C*d^3*e^2*f - 6*b^3*B*c^3*e*f^2 - 4*a*b^2*c^3*C*e*f^2 + 3*A*b^3*c^2*d*e*f
^2 + 23*a*b^2*B*c^2*d*e*f^2 + 23*a^2*b*c^2*C*d*e*f^2 - 12*a*A*b^2*c*d^2*e*f
^2 - 40*a^2*b*B*c*d^2*e*f^2 - 4*a^3*c*C*d^2*e*f^2 + 24*a^2*A*b*d^3*e*f^2 +
8*a^3*B*d^3*e*f^2 + 5*A*b^3*c^3*f^3 + a*b^2*B*c^3*f^3 + a^2*b*c^3*C*f^3 - 1
8*a*A*b^2*c^2*d*f^3 - 4*a^2*b*B*c^2*d*f^3 - 6*a^3*c^2*C*d*f^3 + 24*a^2*A*b*
c*d^2*f^3 + 8*a^3*B*c*d^2*f^3 - 16*a^3*A*d^3*f^3)*ArcTan[(Sqrt[b*c - a*d]*S
qrt[-(b*e) + a*f]*Sqrt[e + f*x])/((b*e - a*f)*Sqrt[c + d*x])])/(8*(b*c - a*
d)^(7/2)*(b*e - a*f)^3*Sqrt[-(b*e) + a*f])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"giac")
```

[Out] Timed out

maple [B] time = 0.31, size = 18802, normalized size = 22.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x+a)^4/(d*x+c)^(1/2)/(f*x+e)^(1/2),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume((a*d-b*c)>0)', see `assume?` for mo
re details)Is (a*d-b*c)    *(a*f-b*e)    positive, negative or zero?
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/((e + f*x)^(1/2)*(a + b*x)^4*(c + d*x)^(1/2)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x+a)**4/(d*x+c)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Timed out
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
```

```
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
```

```
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```